Further Results on Selection Combining of Binary NCFSK Signals in Rayleigh Fading Channels

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***Abstract*—In this paper, we provide an analytical performance comparison of various selection combining (SC) schemes for binary noncoherent frequency-shift keying (NCFSK) signals operating on independent and nonidentically distributed (i.n.d.) Rayleigh fading channels. With this motivation, we first derive the receiver structure for the optimum SC scheme, which combines one out of the available L diversity branches so as to minimize the probability of bit error. We show that the optimum SC scheme chooses the diversity branch having the largest magnitude of the logarithm of the *a posteriori* probability ratio (LAPPR) of the transmitted information bit. We also show that: 1) the optimum noncoherent diversity receiver, for binary NCFSK signals, is equivalent to combining the LAPPRs of all the diversity branches; 2) the SC scheme proposed by Neasmith and Beaulieu is a special case of the optimum SC scheme for independent and identically distributed (i.i.d.) Rayleigh fading; and 3) for i.n.d. fading with dual diversity (i.e., $L = 2$), the performance of the optimum SC scheme is the same as that of the optimum noncoherent diversity receiver, whereas the Neasmith and Beaulieu SC scheme gives the performance of the suboptimum equal gain combining receiver. Bit-error rate (BER) results show that, at 10^{-4} BER, for i.i.d. Rayleigh fading, the proposed optimum SC scheme performs better than the existing SC schemes by 0.5–0.9 dB for $L = 3$ and by 0.8–1.5 dB for $L = 5$, and performs within 0.3 dB of the optimum noncoherent diversity receiver for $L = 3, 5$, and for i.n.d. Rayleigh fading with $L = 5$, the optimum SC scheme gives an additional gain of about 2.0 dB over the Pierce SC scheme. Further, for i.i.d. Rayleigh fading, we derive the bit-error probability expression for a $(3, L)$ selection scheme which combines three branches whose LAPPR magnitudes are the largest among the available L branches. Numerical results for this $(3, L)$ selection scheme show that for $L = 5$ at a BER of 10^{-4}, combining the three branches with the largest LAPPR magnitudes yields almost the full performance of the optimum noncoherent diversity receiver, whereas, for $L = 7$, it is just about 0.2 dB away from the latter.**

***Index Terms*—Fading channels, noncoherent frequency-shift keying (NCFSK), optimum combining, selection diversity.**

I. INTRODUCTION

DIVERSITY reception is a well-known technique for mitigating the effects of fading in wireless communication systems [1], [2]. Diversity reception can improve the link quality and reduce the link budget. Combining of diversity signals can be done either coherently or noncoherently. Typical diversity-combining schemes include maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC), and generalized selection combining (GSC). SC is the simplest scheme of all, as it processes only one of the diversity branches. In this paper, we are concerned with the *optimum SC* of binary noncoherent frequency-shift keying (NCFSK) signals.

For binary NCFSK signaling on independent and identically distributed (i.i.d.) Rayleigh fading channels, Pierce [3] derived the error probability of a predetection SC scheme in which the diversity branch that has the largest instantaneous signal-to-noise ratio (SNR) is chosen for the subsequent signal detection. In [4], Chyi *et al.* analyzed a postdetection SC scheme, known as maximum-output selection combining (MOSC), for M-ary NCFSK signals. In this scheme, the diversity branch with the largest energy detector output is chosen. In [5], Neasmith and Beaulieu propose a postdetection SC scheme in which the branch with the largest magnitude of the energy difference is chosen. However, for independent and nonidentically distributed (i.n.d.) Rayleigh fading, neither the predetection SC scheme of [3] nor the postdetection SC schemes of [4] and [5] are optimal.

Recently, Kim and Kim [6] derived the optimum SC scheme for binary phase-shift keying (BPSK) signals in Rayleigh fading. In this scheme, the branch with the largest magnitude of the logarithm of the *a posteriori* probability ratio (LAPPR) of the transmitted bit is chosen for signal detection. Since the LAPPR for BPSK signals is proportional to the product of the fade amplitude and the phase-compensated matched-filter (MF) output, combining the LAPPRs of all the diversity branches gives the performance of the MRC scheme.

In this paper, we provide an analytical performance comparison of various SC schemes for binary NCFSK signals on i.n.d. as well as i.i.d. Rayleigh fading channels. With this objective, we first derive the receiver structure for the optimum SC scheme, which selects one out of the available L diversity branches so as to minimize the probability of bit error. We show that the optimum SC scheme chooses the diversity branch having the largest magnitude of the LAPPR of the transmitted information bit and proceed to derive its bit-error probability (BEP) performance. For the case of i.i.d. Rayleigh fading, we obtain a closed-form expression for the probability of bit error, whereas a single integral expression is obtained for i.n.d. fading, which can be evaluated using a simple Gauss–Chebychev quadrature (GCQ) rule. We compare the performance of

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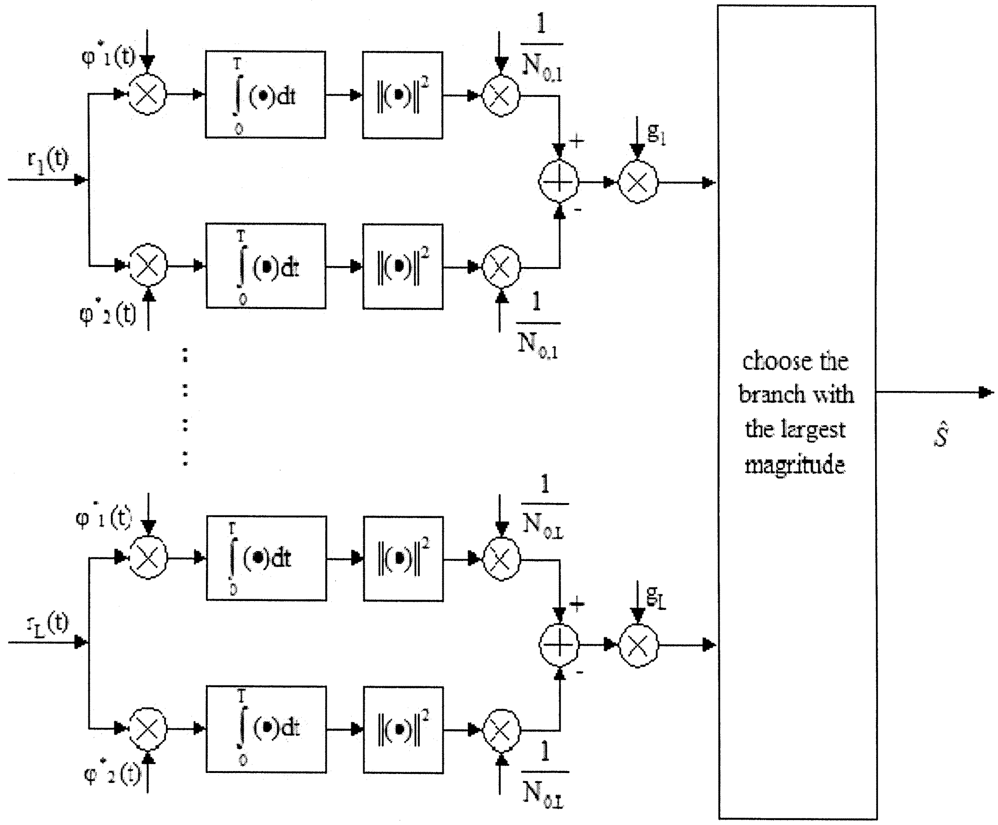

Fig. 1. Receiver structure for the optimum SC for binary NCFSK signals on i.n.d. Rayleigh fading channels. $g_l = \overline{\gamma}_l/1 + \overline{\gamma}_l$, and $N_{0,l}$ are, respectively, the weighting factor and the noise power spectral density on the lth diversity branch.

the optimum SC scheme with that of the schemes proposed by Pierce [3], Chyi *et al.* [4] and Neasmith and Beaulieu [5]. To enable this comparison, we extend the i.i.d. fading analyses of [3]–[5] to the case of i.n.d. Rayleigh fading. We show that: 1) the optimum noncoherent diversity receiver, for binary FSK (BFSK) signals, is equivalent to combining the LAPPRs of all the diversity branches; 2) the Neasmith and Beaulieu SC scheme is a special case of the optimum SC scheme for i.i.d. fading; and 3) for i.n.d. fading with $L = 2$, the optimum SC scheme coincides with the optimum noncoherent diversity receiver, whereas the Neasmith and Beaulieu SC scheme provides the performance of the suboptimum EGC receiver. Our bit-error rate (BER) results show that, at a BER of 10^{-4}, for i.i.d. fading, the optimum SC scheme performs better than the existing SC schemes by 0.5–0.9 dB for $L = 3$ and by 0.8–1.5 dB for $L = 5$, and performs within 0.3 dB of the optimum noncoherent diversity receiver for $L = 3, 5$. For i.n.d. fading with $L = 5$, the optimum SC scheme gives an improvement of about 2.0 dB over the Pierce SC scheme. Finally, we propose a GSC scheme which combines three branches whose LAPPR magnitudes are the largest among the available L branches. We denote this scheme as GSC $(3, L)$. We also obtain a closed-form expression for the error probability of this scheme for i.i.d. Rayleigh fading channels. Numerical results for this scheme show that for $L = 5$, combining the three branches with the largest LAPPR magnitudes yields almost the full performance of the optimum noncoherent diversity receiver, whereas, for $L = 7$, it is just about 0.2 dB away from the performance of the latter.

The rest of the paper is organized as follows. In Section II, we introduce the system model and derive the LAPPR of bit detection. In Section III, the BEPs of the optimum SC scheme, for i.n.d. as well as i.i.d. Rayleigh fading, are derived. In Section IV, we provide i.n.d. fading extensions to the i.i.d. fading analyses of the SC schemes of [3]–[5]. The proposed GSC $(3, L)$ scheme is analyzed in Section V. Section VI gives the comparative performance results of the optimum SC scheme versus the existing SC schemes. Finally, we conclude this paper in Section VII.

II. System Model

We assume that the information bits, $\mathcal{S} \in \{0, 1\}$, are BFSK modulated with $s_1(t) = \sqrt{E_b}\phi_1(t)$ and $s_2(t) = \sqrt{E_b}\phi_2(t)$, where $\phi_1(t) = (1/\sqrt{T})\exp(j2\pi f_1 t)$, $\phi_2(t) = 1/\sqrt{T}\exp(j2\pi f_2 t)$, $j = \sqrt{-1}$, $0 \leq t < T$, denoting the lowpass equivalent representation of the signals transmitted for "1" and "0," respectively. Here, T is the signal duration, E_b is the energy per bit, and f_1 and f_2 are two possible frequency tones chosen such that in the interval $[0, T)$, the two waveforms $\phi_1(t)$ and $\phi_2(t)$ are orthogonal. The transmitted signal is passed through a fading channel and noise is added to it at the receiver front end. We assume that the fading process is slow, frequency nonselective, and remains constant over one signaling interval. Assuming the transmission of a binary "1" and perfect timing at the receiver, the lowpass equivalent representation of the received symbols at the outputs of the correlator, as shown in Fig. 1, are given by

$$r_{c,1}^{(l)} = \alpha^{(l)} \cos\theta^{(l)} \sqrt{E_b} + n_{c,1}^{(l)}, \quad l = 1, \dots, L \tag{1}$$

$$r_{c,2}^{(l)} = \alpha^{(l)} \sin\theta^{(l)} \sqrt{E_b} + n_{c,2}^{(l)}, \quad l = 1, \dots, L \tag{2}$$

$$r_{s,1}^{(l)} = n_{s,1}^{(l)}, \quad l = 1, \dots, L \tag{3}$$

$$r_{s,2}^{(l)} = n_{s,2}^{(l)}, \quad l = 1, \dots, L \tag{4}$$

where $r_c^{(l)} = r_{c,1}^{(l)} + jr_{c,2}^{(l)}$ and $r_s^{(l)} = r_{s,1}^{(l)} + jr_{s,2}^{(l)}$ are the complex-valued outputs due to correlating with $\phi_1(t)$ and $\phi_2(t)$, respectively. In (1) and (2), $\alpha^{(l)}$ and $\theta^{(l)}$ are, respectively, the fade amplitude and the random phase on the lth antenna path. $\theta^{(l)}$ is assumed to be uniformly distributed over $[0, 2\pi]$ and $\alpha^{(l)}$ is modeled as Rayleigh distributed with the probability density function (pdf) given by

$$f_{\alpha^{(l)}}(a) = \frac{2a}{\Omega_l} e^{-a^2/\Omega_l}, \quad a \geq 0 \tag{5}$$

where $\Omega_l = E([\alpha^{(l)}]^2)$ is the average fade power on the lth diversity path, and $n_{c,1}^{(l)}, n_{c,2}^{(l)}, n_{s,1}^{(l)}, n_{s,2}^{(l)}$ are zero-mean, independent Gaussian random variables on the lth antenna path with a variance of $\sigma^2 = N_{0,l}/2$. We define $\gamma_l = [\alpha^{(l)}]^2 E_b/N_{0,l}$ as the instantaneous SNR on the lth branch, whereas $\overline{\gamma}_l = E[\gamma_l] = \Omega_l E_b/N_{0,l}$ is the corresponding average SNR.

We choose the diversity branch whose magnitude of the LAPPR (i.e., $|\text{LAPPR}|$) of the transmitted information bit is the largest. The LAPPR of the transmitted information bit $\mathcal{S}$ on the lth antenna path is given by

$$\text{LAPPR}^{(l)} = \log\left(\frac{\text{Prob}\left(\mathcal{S} = 1 \mid r_c^{(l)}, r_s^{(l)}\right)}{\text{Prob}\left(\mathcal{S} = 0 \mid r_c^{(l)}, r_s^{(l)}\right)}\right). \tag{6}$$

For equally probable message signals, using [2]

$$\begin{aligned} \text{LAPPR}^{(l)} &= \log\left(\frac{f_{r_c^{(l)}, r_s^{(l)}}\left(r_c^{(l)}, r_s^{(l)} \mid \mathcal{S} = 1\right)}{f_{r_c^{(l)}, r_s^{(l)}}\left(r_c^{(l)}, r_s^{(l)} \mid \mathcal{S} = 0\right)}\right) \\ &= \frac{\overline{\gamma}_l}{1 + \overline{\gamma}_l}\left(\frac{\left|r_c^{(l)}\right|^2}{N_{0,l}} - \frac{\left|r_s^{(l)}\right|^2}{N_{0,l}}\right) \\ &= g_l\left(\left[X_1^{(l)}\right]^2 - \left[X_0^{(l)}\right]^2\right) \end{aligned} \tag{7}$$

where

$$\left|r_c^{(l)}\right| = \sqrt{\left[r_{c,1}^{(l)}\right]^2 + \left[r_{c,2}^{(l)}\right]^2}$$

and

$$\left|r_s^{(l)}\right| = \sqrt{\left[r_{s,1}^{(l)}\right]^2 + \left[r_{s,2}^{(l)}\right]^2}$$

are the envelopes at the output of the noncoherent detector,

$$\left[X_1^{(l)}\right]^2 = \left|r_c^{(l)}\right|^2 / N_{0,l}$$

and

$$\left[X_0^{(l)}\right]^2 = \left|r_s^{(l)}\right|^2 / N_{0,l}$$

are the normalized energies, and $g_l = \overline{\gamma}_l/1 + \overline{\gamma}_l$ is the weighting factor on the lth branch, which is a function of only the average SNR on that branch.

In this paper, we propose to choose the diversity branch whose magnitude of the LAPPR in (7) is the largest. In Appendix A, we show that this combining scheme minimizes the error probability of reception, and hence is optimum. It is interesting to note that by combining all the LAPPRs, we obtain the metric

$$\Lambda = \sum_{l=1}^{L}\left(\frac{\overline{\gamma}_l}{1 + \overline{\gamma}_l}\right)\left(\left[X_1^{(l)}\right]^2 - \left[X_0^{(l)}\right]^2\right) \tag{8}$$

and decide that the transmitted bit is "1" if $\Lambda \geq 0$, and is "0" otherwise. This is equivalent to the metric that is used by the optimum noncoherent diversity receiver, with BFSK signaling, on i.n.d. Rayleigh fading channels [2, Eq. (7.31)]. Thus, we conclude that $\sum_{l=1}^{L} \text{LAPPR}^{(l)}$ is the decision statistic for the optimum noncoherent diversity receiver. Similar to the optimum noncoherent diversity receiver, implementation of the optimum SC scheme requires the knowledge of the average channel gains on the individual diversity branches along with the average noise powers $N_{0,l}$ (refer to Fig. 1). For the rest of this paper, we assume that $N_{0,1} = \cdots = N_{0,L} = N_0$.

III. Error-Probability Analysis

In this section, we analyze the BEP performance of the optimum SC scheme for both i.n.d. as well as i.i.d. fading cases. First, we assume that the transmitted bit is a "1." Then, using (1)–(4), the pdfs of $X_1^{(l)}$ and $X_0^{(l)}$ are, respectively, given by

$$\begin{aligned} f_{X_1^{(l)}}(x) &= \frac{2x}{1 + \overline{\gamma}_l} e^{-x^2/1+\overline{\gamma}_l}, \quad x \geq 0 \\ f_{X_0^{(l)}}(x) &= 2xe^{-x^2}, \quad x \geq 0. \end{aligned} \tag{9}$$

The pdf of $\text{LAPPR}^{(l)}$ of (7) can be obtained as

$$f_{\text{LAPPR}^{(l)}}(x) = \begin{cases} \frac{1+\overline{\gamma}_l}{\overline{\gamma}_l(2+\overline{\gamma}_l)} e^{-x/\overline{\gamma}_l}, & x \geq 0 \\ \frac{1+\overline{\gamma}_l}{\overline{\gamma}_l(2+\overline{\gamma}_l)} e^{x(1+\overline{\gamma}_l)/\overline{\gamma}_l}, & x < 0. \end{cases} \tag{10}$$

It is not difficult to show that the pdf and the cumulative distribution function (cdf) of $|\text{LAPPR}^{(l)}|$ are, respectively, given by

$$f_{|\text{LAPPR}^{(l)}|}(x) = \frac{1 + \overline{\gamma}_l}{\overline{\gamma}_l(2 + \overline{\gamma}_l)} \times \left(e^{-x/\overline{\gamma}_l} + e^{-x(1+\overline{\gamma}_l)/\overline{\gamma}_l}\right), \quad x \geq 0 \tag{11}$$

$$F_{|\text{LAPPR}^{(l)}|}(x) = 1 - \frac{1 + \overline{\gamma}_l}{2 + \overline{\gamma}_l} e^{-x/\overline{\gamma}_l} - \frac{1}{2 + \overline{\gamma}_l} e^{-x(1+\overline{\gamma}_l)/\overline{\gamma}_l}, \quad x \geq 0. \tag{12}$$

A. i.n.d. Fading

With the assumption of "1" being transmitted, a bit-detection error occurs if the $\mathrm{LAPPR}^{(l)}$ with the largest magnitude is negative. Accordingly, the error probability of the optimum SC scheme is given by the following expression:

$$\begin{aligned} P_e^{\mathrm{OptimumSC,i.n.d.}} &= \sum_{l=1}^{L} \mathrm{Prob}\Bigg(\max\left(|\mathrm{LAPPR}^{(j),j\neq l}|\right) < |\mathrm{LAPPR}^{(l)}|, \mathrm{LAPPR}^{(l)} < 0 \Bigg) \\ &= \sum_{l=1}^{L} \mathrm{Prob}\Bigg(\max\left(|\mathrm{LAPPR}^{(j),j\neq l}|\right) + \mathrm{LAPPR}^{(l)} < 0 \Bigg) \\ &= \sum_{l=1}^{L} P_e(l) \end{aligned} \tag{13}$$

where

$$\begin{aligned} P_e(l) &= \mathrm{Prob}\Bigg(\max\left(|\mathrm{LAPPR}^{(j),j\neq l}|\right) + \mathrm{LAPPR}^{(l)} < 0 \Bigg) \\ &= \mathrm{Prob}\left(Z_l + \mathrm{LAPPR}^{(l)} < 0 \right) \\ &= \int_{z=0}^{\infty} \int_{x=-\infty}^{-z} f_{Z_l}(z) f_{\mathrm{LAPPR}^{(l)}}(x) dz\, dx \\ &= \frac{1}{2+\overline{\gamma}_l} \int_{z=0}^{\infty} f_{Z_l}(z) e^{-(z(1+\overline{\gamma}_l)/\overline{\gamma}_l)} dz \\ &= \frac{1}{2+\overline{\gamma}_l} \sum_{j=1, j\neq l}^{L} \Phi(l,j). \end{aligned} \tag{14}$$

In the above, $Z_l = \max\left(|\mathrm{LAPPR}^{(j),j\neq l}|\right)$, and $\Phi(l,j)$ is as shown in (15) at the bottom of the page. The derivation of (15) is given in Appendix B. Unfortunately, (15) as given cannot be expressed in a closed form. Nevertheless, the integral $\Phi(l,j)$ can be evaluated efficiently using the standard GCQ rule. Interestingly, for the special case of dual diversity (i.e., $L = 2$), we obtain the following expression for $P_e^{\mathrm{OptimumSC,i.n.d.}}$:

$$P_e^{\mathrm{OptimumSC,i.n.d.}}|_{L=2} = \frac{2\overline{\gamma}_1 + 2\overline{\gamma}_2 + 3\overline{\gamma}_1\overline{\gamma}_2}{(2+\overline{\gamma}_1)(2+\overline{\gamma}_2)(\overline{\gamma}_1 + \overline{\gamma}_2 + \overline{\gamma}_1\overline{\gamma}_2)} \tag{16}$$

which is exactly the performance of the optimum noncoherent diversity receiver [8, Eq. (19)].

B. i.i.d. Fading

With the assumption of i.i.d. fading on each antenna path, we have $\Omega_l = \Omega \; \forall \; l = 1, \ldots, L$, and accordingly, we define $\overline{\gamma} = \overline{\gamma}_1 = \cdots = \overline{\gamma}_L$. Notice that the weighting factors $g_l = \overline{\gamma}_l / 1 + \overline{\gamma}_l$ are now independent of the branch index l. With this, the probability of bit-detection error, $P_e^{\mathrm{OptimumSC,i.i.d.}}$, is given by

$$\begin{aligned} P_e^{\mathrm{OptimumSC,i.i.d.}} &= \sum_{l=1}^{L} \mathrm{Prob}\Bigg(\max\left(|\mathrm{LAPPR}^{(j),j\neq l}|\right) < |\mathrm{LAPPR}^{(l)}|, \mathrm{LAPPR}^{(l)} < 0 \Bigg) \\ &= L \cdot \mathrm{Prob}\Bigg(\max\left(|\mathrm{LAPPR}^{(j),j\neq l}|\right) + \mathrm{LAPPR}^{(l)} < 0 \Bigg) \\ &= L \sum_{k=0}^{L-1} \sum_{j=0}^{k} \binom{L-1}{k} \binom{k}{j} \times \frac{(-1)^k}{(2+\overline{\gamma})^{k+1}} \frac{(1+\overline{\gamma})^{j+1}}{1+k+\overline{\gamma}(k-j+1)}. \end{aligned} \tag{17}$$

The derivation of (17) is given in Appendix C. It is to be noted that, unlike (14) and (15), the expression in (17) is in closed form.

IV. Extensions to Existing Analyses

We would like to compare the error-probability performance of the derived optimum SC scheme with other existing SC schemes, as well as with the L-branch optimum noncoherent combining scheme. We consider the three SC schemes proposed in [3]–[5] for our comparison. In order to enable the comparison, in this section, we extend the analyses of the above SC schemes to the case of i.n.d. fading.

$$\begin{aligned} \Phi(l,j) = \frac{1+\overline{\gamma}_j}{\overline{\gamma}_j(2+\overline{\gamma}_j)} \int_{\psi=0}^{\pi/2} &\left(e^{-(\tan\psi/\overline{\gamma}_j)} + e^{-(\tan\psi(1+\overline{\gamma}_j)/\overline{\gamma}_j)} \right) e^{-(\tan\psi(1+\overline{\gamma}_l)/\overline{\gamma}_l)} \\ &\times \prod_{k=1, k\neq\{j,l\}}^{L} \left(1 - \frac{(1+\overline{\gamma}_k)e^{-(\tan\psi/\overline{\gamma}_k)} + e^{-(\tan\psi(1+\overline{\gamma}_k)/\overline{\gamma}_k)}}{2+\overline{\gamma}_k} \right) \sec^2\psi d\psi \end{aligned} \tag{15}$$

The Pierce SC scheme is of the predetection type, and chooses the branch whose instantaneous SNR is the largest. The BEP with this scheme on i.i.d. Rayleigh fading channel is derived in [3] as

$$P_e^{\text{Pierce,i.i.d.}} = \frac{1}{2}\prod_{l=1}^{L}\frac{l}{l+\frac{\overline{\gamma}}{2}}. \quad (18)$$

We are able to obtain a closed-form expression for the BEP of the Pierce SC scheme for i.n.d. Rayleigh fading as

$$P_e^{\text{Pierce,i.n.d.}} = \sum_{l=1}^{L}\Bigg\{\frac{1}{\overline{\gamma}_l+2} - \sum_{i=1,i\neq l}^{L}\frac{1}{\overline{\gamma}_l+2+\frac{2\overline{\gamma}_l}{\overline{\gamma}_i}} + \sum_{i=1,k=1,i\neq k\neq l}^{L}\frac{1}{\overline{\gamma}_l+2+\frac{2\overline{\gamma}_l}{\overline{\gamma}_i}+\frac{2\overline{\gamma}_l}{\overline{\gamma}_k}} + \cdots + (-1)^{L-1} \times \frac{1}{\overline{\gamma}_l+2+\sum\limits_{i=1,i\neq l}^{L}\frac{2\overline{\gamma}_l}{\overline{\gamma}_i}}\Bigg\}. \quad (19)$$

The derivation of (19) is given in Appendix D.

In the SC scheme by Chyi *et al.* [4], the largest output of the square-law combiner of the available L antenna branches is chosen. This is obviously a postdetection SC scheme, and the error probability for this scheme on i.i.d. Rayleigh fading is derived in [4] as

$$P_e^{\text{Chyi,i.i.d.}} = \sum_{l=0}^{L}(-1)^l\binom{L}{l}\prod_{j=1}^{L}\frac{j}{j+\frac{l}{1+\overline{\gamma}}}. \quad (20)$$

We derive a closed-form expression for the error probability of the Chyi *et al.* SC scheme for i.n.d. Rayleigh fading as

$$P_e^{\text{Chyi,i.n.d.}} = \sum_{k=0}^{L-1}(-1)^k L\binom{L-1}{k} \times \Bigg\{\frac{1}{k+1} - \sum_{i=1}^{L}\frac{1}{k+1+\frac{1}{1+\overline{\gamma}_i}} + \sum_{i=1,j=1,i\neq j}^{L}\frac{1}{k+1+\frac{1}{1+\overline{\gamma}_i}+\frac{1}{1+\overline{\gamma}_j}} + \cdots + (-1)^L\frac{1}{k+1+\sum\limits_{l=1}^{L}\frac{1}{1+\overline{\gamma}_l}}\Bigg\}. \quad (21)$$

The derivation of (21) is given in Appendix E.

In the postdetection SC scheme proposed by Neasmith and Beaulieu [5], the branch with the largest magnitude of the energy difference, $\left[X_1^{(l)}\right]^2 - \left[X_0^{(l)}\right]^2$, is chosen (in [5], this is denoted by "$S+N$ Selection Receiver Model 3"). Clearly, for the case of i.i.d. Rayleigh fading, the error probability of this scheme is the same as that of the optimum SC scheme presented here, and the average probability of error is exactly given by (17), in agreement with [5, Eq. (9)]. Since this scheme does not require the knowledge of the weighting factors g_l, we can use this as a suboptimum implementation of the optimum SC scheme for i.n.d. Rayleigh fading. For this latter case, the error-probability expression for this scheme can be derived in a way similar to Section III-A as

$$P_e^{\text{Neasmith,i.n.d.}} = \sum_{l=1}^{L}\frac{1}{2+\overline{\gamma}_l}\sum_{j=1,j\neq l}^{L}\frac{1}{2+\overline{\gamma}_j} \times \int_{\psi=0}^{\pi/2}\left(e^{-\tan\psi/(1+\overline{\gamma}_j)}+e^{-\tan\psi}\right)e^{-\tan\psi} \times \prod_{k=1,k\neq\{j,l\}}^{L}\left(1-\frac{(1+\overline{\gamma}_k)e^{-\tan\psi/(1+\overline{\gamma}_k)}+e^{-\tan\psi}}{2+\overline{\gamma}_k}\right) \times \sec^2\psi d\psi. \quad (22)$$

Similar to (15), the above expression can be efficiently evaluated using the GCQ formula. For the case of dual diversity, (22) can be simplified to

$$P_e^{\text{Neasmith,i.n.d.}}\big|_{L=2} = \frac{8+5\overline{\gamma}_1+5\overline{\gamma}_2+3\overline{\gamma}_1\overline{\gamma}_2}{(2+\overline{\gamma}_1)^2(2+\overline{\gamma}_2)^2} = \frac{1}{\overline{\gamma}_1-\overline{\gamma}_2}\left[\frac{(1+\overline{\gamma}_1)(3+2\overline{\gamma}_1)}{(2+\overline{\gamma}_1)^2} - \frac{(1+\overline{\gamma}_2)(3+2\overline{\gamma}_2)}{(2+\overline{\gamma}_2)^2}\right] \quad (23)$$

which is exactly the performance of the noncoherent EGC receiver [8, Eq. (12)]. Using [8, Eq. (16)], for fixed $\overline{\gamma}_1$ but as $\overline{\gamma}_2 \to 0$, we can then write the performance difference between the optimum SC and the suboptimum SC (i.e., the Neasmith and Beaulieu receiver) as

$$\frac{P_e^{\text{Neasmith,i.n.d.}}\big|_{L=2} - P_e^{\text{OptimumSC,i.n.d.}}\big|_{L=2}}{P_e^{\text{OptimumSC,i.n.d.}}\big|_{L=2}} = \frac{\overline{\gamma}_1}{4(2+\overline{\gamma}_1)} \quad (24)$$

which is monotonically increasing with the average SNR $\overline{\gamma}_1$ of the first branch. Finally, we use [8, Eqs. (8) and (9)] to compare the performance of the optimum SC scheme against the performance of the optimum noncoherent diversity receiver and the noncoherent EGC receiver, respectively. For simplicity, in the rest of this paper, the optimum noncoherent diversity receiver is referred to as the *optimum receiver*, and the suboptimum noncoherent EGC receiver is given the term *equal gain receiver*.

V. Generalized Selection Combining

In this section, we propose and analyze a GSC scheme which combines K branches whose LAPPR magnitudes are the largest among the available L branches. We denote this combining scheme by $\text{GSC}(K,L)$, and the decision statistic for this scheme is given by

$$Z^{\text{GSC}} = \sum_{k=1}^{K}\text{LAPPR}_{(k)}, |\text{LAPPR}_{(1)}| \geq |\text{LAPPR}_{(2)}| \geq \cdots \geq |\text{LAPPR}_{(K)}| \quad (25)$$

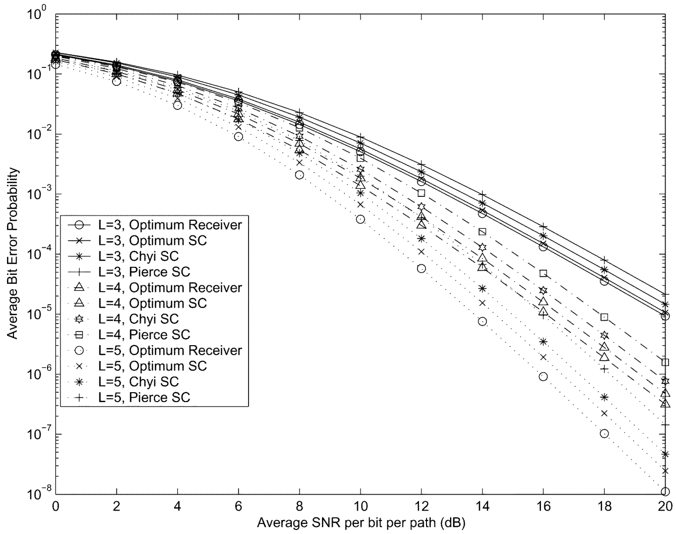

Fig. 2. Comparison of the error performance of various combining schemes on i.i.d. Rayleigh fading for $L = 3$, 4, and 5. The curve corresponding to the legend "optimum receiver" is also the curve for the noncoherent EGC receiver. Similarly, the curve corresponding to the legend "optimum SC" is also the curve for the Neasmith and Beaulieu SC scheme.

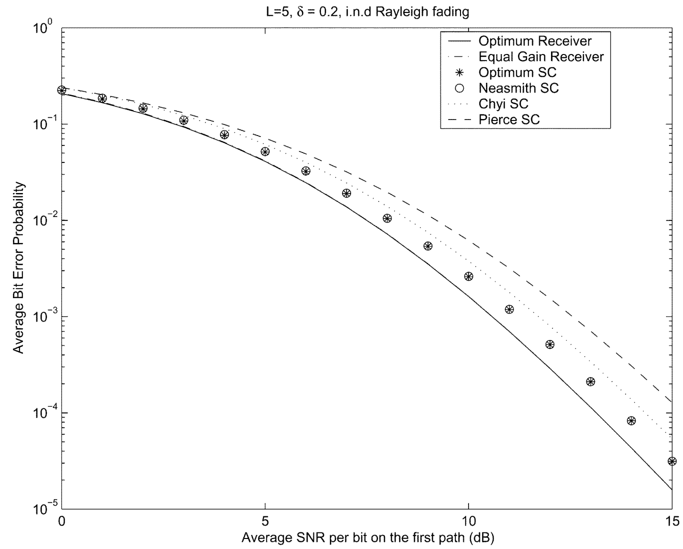

Fig. 3. Comparison of the error performance of various combining schemes on i.n.d. Rayleigh fading channels with $L = 5$ and the MIP decay factor $\delta = 0.2$.

where $\mathrm{LAPPR}_{(1)}, \ldots, \mathrm{LAPPR}_{(L)}$ are the order statistics [9] of the random variables $\mathrm{LAPPR}^{(1)}, \ldots, \mathrm{LAPPR}^{(L)}$. In this paper, we restrict the analysis to the case of i.i.d. Rayleigh fading, as the analysis appears to be prohibitively difficult for i.n.d. Rayleigh fading. As shown in [10], the decision statistic Z^{GSC} does not change its sign for $K = 2$, because $|\mathrm{LAPPR}_{(1)}| \geq |\mathrm{LAPPR}_{(2)}|$. In other words, the sign of the decision statistic Z^{GSC} is same as that of the optimum SC receiver. This implies that the BEP of $\mathrm{GSC}(2, L)$ is same as the BEP of the optimum SC scheme. One difficulty in analyzing the performance of the above $\mathrm{GSC}(K, L)$ receiver for arbitrary K, for a given L, is that the number of decision regions over which the error probability is to be evaluated grows exponentially as a function of K [10]. In what follows, we derive the error-probability expression for arbitrary L, but K is restricted to three, i.e., a $\mathrm{GSC}(3, L)$ scheme.

With the assumption of binary "1" being transmitted, as shown in [10], a decision error occurs under the following conditions:

1) $\mathcal{R}_1 : \mathrm{LAPPR}_{(1)} < 0, \mathrm{LAPPR}_{(2)} < 0$;
2) $\mathcal{R}_2 : \mathrm{LAPPR}_{(1)} < 0, \mathrm{LAPPR}_{(2)} > 0, \mathrm{LAPPR}_{(3)} < 0$;
3) $\mathcal{R}_3 : \mathrm{LAPPR}_{(1)} < 0, \mathrm{LAPPR}_{(2)} > 0, \mathrm{LAPPR}_{(3)} > 0, \mathrm{LAPPR}_{(1)} + \mathrm{LAPPR}_{(2)} + \mathrm{LAPPR}_{(3)} < 0$;
4) $\mathcal{R}_4 : \mathrm{LAPPR}_{(1)} > 0, \mathrm{LAPPR}_{(2)} < 0, \mathrm{LAPPR}_{(3)} < 0, \mathrm{LAPPR}_{(1)} + \mathrm{LAPPR}_{(2)} + \mathrm{LAPPR}_{(3)} < 0$.

It is to be noted that the above error events are mutually exclusive, and therefore, the probability of error is given by

$$P_e^{\mathrm{GSC}(3,L),\mathrm{i.i.d.}} = \mathrm{Prob}(\mathcal{R}_1) + \mathrm{Prob}(\mathcal{R}_2) + \mathrm{Prob}(\mathcal{R}_3) + \mathrm{Prob}(\mathcal{R}_4). \quad (26)$$

In Appendix F, we derive closed-form expressions for the probability of the error events $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$, and $\mathcal{R}_4$.

VI. Results and Discussion

Fig. 2 shows the comparative performance of the optimum SC scheme, and the SC schemes of Pierce and Chyi *et al.*, with i.i.d. Rayleigh fading for $L = 3$, 4, and 5. The performance of the optimum receiver, which combines all the L LAPPRs, is also plotted for comparison. Note that, for i.i.d. Rayleigh fading, the Neasmith and Beaulieu SC scheme coincides with the optimum SC scheme. Similarly, the equal gain receiver has the same performance as that of the optimum receiver. With $L = 3$, at a BER of 10^{-4}, the optimum SC scheme performs 0.3 dB more poorly compared with the optimum receiver, but performs better than the SC schemes of Pierce and Chyi *et al.* by 0.9 and 0.5 dB, respectively. As the order of diversity increases from $L = 3$ to 5, the diversity gain of the optimum SC scheme increases over the Pierce and Chyi *et al.* schemes. For example, for $L = 5$, the diversity gain of the optimum SC scheme is 1.5 dB over the Pierce SC scheme and 0.8 dB over the Chyi *et al.* SC scheme. Recall that the Chyi *et al.* scheme chooses the branch with maximum output energy [see Appendix E, first step of (42)]. This scheme gives a decision error when the output energy of the noise-only branch exceeds the energy of the signal-plus-noise branch. The optimum SC scheme, on the other hand, performs a subtraction of the output energies [signal-plus-noise energy output due to correct hypothesis, and noise-only energy output due to incorrect hypothesis, (7)]. This subtraction basically eliminates the dominance of the noise-only energy output, and this results in an enhanced performance of the optimum SC scheme over the other SC schemes.

Fig. 3 shows the comparative performances of various SC schemes for the case of i.n.d. Rayleigh fading. For comparison purposes, we have also plotted the performances of the optimum receiver and the equal gain receiver. An exponentially decaying multipath intensity profile (MIP) is assumed with the average SNR on the lth branch decaying as $\overline{\gamma}_l = \overline{\gamma}_1 e^{-\delta(l-1)}$, $l = 1, \ldots, L$, where δ is the power decay factor. In Fig. 3, we fix $L = 5$ and $\delta = 0.2$ and notice that, at a BER of 10^{-4}, an improvement of about 2 dB in performance over the Pierce SC

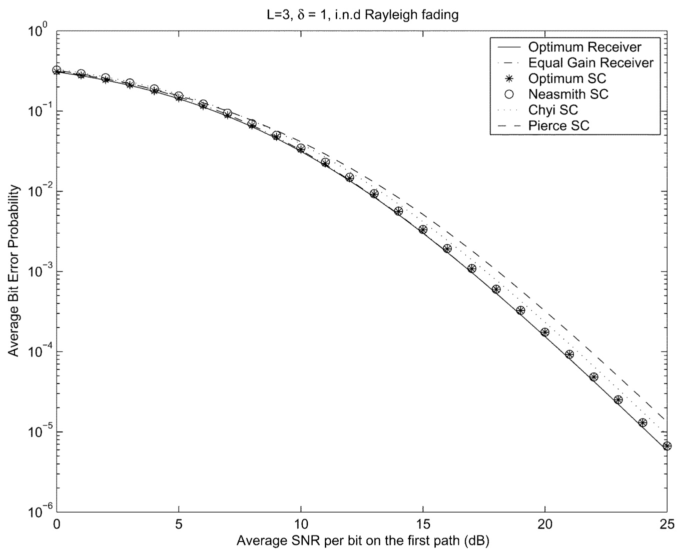

Fig. 4. Comparison of the error performance of various diversity-combining schemes on i.n.d. Rayleigh fading channel with $L = 3$ diversity branches. $\overline{\gamma}_l = \overline{\gamma}_1 e^{-\delta(l-1)}$, $l = 1, 2, 3$, and $\delta = 1$.

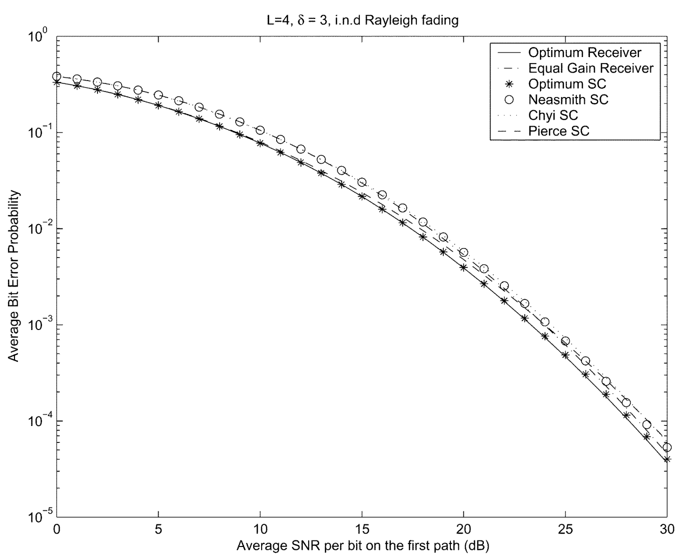

Fig. 5. Comparison of the error performance of various diversity-combining schemes on i.n.d. Rayleigh fading with $L = 4$ antennas. Exponentially decaying MIP is assumed with decay factor $\delta = 3.0$.

scheme can be achieved using the optimum SC scheme. From Fig. 3, we also observe that, with sufficiently large diversity and with small decay constant, the Neasmith and Beaulieu SC scheme performs almost the same as the optimum SC scheme, similar to the case of i.i.d. fading, and both are within 0.5 dB of the optimum receiver.

However, as we see next, at low SNRs and for large δ, the optimum SC scheme performs better than the Neasmith and Beaulieu SC scheme, and the latter one performs more poorly than the Pierce SC scheme. This behavior is more clearly observed in Figs. 4 and 5, which are obtained for $\{L = 3, \delta = 1\}$ and $\{L = 4, \delta = 3\}$, respectively. The reason for this is as follows. At high SNRs, with the approximation $1 + \overline{\gamma}_l \approx \overline{\gamma}_l$ in (7), the weighting factors g_l become independent of the average branch SNR, $\overline{\gamma}_l$. At low SNRs, with the approximation

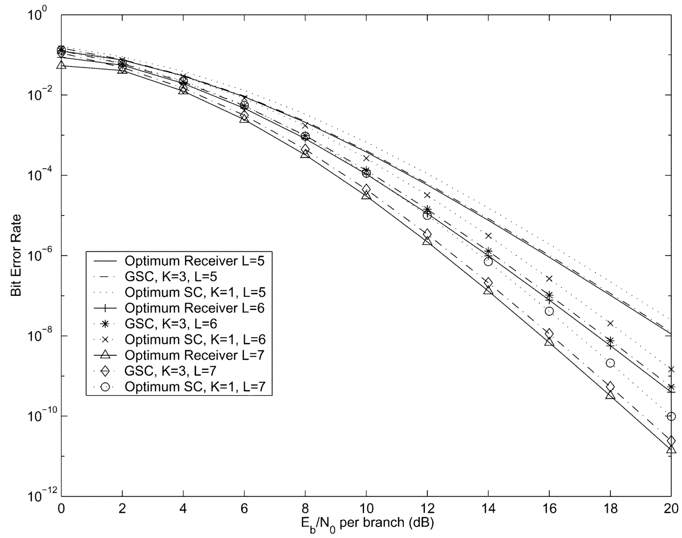

Fig. 6. Comparison of the error performance of the GSC $(3, L)$ scheme versus the optimum SC scheme on i.i.d. Rayleigh fading for $L = 5$, 6, and 7.

$1 + \overline{\gamma}_l \approx 1$, we have $g_l \approx \overline{\gamma}_l$. The Neasmith and Beaulieu SC scheme does not take this gain factor into account, whereas the optimum SC scheme appropriately weights the energy differences before the selection is made, which improves the performance. This illustrates an important conclusion of this paper. That is, while the Neasmith and Beaulieu SC scheme is optimum for i.i.d. Rayleigh fading, it performs more poorly than the Pierce SC scheme for i.n.d. Rayleigh fading at low SNRs and large MIP decay factors, and the SC scheme derived in this paper provides the optimum performance for arbitrary i.n.d. Rayleigh fading. While comparing the performances of various SC receivers, the following aspects of receiver complexity should be kept in mind. First, the predetection SC scheme of Pierce requires estimation of the instantaneous SNR on each diversity branch, and the postdetection optimum SC requires the corresponding average SNRs, whereas the schemes of Chyi *et al.* and Neasmith and Beaulieu can be easily implemented using the outputs of the MF alone. If the fading is close to i.i.d., then it is attractive to go for the Neasmith and Beaulieu SC scheme. However, for severe fading conditions with a nonuniform MIP, and for a large number of diversity channels, the optimum SC receiver provides a very good tradeoff between the performance and the complexity.

Fig. 6 shows the performance of the GSC $(3, L)$ scheme with i.i.d. Rayleigh fading for $L = 5$, 6, and 7. The performance of the optimum receiver is also plotted for comparison. It is observed that, for $L = 5$, at a BER of 10^{-4}, the GSC $(3, L)$ scheme yields almost the same performance as the optimum receiver, whereas for $L = 7$, it is just about 0.2 dB away from the optimum receiver's performance.

VII. Conclusions

In this paper, we provided an analytical performance comparison of various SC schemes for binary NCFSK signals on i.n.d. Rayleigh fading channels. First, we derived the receiver structure for the optimum SC scheme, and showed that the optimum

SC scheme chooses the diversity branch having the largest magnitude of the LAPPR of the transmitted information bit. We also showed that the optimum diversity receiver, for binary NCFSK signals, is equivalent to combining the LAPPRs of all the diversity branches; the SC scheme proposed by Neasmith and Beaulieu is a special case of the optimum SC scheme for i.i.d. Rayleigh fading; and for i.n.d. fading with dual diversity (i.e., $L = 2$), the performance of the optimum SC scheme is the same as that of the optimum noncoherent diversity receiver, whereas the Neasmith and Beaulieu SC scheme gives the performance of the suboptimum EGC receiver. BER results showed that, at 10^{-4} BER, for i.i.d. Rayleigh fading, the optimum SC scheme performs better than the existing SC schemes by 0.5–0.9 dB for $L = 3$, and by 0.8–1.5 dB for $L = 5$, and performs within 0.3 dB of the optimum noncoherent diversity receiver for $L = 3$ or 5. For i.n.d. Rayleigh fading with $L = 5$, the optimum SC scheme gives an additional gain of about 2.0 dB over the Pierce SC scheme. Further, for i.i.d. Rayleigh fading, we derived the BEP expression for a $(3, L)$ selection scheme which combines three branches whose LAPPR magnitudes are the largest among the available L branches. Numerical results for this $(3, L)$ selection scheme showed that for $L = 5$ at a BER of 10^{-4}, combining the three branches with the largest LAPPR magnitudes yields almost the full performance of the optimum noncoherent diversity receiver, whereas for $L = 7$, it is just about 0.2 dB away from the latter.

Appendix A

A. Optimality of the Proposed SC Scheme

In this appendix, we show that choosing the largest LAPPR magnitude among the available L LAPPR magnitudes minimizes the probability of error, thus proving the optimality of the proposed SC scheme. The general expression for the error probability is given by

$$\begin{aligned} P_b &= \int_{\mathbf{X}} \mathrm{Prob}(\widehat{\mathcal{S}} \neq \mathcal{S} \mid \mathbf{X}) f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X} \\ &= 1 - \int_{\mathbf{X}} \mathrm{Prob}(\widehat{\mathcal{S}} = \mathcal{S} \mid \mathbf{X}) f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X} \end{aligned} \tag{27}$$

where $\mathbf{X} = \left(X_0^{(1)}, X_1^{(1)}, X_0^{(2)}, X_1^{(2)}, \ldots, X_0^{(L)}, X_1^{(L)}\right)$, and $\widehat{\mathcal{S}}$ is the detected bit when $\mathcal{S}$ is the transmitted bit. The expression $\mathrm{Prob}(\widehat{\mathcal{S}} = \mathcal{S} \mid \mathbf{X})$ can be obtained as follows:

$$\begin{aligned} &\mathrm{Prob}(\widehat{\mathcal{S}} = \mathcal{S} \mid \mathbf{X}) \\ &\quad = \sum_{l=1}^{L} \mathrm{Prob}\left(\widehat{\mathcal{S}} = \mathcal{S} \mid \mathbf{X}, l^{\text{th}} \text{ antenna is chosen}\right) \\ &\qquad \cdot \mathrm{Prob}\left(l^{\text{th}} \text{ antenna is chosen} \mid \mathbf{X}\right) \\ &\quad = \sum_{l=1}^{L} \mathrm{Prob}\left(\widehat{\mathcal{S}} = \mathcal{S} \mid X_0^{(l)}, X_1^{(l)}\right) \\ &\qquad \cdot \mathrm{Prob}\left(l^{\text{th}} \text{ antenna is chosen} \mid \mathbf{X}\right) \\ &\quad \leq \max\left(\mathrm{Prob}\left(\widehat{\mathcal{S}} = \mathcal{S} \mid X_0^{(l)}, X_1^{(l)}\right)\right). \end{aligned} \tag{28}$$

The last step in the above equation is due to the fact that $\sum_{l=1}^{L} \mathrm{Prob}(l^{\text{th}} \text{ antenna is chosen} \mid \mathbf{X}) = 1$ and $0 \leq \mathrm{Prob}(l^{\text{th}} \text{ antenna is chosen} \mid \mathbf{X}) \leq 1$. It is to be noted that the equality in (28) is achieved, and hence, P_b is minimized, by selecting the branch providing the maximum $\mathrm{Prob}\left(\widehat{\mathcal{S}} = \mathcal{S} \mid X_0^{(l)}, X_1^{(l)}\right)$. With this, we proceed to derive $\mathrm{Prob}\left(\widehat{\mathcal{S}} = \mathcal{S} \mid X_0^{(l)}, X_1^{(l)}\right)$ and show that it is a monotonically increasing function of $|\mathrm{LAPPR}^{(l)}|$.

The expression $\mathrm{Prob}\left(\widehat{\mathcal{S}} = \mathcal{S} \mid X_0^{(l)}, X_1^{(l)}\right)$ can be calculated as

$$\begin{aligned} &\mathrm{Prob}\left(\widehat{\mathcal{S}} = \mathcal{S} \mid X_0^{(l)}, X_1^{(l)}\right) \\ &\qquad = \mathrm{Prob}\left(\widehat{\mathcal{S}} = 0, \mathcal{S} = 0 \mid X_0^{(l)}, X_1^{(l)}\right) \\ &\qquad\quad + \mathrm{Prob}\left(\widehat{\mathcal{S}} = 1, \mathcal{S} = 1 \mid X_0^{(l)}, X_1^{(l)}\right). \end{aligned} \tag{29}$$

From (7), we obtain

$$\begin{aligned} \mathrm{Prob}\left(\widehat{\mathcal{S}} = 1 \mid X_0^{(l)}, X_1^{(l)}\right) &= \frac{1}{1 + e^{-\mathrm{LAPPR}^{(l)}}} \\ \mathrm{Prob}\left(\widehat{\mathcal{S}} = 0 \mid X_0^{(l)}, X_1^{(l)}\right) &= \frac{1}{1 + e^{\mathrm{LAPPR}^{(l)}}}. \end{aligned} \tag{30}$$

Noting that $\mathrm{LAPPR}^{(l)} \geq 0$ when $\widehat{\mathcal{S}} = 1$ and $\mathcal{S} = 1$, and $\mathrm{LAPPR}^{(l)} < 0$ when $\widehat{\mathcal{S}} = 0$ and $\mathcal{S} = 0$, (30) can be simplified as

$$\mathrm{Prob}\left(\widehat{\mathcal{S}} = \mathcal{S} \mid X_0^{(l)}, X_1^{(l)}\right) = \frac{1}{1 + e^{-|\mathrm{LAPPR}^{(l)}|}}. \tag{31}$$

The above expression is clearly a monotonically increasing function of $|\mathrm{LAPPR}^{(l)}|$. Therefore, the BEP is minimized by choosing the branch with the largest magnitude of the LAPPR.

Appendix B

A. Derivation of $P_e^{\mathrm{OptimumSC,i.n.d.}}$

We first find the pdf of $Z_l = \max(|\mathrm{LAPPR}^{(j),j \neq l}|)$ and use this in step 2 of (14) to obtain (15)

$$\begin{aligned} F_{Z_l}(z) &= \mathrm{Prob}\left(\max\left(|\mathrm{LAPPR}^{(j),j\neq l}|\right) \leq z\right) \\ &= \prod_{j=1, j\neq l}^{L} F_{|\mathrm{LAPPR}^{(j)}|}(z). \end{aligned} \tag{32}$$

Taking logarithms on both sides, differentiating with respect to z, and then rearranging, we arrive at

$$f_{Z_l}(z) = \sum_{j=1, j\neq l}^{L} f_{|\mathrm{LAPPR}^{(j)}|}(z) \prod_{k=1, k\neq \{j,l\}}^{L} F_{|\mathrm{LAPPR}^{(k)}|}(z). \tag{33}$$

The expression $P_e(l)$ in (13) is derived as follows:

$$\begin{aligned}
&P_e(l)\\
&= \text{Prob}\left(Z_l + \text{LAPPR}^{(l)} < 0\right)\\
&= \int_{z=0}^{\infty}\int_{x=-\infty}^{-z} f_{Z_l}(z) f_{\text{LAPPR}^{(l)}}(x)\,dz\,dx\\
&= \frac{1}{2+\overline{\gamma}_l}\int_{z=0}^{\infty} f_{Z_l}(z) e^{-(z(1+\overline{\gamma}_l)/\overline{\gamma}_l)}dz\\
&= \frac{1}{2+\overline{\gamma}_l}\sum_{j=1,j\neq l}^{L}\frac{1+\overline{\gamma}_j}{\overline{\gamma}_j(2+\overline{\gamma}_j)}\\
&\quad\times\int_{z=0}^{\infty}\left(e^{-(z/\overline{\gamma}_j)} + e^{-(z(1+\overline{\gamma}_j)/\overline{\gamma}_j)}\right)e^{-(z(1+\overline{\gamma}_l)/\overline{\gamma}_l)}\\
&\quad\times\prod_{k=1,k\neq\{j,l\}}^{L}\left(1-\frac{(1+\overline{\gamma}_k)e^{-(z/\overline{\gamma}_k)} + e^{-(z(1+\overline{\gamma}_k)/\overline{\gamma}_k)}}{2+\overline{\gamma}_k}\right)\times dz\\
&= \frac{1}{2+\overline{\gamma}_l}\sum_{j=1,j\neq l}^{L}\Phi(l,j) \qquad (34)
\end{aligned}$$

where in step 3 of (34) we have made use of the expressions for $f_{Z_l}(z)$ of (33). Upon substituting $z = \tan\psi$, we arrive at (15).

Appendix C

A. Derivation of $P_e^{\text{OptimumSC,i.i.d.}}$

$$\begin{aligned}
&P_e^{OptimumSC,i.i.d.}\\
&= \sum_{l=1}^{L}\text{Prob}\left(\max\left(|\text{LAPPR}^{(j),j\neq l}|\right) < |\text{LAPPR}^{(l)}|, \text{LAPPR}^{(l)} < 0\right)\\
&= L\times\text{Prob}\left(Z_l + \text{LAPPR}^{(l)} < 0\right)\\
&= L\int_{z=0}^{\infty}\int_{y=-\infty}^{-z} f_{Z_l}(z) f_{\text{LAPPR}^{(l)}}(y)\,dz\,dy\\
&= \frac{L}{2+\overline{\gamma}}\int_{z=0}^{\infty} f_{Z_l}(z) e^{-(z(1+\overline{\gamma})/\overline{\gamma})}dz. \qquad (35)
\end{aligned}$$

Since $|\text{LAPPR}^{(j)}|$, $j = 1,\ldots,L$, $j \neq i$, are i.i.d., the density function of $Z_l = \max\left(|\text{LAPPR}^{(j),j\neq l}|\right)$ can easily be obtained as

$$\begin{aligned}
f_{Z_l}(z) &= \frac{d}{dz}\left[\text{Prob}(Z_l \le z)\right]\\
&= (L-1)\left(F_{|\text{LAPPR}|}(z)\right)^{L-2} f_{|\text{LAPPR}|}(z)\\
&= (L-1)\frac{1+\overline{\gamma}}{\overline{\gamma}(2+\overline{\gamma})}\\
&\quad\times\left(1-\frac{(1+\overline{\gamma})e^{-(z(1+\overline{\gamma})/\overline{\gamma})} + e^{-(z/\overline{\gamma})}}{2+\overline{\gamma}}\right)^{L-2}\\
&\quad\times\left(e^{-(z(1+\overline{\gamma})/\overline{\gamma})} + e^{-(z/\overline{\gamma})}\right)\\
&= (L-1)\sum_{k=0}^{L-2}\sum_{j=0}^{k}\frac{(-1)^k(1+\overline{\gamma})^{j+1}}{(2+\overline{\gamma})^{k+1}}\\
&\quad\times\binom{L-2}{k}\binom{k}{j}\\
&\quad\times\frac{1}{\overline{\gamma}}\left(e^{-(z(1+k+(1+j)\overline{\gamma})/\overline{\gamma})}\right.\\
&\qquad\left.+e^{-(z(1+k+j\overline{\gamma})/\overline{\gamma})}\right). \qquad (36)
\end{aligned}$$

Substituting (36) and performing the integration along with simplification, we arrive at (17).

Appendix D

A. Derivation of $P_e^{\text{Pierce,i.n.d.}}$

According to the system model in [3], we have

$$\begin{aligned}
P_e^{\text{Pierce,i.n.d.}} &= E_\gamma[P_e(\gamma)] = E_\gamma\left[\frac{1}{2}e^{-\gamma/2}\right]\\
&= \frac{1}{2}\int_{x=0}^{\infty} e^{-x/2} f_\gamma(x)dx \qquad (37)
\end{aligned}$$

where $\gamma = \max(\gamma_1,\gamma_2,\ldots,\gamma_L)$. By making use of the approach given in Appendix B, we can readily obtain the pdf of γ as follows:

$$\begin{aligned}
f_\gamma(x) &= \sum_{l=1}^{L} f_{\gamma_l}(x)\prod_{j=1,j\neq l}^{L}F_{\gamma_j}(x)\\
&= \sum_{l=1}^{L}\frac{1}{\overline{\gamma}_l}e^{-(x/\overline{\gamma}_l)}\prod_{j=1,j\neq l}^{L}\left(1-e^{-(x/\overline{\gamma}_j)}\right). \qquad (38)
\end{aligned}$$

Substituting (38) in (37), we obtain

$$\begin{aligned}
P_e^{\text{Pierce,i.n.d.}} &= \sum_{l=1}^{L}\frac{1}{2\overline{\gamma}_l}\int_{x=0}^{\infty}e^{-(1/2+1/\overline{\gamma}_l)x}\\
&\quad\times\prod_{j=1,j\neq l}^{L}\left(1-e^{-(x/\overline{\gamma}_j)}\right)dx. \qquad (39)
\end{aligned}$$

In order to obtain a closed-form solution for (39), we use the following simple identity [7]:

$$\prod_{j=1,j\neq l}^{L}\left(1-e^{-(x/\overline{\gamma}_j)}\right)=1-\sum_{i=1,i\neq l}^{L} e^{-(x/\overline{\gamma}_i)} + \sum_{i=1,k=1,i\neq k\neq l}^{L} e^{-((x/\overline{\gamma}_i)+(x/\overline{\gamma}_k))} + \cdots + (-1)^{L-1} e^{-\sum_{i=1,i\neq l}^{L} x/\overline{\gamma}_i}. \tag{40}$$

Using (40) in (39) and performing the simple integration, we obtain a closed-form expression for $P_e^{\text{Pierce,i.n.d.}}$ as

$$P_e^{\text{Pierce,i.n.d.}} = \sum_{l=1}^{L}\frac{1}{\overline{\gamma}_l+2} - \sum_{l=1}^{L}\sum_{i=1,i\neq l}^{L}\frac{1}{\overline{\gamma}_l+2+\frac{2\overline{\gamma}_l}{\overline{\gamma}_i}} + \sum_{l=1}^{L}\sum_{i=1,k=1,i\neq k\neq l}^{L}\frac{1}{\overline{\gamma}_l+2+\frac{2\overline{\gamma}_l}{\overline{\gamma}_i}+\frac{2\overline{\gamma}_l}{\overline{\gamma}_k}} + \cdots + (-1)^{L-1} \times \sum_{l=1}^{L}\frac{1}{\overline{\gamma}_l+2+\sum_{i=1,i\neq l}^{L}\frac{2\overline{\gamma}_l}{\overline{\gamma}_i}}. \tag{41}$$

Appendix E

A. Derivation of $P_e^{\text{Chyi,i.n.d.}}$

The error-probability expression with the system model in [4], assuming a binary "1" is transmitted, is given by

$$P_e^{\text{Chyi,i.n.d.}} = \text{Prob}\left(\max\left(\left[X_1^{(1)}\right]^2,\ldots,\left[X_1^{(L)}\right]^2\right) < \max\left(\left[X_0^{(1)}\right]^2,\ldots,\left[X_0^{(L)}\right]^2\right)\right) = \text{Prob}(Z_1<Z_2) = \int_{z=0}^{\infty} F_{Z_1}(z) f_{Z_2}(z)\,dz \tag{42}$$

where

$$Z_1 = \max\left(\left[X_1^{(1)}\right]^2,\ldots,\left[X_1^{(L)}\right]^2\right)$$

$$Z_2 = \max\left(\left[X_0^{(1)}\right]^2,\ldots,\left[X_0^{(L)}\right]^2\right).$$

Since all $\left[X_0^{(l)}\right]^2$'s are i.i.d. with pdf $f_{\left[X_0^{(l)}\right]^2}(x) = e^{-x}, x\geq 0$, we write

$$f_{Z_2}(z) = Le^{-z}(1-e^{-z})^{L-1}, \quad z\geq 0. \tag{43}$$

Since $\left[X_1^{(l)}\right]^2$ is exponentially distributed with mean $1+\overline{\gamma}_l$, the cdf of Z_2 can easily be shown to be

$$F_{Z_1}(z) = \prod_{l=1}^{L}\left(1-e^{-z/(1+\overline{\gamma}_l)}\right), \quad z\geq 0. \tag{44}$$

Substituting (43) and (44) in (42), we obtain

$$P_e^{\text{Chyi,i.n.d.}} = \int_{z=0}^{\infty} Le^{-z}\left(1-e^{-z}\right)^{L-1} \times \prod_{l=1}^{L}\left(1-e^{-z/(1+\overline{\gamma}_l)}\right)dz = \sum_{k=0}^{L-1}(-1)^k\binom{L-1}{k}L \times \int_{z=0}^{\infty} e^{-z(k+1)}\prod_{l=1}^{L}\left(1-e^{-z/(1+\overline{\gamma}_l)}\right)dz. \tag{45}$$

Using (40) to simplify the product term of (45), and after some elementary integration, we obtain the following closed-form expression for $P_e^{\text{Chyi,i.n.d.}}$:

$$P_e^{\text{Chyi,i.n.d.}} = \sum_{k=0}^{L-1}(-1)^k L\binom{L-1}{k} \times\left\{\frac{1}{k+1} - \sum_{i=1}^{L}\frac{1}{k+1+\frac{1}{1+\overline{\gamma}_i}} + \sum_{i=1,j=1,i\neq j}^{L}\frac{1}{k+1+\frac{1}{1+\overline{\gamma}_i}+\frac{1}{1+\overline{\gamma}_j}} + \cdots + (-1)^L\frac{1}{k+1+\sum_{l=1}^{L}\frac{1}{1+\overline{\gamma}_l}}\right\}. \tag{46}$$

Appendix F

A. Derivation of $P_e^{\text{GSC}(3,L),\text{i.i.d.}}$

We introduce the following notation [10]. Let $\{\text{LAPPR}_{[1]},\text{LAPPR}_{[2]},\ldots,\text{LAPPR}_{[L]}\}$ be the order statistics[1] of the original random variables

[1]It is to be noted that the set $\{\text{LAPPR}_{[1]},\text{LAPPR}_{[2]},\ldots,\text{LAPPR}_{[L]}\}$ is different from the set $\{\text{LAPPR}_{(1)},\text{LAPPR}_{(2)},\ldots,\geq\text{LAPPR}_{(L)}\}$ of Section V.

$\{\mathrm{LAPPR}^{(1)}, \mathrm{LAPPR}^{(2)}, \ldots, \mathrm{LAPPR}^{(L)}\}$ such that $\mathrm{LAPPR}_{[1]} \geq \mathrm{LAPPR}_{[2]}, \ldots, \mathrm{LAPPR}_{[L]}$. From [10], the error events $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$, $\mathcal{R}_4$ can be further simplified as follows:

$$\begin{aligned}\mathrm{Prob}(\mathcal{R}_1) &= \mathrm{Prob}(\mathrm{LAPPR}_{(1)} < 0, \mathrm{LAPPR}_{(2)} < 0) \\ &= \mathrm{Prob}(\mathrm{LAPPR}_{[1]} < 0) \\ &\quad + \mathrm{Prob}\Big(\mathrm{LAPPR}_{[1]} > 0, \mathrm{LAPPR}_{[L-1]} \\ &\qquad < -\mathrm{LAPPR}_{[1]}\Big) \\ &= \int_{x=-\infty}^{0} f_{\mathrm{LAPPR}_{[1]}}(x) dx \\ &\quad + \int_{x=0}^{\infty} \int_{y=-\infty}^{-x} f_{\mathrm{LAPPR}_{[1]}, \mathrm{LAPPR}_{[L-1]}} \\ &\quad \times (x,y) dx dy. \end{aligned} \tag{47}$$

From order statistics [9], we have

$$f_{\mathrm{LAPPR}_{[1]}}(x) = \frac{L!}{(L-1)!} f_{\mathrm{LAPPR}}(x) [F_{\mathrm{LAPPR}}(x)]^{L-1} \tag{48}$$

and

$$\begin{aligned} f_{\mathrm{LAPPR}_{[1]}, \mathrm{LAPPR}_{[L-1]}}(x,y) = \frac{L!}{(L-3)!} & F_{\mathrm{LAPPR}}(y) \\ & \times f_{\mathrm{LAPPR}}(y) f_{\mathrm{LAPPR}}(x) \\ & \times [F_{\mathrm{LAPPR}}(x) \\ & \quad - F_{\mathrm{LAPPR}}(y)]^{L-3}. \end{aligned} \tag{49}$$

Substituting (48) and (49) in (47) and performing the integration, we obtain

$$\begin{aligned} \mathrm{Prob}(\mathcal{R}_1) = \left(\frac{1}{2+\overline{\gamma}}\right)^L + \frac{L!}{(L-3)!} & \\ \cdot \sum_{k=0}^{L-3} \sum_{j=0}^{L-3-k} (-1)^{k+j} & \binom{L-3}{k} \\ \cdot \binom{L-3-k}{j} & \left(\frac{1}{2+\overline{\gamma}}\right)^{k+3} \\ \cdot \frac{1}{k+2} & \left(\frac{1+\overline{\gamma}}{2+\overline{\gamma}}\right)^j \\ \cdot \frac{1+\overline{\gamma}}{j+1+(1+\overline{\gamma})(k+2)}. & \end{aligned} \tag{50}$$

$$\begin{aligned} \mathrm{Prob}(\mathcal{R}_2) &= \mathrm{Prob}\Big(\mathrm{LAPPR}_{(1)} < 0, \mathrm{LAPPR}_{(2)} > 0, \mathrm{LAPPR}_{(3)} < 0\Big) \\ &= \mathrm{Prob}\Big(\mathrm{LAPPR}_{[1]} + \mathrm{LAPPR}_{[L]} < 0, \mathrm{LAPPR}_{[1]} + \mathrm{LAPPR}_{[L-1]} > 0, \\ &\qquad \mathrm{LAPPR}_{[2]} + \mathrm{LAPPR}_{[L-1]} < 0, \mathrm{LAPPR}_{[2]} > 0\Big) \\ &\quad + \mathrm{Prob}\Big(\mathrm{LAPPR}_{[1]} + \mathrm{LAPPR}_{[L]} < 0, \mathrm{LAPPR}_{[1]} + \mathrm{LAPPR}_{[L-1]} > 0, \\ &\qquad \mathrm{LAPPR}_{[2]} < 0\Big) \\ &= \int_{x=0}^{\infty} \int_{z=-x}^{0} \int_{y=0}^{-z} \int_{w=-\infty}^{-x} f_{\mathrm{LAPPR}_{[1]}, \mathrm{LAPPR}_{[2]}, \mathrm{LAPPR}_{[L-1]}, \mathrm{LAPPR}_{[L]}}(x,y,z,w) dw dy dz dx \\ &\quad + \int_{x=0}^{\infty} \int_{z=-x}^{0} \int_{y=z}^{0} \int_{w=-\infty}^{-x} f_{\mathrm{LAPPR}_{[1]}, \mathrm{LAPPR}_{[2]}, \mathrm{LAPPR}_{[L-1]}, \mathrm{LAPPR}_{[L]}}(x,y,z,w) dw dy dz dx. \end{aligned} \tag{51}$$

To derive $\mathrm{Prob}(\mathcal{R}_2)$, we simplify the expression for $\mathrm{Prob}(\mathcal{R}_2)$ as shown in (51) at the bottom of the previous page [10]. From [9], we have

$$\begin{aligned} &f_{\mathrm{LAPPR}_{[1]},\mathrm{LAPPR}_{[2]},\mathrm{LAPPR}_{[L-1]},\mathrm{LAPPR}_{[L]}}(x,y,z,w) \\ &= \frac{L!}{(L-4)!} f_{\mathrm{LAPPR}}(x) f_{\mathrm{LAPPR}}(y) f_{\mathrm{LAPPR}}(z) \\ &\quad \times f_{\mathrm{LAPPR}}(w) \left[F_{\mathrm{LAPPR}}(y) - F_{\mathrm{LAPPR}}(z)\right]^{L-4}. \end{aligned} \tag{52}$$

Substituting (52) in (51) and performing the integration, we obtain

$$\begin{aligned} \mathrm{Prob}(\mathcal{R}_2) = &\frac{L!}{(L-4)!} \sum_{k=0}^{L-4} \sum_{j=0}^{L-4-k} (-1)^{j+k} \binom{L-4}{k} \\ &\times \binom{L-4-k}{j} \left(\frac{1}{2+\overline{\gamma}}\right)^{k+2} \\ &\times \left(\frac{1+\overline{\gamma}}{2+\overline{\gamma}}\right)^{j+3} \frac{1}{(1+\overline{\gamma})(k+2)+1} \\ &\times \frac{1}{(1+\overline{\gamma})(k+2)+(j+2)} \\ &+ \frac{L!}{(L-4)!} \sum_{k=0}^{L-4} (-1)^{k} \binom{L-4}{k} \\ &\times \left(\frac{1}{2+\overline{\gamma}}\right)^{L-2} \left(\frac{1+\overline{\gamma}}{2+\overline{\gamma}}\right)^{3} \\ &\times \frac{1}{(1+\overline{\gamma})(k+2)+1} \\ &\times \frac{1}{(1+\overline{\gamma})(L-1)+1}. \end{aligned} \tag{53}$$

To derive $\mathrm{Prob}(\mathcal{R}_3)$, the expression $\mathrm{Prob}(\mathcal{R}_3)$ can be simplified as shown in (54) at the bottom of the next page. Again, substituting (52) in (54) and upon the integration, we get

$$\begin{aligned} \mathrm{Prob}(\mathcal{R}_3) = &\frac{L!}{(L-4)!} \sum_{k=0}^{L-4} \sum_{j=0}^{k} \sum_{l=0}^{L-4-k} \binom{L-4}{k} \\ &\times \binom{L-4-k}{l} \binom{k}{j} \left(\frac{1}{2+\overline{\gamma}}\right)^{2} \\ &\times \left(\frac{1+\overline{\gamma}}{2+\overline{\gamma}}\right)^{l+j+3} \times \frac{1}{l+2+2(1+\overline{\gamma})} \\ &\times \frac{1}{3+l+j+2(1+\overline{\gamma})} \\ &+ \frac{L!}{(L-4)!} \sum_{k=0}^{L-4} \sum_{j=0}^{L-4-k} (-1)^{k+j} \binom{L-4}{k} \\ &\times \binom{L-4-k}{j} \left(\frac{1}{2+\overline{\gamma}}\right)^{k+2} \\ &\times \left(\frac{1+\overline{\gamma}}{2+\overline{\gamma}}\right)^{j+3} \times \frac{1}{j+2+2(1+\overline{\gamma})} \\ &\times \frac{1}{j+2+(k+3)(1+\overline{\gamma})}. \end{aligned} \tag{55}$$

Finally, the expression for $\mathrm{Prob}(\mathcal{R}_4)$ can be obtained as shown in (56) at the top of the next page. Substituting (52) in (56) and upon the integration, we obtain (57), as shown at the top of the next page. Combining (50), (53), (55), and (57), we obtain the final expression for $P_e^{\mathrm{GSC}(3,L),\mathrm{i.i.d.}}$.

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$$\begin{aligned} \mathrm{Prob}(\mathcal{R}_3) = &\,\mathrm{Prob}\Big(\mathrm{LAPPR}_{(1)} < 0, \mathrm{LAPPR}_{(2)} > 0, \mathrm{LAPPR}_{(3)} > 0, \mathrm{LAPPR}_{(1)} \\ &\qquad + \mathrm{LAPPR}_{(2)} + \mathrm{LAPPR}_{(3)} < 0\Big) \\ = &\,\mathrm{Prob}\Big(\mathrm{LAPPR}_{[1]} + \mathrm{LAPPR}_{[L]} < 0, \\ &\qquad \mathrm{LAPPR}_{[1]} + \mathrm{LAPPR}_{[2]} + \mathrm{LAPPR}_{[L]} < 0, \mathrm{LAPPR}_{[L-1]} > 0\Big) \\ &+ \mathrm{Prob}\Big(\mathrm{LAPPR}_{[1]} + \mathrm{LAPPR}_{[L]} < 0, \mathrm{LAPPR}_{[1]} + \mathrm{LAPPR}_{[2]} + \mathrm{LAPPR}_{[L]} < 0, \\ &\qquad \mathrm{LAPPR}_{[2]} + \mathrm{LAPPR}_{[L-1]} > 0, \mathrm{LAPPR}_{[L-1]} < 0\Big) \\ = &\int_{x=0}^{\infty} \int_{y=0}^{x} \int_{z=0}^{y} \int_{w=-\infty}^{-x-y} f_{\mathrm{LAPPR}_{[1]},\mathrm{LAPPR}_{[2]},\mathrm{LAPPR}_{[L-1]},\mathrm{LAPPR}_{[L]}}(x,y,z,w)\, dw\, dz\, dy\, dx \\ &+ \int_{x=0}^{\infty} \int_{y=0}^{x} \int_{z=-y}^{0} \int_{w=-\infty}^{-x-y} f_{\mathrm{LAPPR}_{[1]},\mathrm{LAPPR}_{[2]},\mathrm{LAPPR}_{[L-1]},\mathrm{LAPPR}_{[L]}}(x,y,z,w)\, dw\, dz\, dy\, dx. \end{aligned} \tag{54}$$

$$\begin{aligned}\text{Prob}(\mathcal{R}_4) &= \text{Prob}\Big(\text{LAPPR}_{(1)} > 0, \text{LAPPR}_{(2)} < 0, \text{LAPPR}_{(3)} < 0,\\ &\qquad \text{LAPPR}_{(1)} + \text{LAPPR}_{(2)} + \text{LAPPR}_{(3)} < 0\Big)\\ &= \text{Prob}\Big(\text{LAPPR}_{[1]} + \text{LAPPR}_{[L]} > 0, \text{LAPPR}_{[2]} + \text{LAPPR}_{[L-1]} < 0,\\ &\qquad \text{LAPPR}_{[1]} + \text{LAPPR}_{[L-1]} + \text{LAPPR}_{[L]} < 0, \text{LAPPR}_{[2]} > 0\Big)\\ &\quad + \text{Prob}\Big(\text{LAPPR}_{[1]} + \text{LAPPR}_{[L]} > 0, \text{LAPPR}_{[1]}\\ &\qquad + \text{LAPPR}_{[L-1]} + \text{LAPPR}_{[L]} < 0, \text{LAPPR}_{[2]} < 0\Big)\\ &= \int_{w=-\infty}^{0}\int_{z=w}^{0}\int_{y=0}^{-z}\int_{x=-w}^{-z-w} f_{\text{LAPPR}_{[1]},\text{LAPPR}_{[2]},\text{LAPPR}_{[L-1]},\text{LAPPR}_{[L]}}(x,y,z,w)\,dx\,dy\,dz\,dw\\ &\quad + \int_{w=-\infty}^{0}\int_{z=w}^{0}\int_{y=z}^{0}\int_{x=-w}^{-z-w} f_{\text{LAPPR}_{[1]},\text{LAPPR}_{[2]},\text{LAPPR}_{[L-1]},\text{LAPPR}_{[L]}}(x,y,z,w)\,dx\,dy\,dz\,dw \end{aligned} \tag{56}$$

$$\begin{aligned}\text{Prob}(\mathcal{R}_4) &= \frac{L!}{(L-4)!}\sum_{k=0}^{L-4}\sum_{j=0}^{L-4-k}(-1)^{j+k}\binom{L-4}{k}\binom{L-4-k}{j}\left(\frac{1}{2+\overline{\gamma}}\right)^{k+1} \times \left(\frac{1+\overline{\gamma}}{2+\overline{\gamma}}\right)^{j+4} \times \frac{1}{j+1}\\ &\quad \times \left[\frac{1}{(1+(k+2)(1+\overline{\gamma}))(2+(k+2)(1+\overline{\gamma}))} - \frac{1}{(j+2+(k+2)(1+\overline{\gamma}))(j+3+(k+2)(1+\overline{\gamma}))}\right]\\ &\quad + \frac{L!}{(L-4)!}\sum_{k=0}^{L-4}(-1)^k\binom{L-4}{k}\left(\frac{1}{2+\overline{\gamma}}\right)^{L-2} \times \left(\frac{1+\overline{\gamma}}{2+\overline{\gamma}}\right)^{3} \times \frac{1}{(L-3-k)}\\ &\quad \cdot \left[\frac{1}{(1+(k+2)(1+\overline{\gamma}))(2+(k+2)(1+\overline{\gamma}))} - \frac{1}{(1+(L-1)(1+\overline{\gamma}))(2+(L-1)(1+\overline{\gamma}))}\right]\end{aligned} \tag{57}$$

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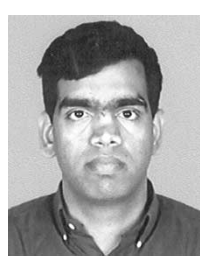

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