Single-Symbol ML Decodable Distributed STBCs for Partially-Coherent Cooperative Networks

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***Abstract*—A relay network with N relays and a single source-destination pair is called a partially-coherent relay channel (PCRC) if the destination has perfect channel state information (CSI) of all the channels and the relays have only the phase information of the source-to-relay channels. In this paper, first, a new set of necessary and sufficient conditions for a space-time block code (STBC) to be single-symbol decodable (SSD) for co-located multiple antenna communication is obtained. Then, this is extended to a set of necessary and sufficient conditions for a distributed STBC (DSTBC) to be SSD for a PCRC. Using this, several SSD DSTBCs for PCRC are identified. It is proved that even if a SSD STBC for a co-located MIMO channel does not satisfy the additional conditions for the code to be SSD for a PCRC, single-symbol decoding of it in a PCRC gives full-diversity and only coding gain is lost. It is shown that when a DSTBC is SSD for a PCRC, then arbitrary coordinate interleaving of the in-phase and quadrature-phase components of the variables does not disturb its SSD property for PCRC. Finally, it is shown that the possibility of *channel phase compensation* operation at the relay nodes using partial CSI at the relays increases the possible rate of SSD DSTBCs from $\frac{2}{N}$ when the relays do not have CSI to $\frac{1}{2}$, which is independent of N.**

***Index Terms*—Amplify-and-forward protocol, cooperative communication, distributed STBC, single-symbol decoding.**

I. Introduction

THE problem of fading and the ways to combat it through spatial diversity techniques have been an active area of research. Multiple-input multiple-output (MIMO) techniques have become popular in realizing spatial diversity and high data rates through the use of multiple transmit antennas. For such co-located multiple transmit antenna systems low maximum-likelihood (ML) decoding complexity space-time block codes (STBCs) have been studied by several researchers [1]-[10] which include the well known complex orthogonal designs (CODs) and their generalizations. Recent research has shown that the advantages of spatial diversity could be realized in single-antenna user nodes through user cooperation [11],[12] via relaying. A simple wireless relay network of $N+2$ nodes consists of a single source-destination pair with N relays. For such relay channels, use of CODs [1],[2] has been studied in [13]. CODs are attractive for cooperative communications for the following reasons: i) they offer full diversity gain and coding gain, ii) they are 'scale free' in the sense that deleting some rows does not affect the orthogonality, iii) entries are linear combination of the information symbols and their conjugates which means only linear processing is required at the relays, and iv) they admit very fast ML decoding (single-symbol decoding (SSD)). However, it should be noted that the last property applies only to the decode-and-forward (DF) policy at the relay node.

In a scenario where the relays amplify and forward (AF) the signal, it is known that the orthogonality is lost, and hence the destination has to use a complex multi-symbol ML decoding or sphere decoding [13],[14]. It should be noted that the AF policy is attractive for two reasons: i) the complexity at the relay is greatly reduced, and ii) the restrictions on the rate because the relay has to decode is avoided [15].

In order to avoid the complex ML decoding at the destination, in [16], the authors propose an alternative code design strategy and propose a SSD code for 2 and 4 relays. For arbitrary number of relays, recently in [17], distributed orthogonal STBCs (DOSTBC) have been studied and it is shown that if the destination has the complete channel state information (CSI) of all the source-to-relay channels and the relay-to-destination channels, then the maximum possible rate is upper bounded by $\frac{2}{N}$ complex symbols per channel use for N relays. Towards improving the rate of transmission and achieving simultaneously both full-diversity as well as SSD at the destination, in this paper, we study relay channels with the assumption that the relays have the phase information of the source-to-relay channels and the destination has the CSI of all the channels. Coding for partially-coherent relay channel (PCRC, Section II-B) has been studied in [18], where a sufficient condition for SSD has been presented.

The contributions of this paper can be summarized as follows:

- First, a new set of necessary and sufficient conditions for a STBC to be SSD for co-located multiple antenna communication is obtained. The known set of necessary and sufficient conditions in [8] is in terms of the dispersion matrices (weight matrices) of the code, whereas

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our new set of conditions is in terms of the column vector representation matrices [5] of the code and is a generalization of the conditions given in [5].

- A set of necessary and sufficient conditions for a distributed STBC (DSTBC) to be SSD for a PCRC is obtained by identifying the additional conditions. Using this, several SSD DSTBCs for PCRC are identified among the known classes of STBCs for co-located multiple antenna system.
- It is proved that even if a SSD STBC for a co-located MIMO channel does not satisfy the additional conditions for the code to be SSD for a PCRC, single-symbol decoding of it in a PCRC gives full-diversity and only coding gain is lost.
- It is shown that when a DSTBC is SSD for a PCRC, then arbitrary coordinate interleaving of the in-phase and quadrature-phase components of the variables does not disturb its SSD property for PCRC.
- It is shown that the possibility of *channel phase compensation* operation at the relay nodes using partial CSI at the relays increases the possible rate of SSD DSTBCs from $\frac{2}{N}$ when the relays do not have CSI to $\frac{1}{2}$, which is independent of N.

The remaining part of the paper is organized as follows: In Section II, the signal model for a PCRC is developed. Using this model, in Section III, a new set of necessary and sufficient conditions for a STBC to be SSD in a co-located MIMO is presented. Several classes of SSD codes are discussed and conditions for full-diversity of a subclass of SSD codes is obtained. Then, in Section IV, SSD DSTBCs for PCRC are characterized by identifying a set of necessary and sufficient conditions. It is shown that the SSD property is invariant under coordinate interleaving operations which leads to a class of SSD DSTBCs for PCRC. The class of rate-$\frac{1}{2}$ CODs obtained from rate-1 real orthogonal designs (ROD) by stacking construction [1] is shown to be SSD for PCRC. Also, it is shown that SSD codes for co-located MIMO, under suboptimal SSD decoder for PCRC offer full diversity. Simulation results and discussion constitute Section V. Conclusions and scope for further work are presented in Section VI.

II. System Model

Consider a wireless network with $N+2$ nodes consisting of a source, a destination, and N relays[1], as shown in Fig. 1. All nodes are half-duplex nodes, i.e., a node can either transmit or receive at a time on a specific frequency. We consider amplify-and-forward (AF) transmission at the relays. Transmission from the source to the destination is carried out in two phases. In the first phase, the source transmits information symbols $x^{(i)}, 1 \leq i \leq T_1$ in T_1 time slots. All the N relays receive these T_1 symbols. This phase is called the *broadcast phase*. In the second phase, all the N relays perform distributed space-time block encoding on their received vectors and transmit the resulting encoded vectors in T_2 time slots. That is, each relay will transmit a column (with T_2 entries) of a distributed STBC matrix of size $T_2 \times N$. The destination receives a faded and noise added version of this matrix. This phase is called the *relay phase*. We assume that the source-to-relay channels remain static over T_1 time slots, and the relay-to-destination channels remain static over T_2 time slots.

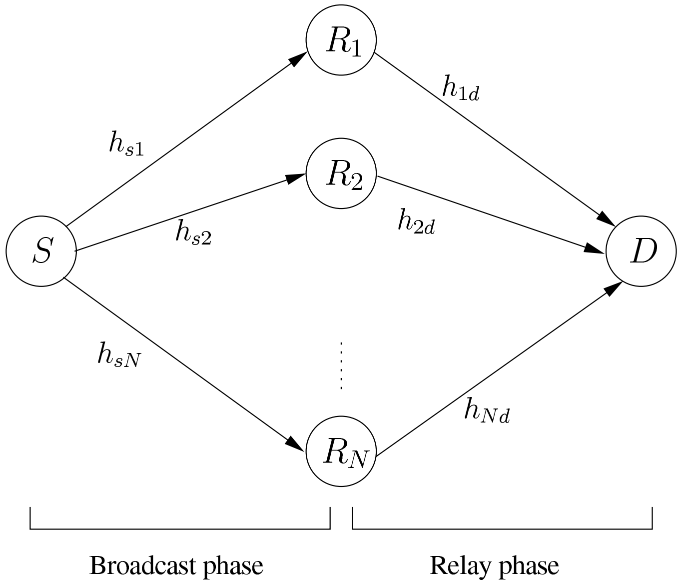

Fig. 1. A cooperative relay network.

A. No CSI at the relays

The received signal at the jth relay, $j = 1, \cdots, N$, in the ith time slot, $i = 1, \cdots, T_1$, denoted by $v_j^{(i)}$, can be written as[2] $v_j^{(i)} = \sqrt{E_1} h_{sj} x^{(i)} + z_j^{(i)}$, where h_{sj} is $\mathcal{N}(0,1)$ distributed complex channel gain from the source s to the jth relay, $z_j^{(i)}$ is complex additive white Gaussian noise with zero mean and unit variance at relay j, E_1 is the transmit energy per symbol in the broadcast phase, and $\mathbb{E}\left[\left(x^{(i)}\right)^* x^{(i)}\right] = 1$. Under the assumption of no CSI at the relays, the amplified ith received signal at the jth relay can be written as [13]

$$\widehat{v}_j^{(i)} = \underbrace{\sqrt{\frac{E_2}{E_1+1}}}_{\triangleq G} v_j^{(i)}, \tag{1}$$

where E_2 is the transmit energy per transmission of a symbol in the relay phase, and G is the amplification factor at the relay that makes the total transmission energy per symbol in the relay phase to be equal to E_2. Let E_t denote the total energy per symbol in both the phases put together. Then, it is shown in [15] that the optimum energy allocation that maximizes the receive SNR at the destination is when half the energy is spent in the broadcast phase and the remaining half in the relay phase when the time allocations for the relay and broadcast phase are same, i.e., $T_1 = T_2$. We also assume that the energy is distributed equally, i.e., $E_1 = \frac{E_t}{2}$ and $E_2 = \frac{E_t}{2M}$, where M

[1]In the system model considered here, we assume that there is no direct link between source and destination. However, the results we show here can be extended to a system model with a direct link between source and destination.

[2]We use the following notation: Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. Superscripts T and $\mathcal{H}$ denote transpose and conjugate transpose operations, respectively, and $*$ denotes matrix conjugation operation.

is the number of symbols per transmission of the STBC. For the unequal-time allocation ($T_1 \neq T_2$,) this distribution might not be optimal.

At relay j, a $2T_1 \times 1$ real vector $\widehat{\mathbf{v}}_j$ given by

$$\widehat{\mathbf{v}}_j = \left[\widehat{v}_{jI}^{(1)}, \widehat{v}_{jQ}^{(1)}, \widehat{v}_{jI}^{(2)}, \widehat{v}_{jQ}^{(2)}, \cdots, \widehat{v}_{jI}^{(T_1)}, \widehat{v}_{jQ}^{(T_1)}\right]^T, \quad (2)$$

is formed, where $\widehat{v}_{jI}^{(i)}$ and $\widehat{v}_{jQ}^{(i)}$, respectively, are the in-phase (real part) and quadrature (imaginary part) components of $\widehat{v}_j^{(i)}$. Now, (2) can be written in the form

$$\widehat{\mathbf{v}}_j = G\sqrt{E_1}\,\mathbf{H}_{sj}\,\mathbf{x} + \widehat{\mathbf{z}}_j, \quad (3)$$

where $\mathbf{x}$ is the $2T_1 \times 1$ data symbol real vector, given by

$$\mathbf{x} = \left[x_I^{(1)}, x_Q^{(1)}, x_I^{(2)}, x_Q^{(2)}, \cdots, x_I^{(T_1)}, x_Q^{(T_1)}\right]^T, \quad (4)$$

$\widehat{\mathbf{z}}_j$ is the $2T_1 \times 1$ noise vector, given by $\widehat{\mathbf{z}}_j = \left[\widehat{z}_{jI}^{(1)}, \widehat{z}_{jQ}^{(1)}, \widehat{z}_{jI}^{(2)}, \widehat{z}_{jQ}^{(2)}, \cdots, \widehat{z}_{jI}^{(T_1)}, \widehat{z}_{jQ}^{(T_1)}\right]^T$, where $\widehat{z}_j^{(i)} = G\,z_j^{(i)}$, and $\mathbf{H}_{sj}$ is a $2T_1 \times 2T_1$ block-diagonal matrix, given by

$$\mathbf{H}_{sj} = \begin{bmatrix} \begin{bmatrix} h_{sjI} & -h_{sjQ} \\ h_{sjQ} & h_{sjI} \end{bmatrix} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \begin{bmatrix} h_{sjI} & -h_{sjQ} \\ h_{sjQ} & h_{sjI} \end{bmatrix} \end{bmatrix} \quad (5)$$

Let $\mathbf{C} = \left[\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_N\right]$ denote the $T_2 \times N$ DSTBC matrix to be sent in the relay phase jointly by all N relays, where $\mathbf{c}_j$ denotes the jth column of $\mathbf{C}$. The jth column $\mathbf{c}_j$ is manufactured by the jth relay as

$$\mathbf{c}_j = \mathbf{A}_j \widehat{\mathbf{v}}_j = \underbrace{G\sqrt{E_1}\mathbf{A}_j\,\mathbf{H}_{sj}}_{\mathbf{B}_j}\,\mathbf{x} + \mathbf{A}_j\,\widehat{\mathbf{z}}_j, \quad (6)$$

where $\mathbf{A}_j$ is the matrix of size $T_2 \times 2T_1$ for the jth relay, called the *relay matrix,* and $\mathbf{B}_j$ can be viewed as the column vector representation matrix [5] for the DSTBC with the difference that in our case the vector $\mathbf{x}$ is real whereas in [5] it is complex. For example, for the 2-relay case (i.e., $N = 2$), with $T_1 = T_2 = 2$, using Alamouti code, the relay matrices are

$$\mathbf{A}_1 = \begin{bmatrix} 1 & \mathbf{j} & 0 & 0 \\ 0 & 0 & -1 & \mathbf{j} \end{bmatrix};\ \mathbf{A}_2 = \begin{bmatrix} 0 & 0 & 1 & \mathbf{j} \\ 1 & -\mathbf{j} & 0 & 0 \end{bmatrix}. \quad (7)$$

Let $\mathbf{y}$ denote the $T_2 \times 1$ received signal vector at the destination in T_2 time slots. Then, $\mathbf{y}$ can be written as $\mathbf{y} = \sum_{j=1}^{N} h_{jd}\mathbf{c}_j + \mathbf{z}_d$, where h_{jd} is the complex channel gain from the jth relay to the destination, and $\mathbf{z}_d$ is the AWGN noise vector at the destination with zero mean and $E[\mathbf{z}_d\,\mathbf{z}_d^{\mathcal{H}}] = \mathbf{I}$. Substituting (6) in this equation, we get

$$\mathbf{y} = G\sqrt{E_1}\left(\sum_{j=1}^{N} h_{jd}\mathbf{A}_j\mathbf{H}_{sj}\right)\mathbf{x} + \sum_{j=1}^{N} h_{jd}\mathbf{A}_j\widehat{\mathbf{z}}_j + \mathbf{z}_d. \quad (8)$$

B. With phase only information at the relays

In this subsection, we obtain a signal model for the case of partial CSI at the relays, where we assume that each relay has the knowledge of the channel phase on the link between the source and itself in the broadcast phase. That is, defining the channel gain from source to relay j as $h_{sj} = \alpha_{sj}e^{\mathbf{j}\theta_{sj}}$, we assume that relay j has perfect knowledge of only θ_{sj} and does not have the knowledge of α_{sj}.

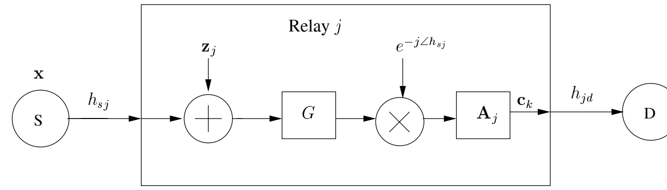

Fig. 2. Processing at the jth relay in the proposed phase compensation scheme.

In the proposed scheme, we perform a phase compensation operation on the amplified received signals at the relays, and space-time encoding is done on these phase-compensated signals. That is, we multiply $\widehat{v}_j^{(i)}$ in (1) by $e^{-\mathbf{j}\theta_{sj}}$ before space-time encoding. Note that multiplication by $e^{-\mathbf{j}\theta_{sj}}$ does not change the statistics of $z_j^{(i)}$. Therefore, with this phase compensation, the $\widehat{\mathbf{v}}_j$ vector in (3) becomes $\widehat{\mathbf{v}}_j = \left(G\sqrt{E_1}\,\mathbf{H}_{sj}\,\mathbf{x} + \widehat{\mathbf{z}}_j\right)e^{-\mathbf{j}\theta_{sj}} = G\sqrt{E_1}\,|h_{s_j}|\,\mathbf{x} + \widehat{\mathbf{z}}_j$. Consequently, the $\mathbf{c}_j$ vector generated by relay j is given by

$$\mathbf{c}_j = \mathbf{A}_j\widehat{\mathbf{v}}_j = \underbrace{G\sqrt{E_1}\mathbf{A}_j\,|h_{sj}|}_{\triangleq\,\mathbf{B}'_j}\,\mathbf{x} + \mathbf{A}_j\,\widehat{\mathbf{z}}_j, \quad (9)$$

where $\mathbf{B}'_j$ is the equivalent weight matrix with phase compensation. Now, we can write the received vector $\mathbf{y}$ as

$$\mathbf{y} = G\sqrt{E_1}\left(\sum_{j=1}^{N} h_{jd}|h_{sj}|\mathbf{A}_j\right)\mathbf{x} + \underbrace{\sum_{j=1}^{N} h_{jd}\mathbf{A}_j\widehat{\mathbf{z}}_j + \mathbf{z}_d}_{\tilde{\mathbf{z}}_d:\ \text{total noise}}. \quad (10)$$

Figure 2 shows the processing at the jth relay in the proposed phase compensation scheme. Such systems will be referred as *partially-coherent relay channels* (PCRC). A distributed STBC which is SSD for a PCRC will be referred as SSD-DSTBC-PCRC.

III. Conditions for SSD and Full-Diversity for Co-located MIMO

The class of SSD codes, including the well known CODs, for co-located MIMO has been studied in [8], where a set of necessary and sufficient conditions for an arbitrary linear STBC to be SSD has been obtained in terms of the dispersion matrices [19], also known as weight matrices. In this section, a new set of necessary and sufficient conditions in terms of the column vector representation matrices [5] of the code is obtained that are amenable for extension to PCRC. This is a generalization of the conditions given in [5] in terms of column vector representation matrices for CODs. Towards this end, the received vector $\mathbf{y}$ in a co-located MIMO setup can be written as

$$\mathbf{y} = \sqrt{E_t}\left(\sum_{j=1}^{N} h_{jd}\mathbf{A}_j\right)\mathbf{x} + \mathbf{z}_d. \quad (11)$$

Theorem 1: For co-located MIMO with N transmit antennas, the linear STBC as given in (11) is SSD iff (12) is

$$\begin{aligned}\mathbf{A}_{jI}^T\mathbf{A}_{jI}+\mathbf{A}_{jQ}^T\mathbf{A}_{jQ} &= \mathbf{D}_{jj}^{(1)}; \quad j=1,2,\cdots,N\\ \mathbf{A}_{jI}^T\mathbf{A}_{iI}+\mathbf{A}_{jQ}^T\mathbf{A}_{iQ}+\mathbf{A}_{iI}^T\mathbf{A}_{jI}+\mathbf{A}_{iQ}^T\mathbf{A}_{jQ} &= \mathbf{D}_{ij}^{(2)}; \quad 1\le i\ne j\le N\\ \mathbf{A}_{jI}^T\mathbf{A}_{iQ}+\mathbf{A}_{jQ}^T\mathbf{A}_{iI}-\mathbf{A}_{iI}^T\mathbf{A}_{jQ}-\mathbf{A}_{iQ}^T\mathbf{A}_{jI} &= \mathbf{D}_{ij}^{(3)}; \quad 1\le i\ne j\le N\end{aligned} \tag{12}$$

$$\mathbf{D}_{ij}^{(k)} = \begin{bmatrix} \underbrace{\begin{bmatrix} a_{ij,1}^{(k)} & b_{ij,1}^{(k)} \\ b_{ij,1}^{(k)} & c_{ij,1}^{(k)} \end{bmatrix}}_{\mathbf{D}_{ij,1}^{(k)}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \underbrace{\begin{bmatrix} a_{ij,2}^{(k)} & b_{ij,2}^{(k)} \\ b_{ij,2}^{(k)} & c_{ij,2}^{(k)} \end{bmatrix}}_{\mathbf{D}_{ij,2}^{(k)}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \underbrace{\begin{bmatrix} a_{ij,T_1}^{(k)} & b_{ij,T_1}^{(k)} \\ b_{ij,T_1}^{(k)} & c_{ij,T_1}^{(k)} \end{bmatrix}}_{\mathbf{D}_{ij,T_1}^{(k)}} \end{bmatrix} \tag{13}$$

satisfied, where $\mathbf{A}_j = \mathbf{A}_{jI} + \mathbf{j}\mathbf{A}_{jQ},\ j = 1, 2, \cdots, N$, where $\mathbf{A}_{jI}$ and $\mathbf{A}_{jQ}$ are real matrices, and $\mathbf{D}_{jj}^{(1)}, \mathbf{D}_{ij}^{(2)}$ and $\mathbf{D}_{ij}^{(3)}$ are block diagonal matrices of the form shown in (13), where it is understood that whenever the superscript is (1) as in $\mathbf{D}_{ij}^{(1)}$, then $i = j$. T_1 is the number of complex variables in the STBC.

Proof: Letting $\mathbf{H}_{eq} = \sqrt{E_t}\sum_{j=1}^{N} h_{jd}\mathbf{A}_j$ in (8) and computing $\widehat{\mathbf{x}} = \arg\min ||\mathbf{y} - \mathbf{H}_{eq}\mathbf{x}||^2$ gives the result. A detailed proof is available in [20]. □

Notice that $\mathbf{D}_{ij}^{(k)} = \mathbf{D}_{ji}^{(k)}$ for all i, j, k. The conditions for achieving maximum diversity depend on the $\mathbf{D}_{ij}^{(k)}$ matrices as well as the signal constellation used for the variables. It can be easily verified that, for co-located MIMO case, for the class of well known CODs [1],[2],[5], we get $\mathbf{D}_{ij}^{(2)} = \mathbf{D}_{ij}^{(3)} = \mathbf{0}\ \forall\, i, j$ and $\mathbf{D}_{jj}^{(1)}$ is the identity matrix $\forall\, j$ for any arbitrary constellations for the variables. Similarly, for the class of coordinate interleaved orthogonal designs (CIODs) of [8], we get $\mathbf{D}_{ij}^{(2)} = \mathbf{D}_{ij}^{(3)} = \mathbf{0}\,\forall\, i, j$. But, $\mathbf{D}_{jj}^{(1)}$ is not the identity matrix $\forall\, j$ which distinguishes CIODs from CODs. Furthermore, it can be verified that for Clifford UW-SSD codes discussed in [10], we get $\mathbf{D}_{ij}^{(2)} = \mathbf{0}\,\forall i, j$, and the matrices $\mathbf{D}_{ij}^{(3)}\,\forall\, i, j$ and $\mathbf{D}_{jj}^{(1)}\,\forall\, j$ are of the form (13).

For the co-located case, SSD conditions have been presented in [8] in terms of the linear dispersion matrices (also called weight matrices). Our SSD conditions given in Theorem 1 is in terms of 'column vector representation matrices' [5]. The significance of our version as in Theorem 1 is that it is instrumental in proving Theorems 2 to 6.

A. *Conditions for full-diversity*

In the previous subsection, we saw several classes of SSD codes. The problem of identifying all possible classes of SSD codes is a open problem [10]. Moreover, different classes of SSD codes may give full-diversity for different sets of signal sets. The following lemma obtains a set of necessary and sufficient conditions for the subclass of SSD codes characterized by $\mathbf{D}_{ij}^{(2)} = \mathbf{D}_{ij}^{(3)} = \mathbf{0}$ (CODs and CIODs, for example) to offer full-diversity for all complex constellations.

Lemma 1: For co-located MIMO, the linear STBC as given in (11) with the $\mathbf{D}_{ij}^{(k)}$ matrices in (12) satisfying $\mathbf{D}_{ij}^{(2)} = \mathbf{D}_{ij}^{(3)} = \mathbf{0}$ achieves maximum diversity for all signal constellations *iff*

$$a_{jj,l}^{(1)} c_{jj,l}^{(1)} - {b_{jj,l}^{(1)}}^2 > 0, \quad 1 \le j \le N; \quad 1 \le l \le T_1, \tag{14}$$

i.e., $\mathbf{D}_{jj,l}^{(1)}$ is positive definite for all j, l.

Proof: The proof follows from the usual way of analyzing the Chernoff bound for the pairwise probability, and is explicitly given in [20]. □

For CODs, by definition, $b_{jj,l}^{(1)} = 0$ and $a_{jj,l}^{(1)} = c_{jj,i}^{(1)} = 1 \forall j, i$. Hence, the condition in (14) is readily satisfied, and hence full diversity is achieved for all signal constellations. However, for CIODs, the condition (14) is not satisfied, which can be easily verified for the code $CIOD_4$. This does not mean that the code can not give full diversity; it only means that it can not give full diversity for all complex constellations as mentioned in Lemma 1. The constellations for which this code offers full diversity can be obtained by choosing the signal constellation such that for any two constellation points, $\Delta\mathbf{x}_I^{(i)}$ and $\Delta\mathbf{x}_Q^{(i)}$ are both non-zero [8].

IV. SSD CODES FOR PCRC

In the previous section, we saw that SSD is achieved if the relay matrices satisfy the condition (12). However, to achieve SSD in the case of distributed STBC with AF protocol, the equivalent weight matrices $\mathbf{B}_j$'s must satisfy the condition in (12). It can be seen that for any $\mathbf{A}_j$ that satisfies the condition in (12), the corresponding $\mathbf{B}_j$'s need not satisfy (12). For example, for the weight matrices in (7), the corresponding equivalent weight matrices $\mathbf{B}_1$ and $\mathbf{B}_2$ do not satisfy the condition in (12). That is, the Alamouti code is not SSD as a distributed STBC with AF protocol. We note that, in [16], code designs which retain the SSD feature have been obtained for no CSI at the relays, but only for $N = 2$ and 4. A key contribution in this paper is that by using partial CSI at the relays (i.e., only the channel phase information of the source-to-relay links), the SSD feature at the destination can be restored for a large subclass of SSD codes for co-located MIMO communication. This key result is given in the following theorem, which characterizes the class of SSD codes for PCRC.

Theorem 2: A code as given by (9) is SSD-DSTBC-PCRC iff the relay matrices $\mathbf{A}_j,\ j = 1, 2, \cdots, N$, satisfy (12) (i.e., the code is SSD for a co-located MIMO set up), and, in addition, satisfy (15) and (16) for any three relays with indices j_1, j_2, j_3, where $j_1, j_2, j_3 \in \{1, 2, \cdots, N\}$, and $\mathbf{D}^{'}_{j_1,j_2,j_3}$ and $\mathbf{D}^{''}_{j_1,j_2,j_3}$ are block diagonal matrices of the form in (13).

Proof: First we show the sufficiency part. It can be seen that the matrices $\mathbf{B}'_j = G\sqrt{E_1}\mathbf{A}_j\ |h_{sj}|,\ j = 1, 2, \cdots, N$ satisfy the condition (12) in spite of the fact that $|h_{sj}|$ are random variables (since $\mathbf{B}'_j$ matrices are scaled versions of the $\mathbf{A}_j$ matrices). Let $\mathbf{H}_{eq}^{(pc)} \stackrel{\triangle}{=} G\sqrt{E_1}\sum_{j=1}^{N}|h_{sj}|h_{jd}\mathbf{A}_j$. It can be seen that $\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\mathbf{H}_{eq}^{(pc)}\right)$ is block diagonal of the form in (13). This implies that each element of the $K \times 1$ vector $\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\mathbf{y}\right)$ is affected by only one information symbol (i.e., there will be no information symbol entanglement in each element). Hence, for single symbol decodability, it suffices to show that noise in each of these terms are uncorrelated, i.e., the DSTBC is SSD *iff* $E\left[\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)^T\right]$ is a block diagonal matrix of the form (13). Expanding $E\left[\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)^T\right]$, we arrive, after some manipulation, at (17) where, in terms of notation, h_{j_1dI} and h_{j_1dQ} denote the real and imaginary parts of the channel gains from the relay j_1 to destination d (i.e., the real and imaginary parts of h_{j_1d}), respectively. Since (17) turns out to be a linear combination of the $\mathbf{D}^{'}_{j_1,j_2,j_3}$ and $\mathbf{D}^{''}_{j_1,j_2,j_3}$ matrices in (15) and (16), the covariance matrix is of the form (13). Hence, along with (12) the conditions in (15) and (16) constitute a set of sufficient conditions.

To show the 'necessary part,' since the terms $h_{sj_1}||h_{j_2d}|^2|h_{sj_3}|(h_{j_1dI}h_{j_3dI} + h_{j_1dQ}h_{j_3dQ})$ and $h_{sj_1}||h_{j_2d}|^2|h_{sj_3}|(h_{j_1dI}h_{j_3dQ} + h_{j_1dQ}h_{j_3dI})$ are independent and if the co-variance matrix has to be block diagonal for all the realizations of h_{sj} and h_{rj}, then the conditions in (15) and (16) have to be necessarily satisfied. Also, in the similar lines of the proof for *Theorem 1*, it can be deduced that $\mathbf{B}'_j$ satisfying condition (12) is necessary to achieve un-entanging of information symbols in the elements of the vector $\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\mathbf{y}\right)$. □

In [18], partially-coherent distributed set up has been studied and a sufficient condition has been identified for a distributed STBC to be SSD. In the following corollary, it is shown that Theorem 2 subsumes this sufficient condition as a special case.

Corollary 3: The sufficient condition in [18], i.e., the noise co-variance to be a scaled identity matrix, is a subset of the conditions (15) and (16).

Proof: It can be observed that the $\mathbf{Z}_j$ matrix in [18], when written in our notation, is $\mathbf{Z}_j = \begin{bmatrix} \mathbf{A}_{jI} \\ \mathbf{A}_{jQ} \end{bmatrix}$. Hence, if $\mathbf{Z}_j\mathbf{Z}_j^T = \alpha\mathbf{I}\ \forall j$, where α is a scalar, then, $\mathbf{A}_{jI}{\mathbf{A}_{jI}}^T = \alpha\mathbf{I}$, $\mathbf{A}_{jQ}{\mathbf{A}_{jQ}}^T = \alpha\mathbf{I}$, $\mathbf{A}_{jQ}{\mathbf{A}_{jI}}^T = \mathbf{0}$, and $\mathbf{A}_{jI}{\mathbf{A}_{jQ}}^T = \mathbf{0}$. Substituting this in (15) and (16), we get the left hand side of (15) to be $\alpha\left({\mathbf{A}_{j_1I}}^T\mathbf{A}_{j_3I} + {\mathbf{A}_{j_3I}}^T\mathbf{A}_{j_1I} + {\mathbf{A}_{j_1Q}}^T\mathbf{A}_{j_3Q} + {\mathbf{A}_{j_3Q}}^T\mathbf{A}_{j_3Q}\right)$ which, by (12), is always a block diagonal matrix of the form (13). Also, the left hand side of (16) is $\mathbf{0}$. Hence, $\mathbf{A}_{jI}{\mathbf{A}_{jI}}^T = \mathbf{A}_{jQ}{\mathbf{A}_{jQ}}^T = \alpha\mathbf{I}$ and $\mathbf{A}_{jI}{\mathbf{A}_{jQ}}^T = \mathbf{A}_{jQ}{\mathbf{A}_{jI}}^T = \mathbf{0}\ \forall j$ is a sufficient condition for a DSTBC to be SSD. □

In [18], it is shown that the 8-antenna code given by (18), which we denote by RR_8, does not satisfy the sufficient condition discussed in that paper for SSD in PCRC, and hence not claimed to be SSD. However, it can be verified that RR_8 satisfies (12), (15) and (16), and hence SSD-DSTBC-PCRC.

A. Invariance of SSD under coordinate interleaving

In this subsection, we show that the property of SSD of a DSTBC for PCRC is invariant under coordinate interleaving of the data symbols. To illustrate the usefulness of this result we first show the following lemma.

Lemma 2: If $\mathbf{G}(x_1, \cdots, x_{T_1})$ is a SSD design in T_1 variables and N relay nodes that satisfies (12), (15) and (16), then the design in $2T_1$ variables and $2N$ relay nodes given by

$$\bar{\mathbf{G}}(x_1, \cdots, x_{2T_1}) = \begin{bmatrix} \mathbf{G}(x_1, \cdots, x_{T_1}) & \mathbf{0} \\ \mathbf{0} & \mathbf{G}(x_{T_1+1}, \cdots, x_{2T_1}) \end{bmatrix} \quad (19)$$

also satisfies (12), (15) and (16).

Proof: If $\mathbf{A}_j,\ 1 \le j \le N$ are the relay matrices of $\mathbf{G}$, then the corresponding $\bar{\mathbf{A}}_j$ matrices for $\bar{\mathbf{G}}$ are $\bar{\mathbf{A}}_j = \begin{bmatrix} \mathbf{A}_j & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, $1 \le j \le N$ and $\bar{\mathbf{A}}_j = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_j \end{bmatrix}$, $N+1 \le j \le 2N$. It is easily verified that if $\mathbf{A}_j$ satisfies (12), (15) and (16), then so do the matrices $\bar{\mathbf{A}}_j$. □

As an example, if we choose $\mathbf{G}(x_1, x_2)$ to be the Alamouti code in the lemma above then we get the code

$$\begin{bmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \\ 0 & 0 & x_3 & x_4 \\ 0 & 0 & -x_4^* & x_3^* \end{bmatrix}. \quad (20)$$

This code is SSD for PCRC. Note that a 4-antenna COD has rate only $\frac{3}{4}$ whereas this code has rate one. However, it is easily shown that this code does not give full-diversity. But, coordinate interleaving for this example results in $CIOD_4$ which gives full-diversity for any signal set with coordinate product distance zero, and we have already seen that $CIOD_4$ has the SSD property for PCRC. The following theorem shows that it is the property of coordinate interleaving to leave the SSD property of any arbitrary STBC for PCRC intact.

Theorem 4: If an STBC with T_1 variables $x_1, x_2, \cdots, x_{T_1}$, satisfy (12), (15) and (16), the SSD property is unaffected

$$\begin{aligned}
&{\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3I}+{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1I}+\\
&{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3Q}+{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1Q} = \mathbf{D}'_{j_1,j_2,j_3}, \qquad (15)
\end{aligned}$$

$$\begin{aligned}
&{\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3Q}+{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1I}+\\
&{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3I}+{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1Q} = \mathbf{D}''_{j_1,j_2,j_3} \qquad (16)
\end{aligned}$$

$$\begin{aligned}
E\left[\Re\left(\mathbf{H}_{eq}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)\Re\left(\mathbf{H}_{eq}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)^T\right] &= \sum_{j_1=1}^{N}\sum_{j_2=1}^{N}\sum_{j_3=1}^{N}|h_{sj_1}||h_{j_2d}|^2|h_{sj_3}|\left(h_{j_1dI}h_{j_3dI}+h_{j_1dQ}h_{j_3dQ}\right)\\
&\left[{\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3I}+{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1I}\right.\\
&\left.+{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3Q}+{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1Q}\right]\\
&+\sum_{j_1=1}^{N}\sum_{j_2=1}^{N}\sum_{j_3=1}^{N}|h_{sj_1}||h_{j_2d}|^2|h_{sj_3}|\left(h_{j_1dI}h_{j_3dQ}+h_{j_1dQ}h_{j_3dI}\right)\\
&\left[{\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3Q}+{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1I}\right.\\
&\left.+{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3I}+{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1Q}\right]\\
&= \sum_{j_1=1}^{N}\sum_{j_2=1}^{N}\sum_{j_3=1}^{N}|h_{sj_1}||h_{j_2d}|^2|h_{sj_3}|\left(h_{j_1dI}h_{j_3dI}+h_{j_1dQ}h_{j_3dQ}\right)\mathbf{D}'_{j_1,j_2,j_3}\\
&\quad+\sum_{j_1=1}^{N}\sum_{j_2=1}^{N}\sum_{j_3=1}^{N}|h_{sj_1}||h_{j_2d}|^2|h_{sj_3}|\left(h_{j_1dI}h_{j_3dQ}+h_{j_1dQ}h_{j_3dI}\right)\mathbf{D}''_{j_1,j_2,j_3} \qquad (17)
\end{aligned}$$

$$\begin{bmatrix}
x_{1I}-\mathbf{j}x_{4Q} & x_{2I}+\mathbf{j}x_{3I} & x_{4I}+\mathbf{j}x_{1Q} & -x_{3Q}+\mathbf{j}x_{2Q} & 0 & 0 & 0 & 0\\
-x_{2I}+\mathbf{j}x_{3I} & x_{1I}+\mathbf{j}x_{4Q} & -x_{3Q}-\mathbf{j}x_{2Q} & -x_{4I}+\mathbf{j}x_{1Q} & 0 & 0 & 0 & 0\\
-x_{4I}-\mathbf{j}x_{1Q} & x_{3Q}-\mathbf{j}x_{2Q} & x_{1I}-\mathbf{j}x_{4Q} & x_{2I}+\mathbf{j}x_{3I} & 0 & 0 & 0 & 0\\
x_{3Q}+\mathbf{j}x_{2Q} & x_{4I}-\mathbf{j}x_{1Q} & -x_{2I}+\mathbf{j}x_{3I} & x_{1I}+\mathbf{j}x_{4Q} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & x_{5I}-\mathbf{j}x_{8Q} & x_{6I}+\mathbf{j}x_{7I} & x_{8I}+\mathbf{j}x_{5Q} & -x_{7Q}+\mathbf{j}x_{6Q}\\
0 & 0 & 0 & 0 & -x_{6I}+\mathbf{j}x_{7I} & x_{5I}+\mathbf{j}x_{8Q} & -x_{7Q}-\mathbf{j}x_{6Q} & -x_{8I}+\mathbf{j}x_{5Q}\\
0 & 0 & 0 & 0 & -x_{8I}-\mathbf{j}x_{5Q} & x_{7Q}-\mathbf{j}x_{6Q} & x_{5I}-\mathbf{j}x_{8Q} & x_{6I}+\mathbf{j}x_{7I}\\
0 & 0 & 0 & 0 & x_{7Q}+\mathbf{j}x_{6Q} & x_{8I}-\mathbf{j}x_{5Q} & -x_{6I}+\mathbf{j}x_{7I} & x_{5I}+\mathbf{j}x_{8Q}
\end{bmatrix} \qquad (18)$$

by doing arbitrary coordinate interleaving among all real and imaginary components of x_i.[3]

Proof: The data-symbol vector in (4) after interleaving can be written as $\tilde{\mathbf{x}} = \tilde{\mathbf{I}}\mathbf{x}$ where $\tilde{\mathbf{I}}$ is the interleaving matrix, which is a permutation matrix obtained by permuting the rows (/columns) of the identity matrix $\mathbf{I}$ to reflect the coordinate interleaving operation. It can be easily checked that $\tilde{\mathbf{I}}^2 = \mathbf{I}$. Also, if $\mathbf{D}$ is a block diagonal matrix of the form (5), then so is the matrix $\tilde{\mathbf{I}}\mathbf{D}\tilde{\mathbf{I}}$. Hence, for PCRC with co-ordinate interleaving (9) can be written as $\mathbf{c}_j = \mathbf{A}_j\widehat{\mathbf{v}}_j = G\sqrt{E_1}\mathbf{A}_j|h_{sj}|\tilde{\mathbf{I}}\mathbf{x} + \mathbf{A}_j\widehat{\mathbf{z}}_j$, which means that after interleaving, the equivalent linear processing matrix is $\mathbf{A}_j\tilde{\mathbf{I}}$. It is easily verified that if $\mathbf{A}_j$ satisfies (12), (15) and (16), then so does $\mathbf{A}_j\tilde{\mathbf{I}}$ also. □

[3]It should be noted that neither the source nor the relay does an explicit interleaving, but the net effect of the relay matrices is such that the output of relays is an interleaved version of the information symbols.

As an example, consider the Alamouti code $\begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$, whose relay matrices are given by (7). For this case, $N = T_1 = T_2 = 2$. The permutation matrix $\tilde{\mathbf{I}}$ for the coordinate interleaving operation is $\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0 \end{bmatrix}$. The relay matrices for the coordinate interleaved code are $\mathbf{A}_1\tilde{\mathbf{I}} = \begin{bmatrix} 1 & 0 & 0 & \mathbf{j} \\ 0 & \mathbf{j} & -1 & 0 \end{bmatrix}$ and $\mathbf{A}_2\tilde{\mathbf{I}} = \begin{bmatrix} 0 & \mathbf{j} & 1 & 0 \\ 1 & 0 & 0 & -\mathbf{j} \end{bmatrix}$, and the resulting code is $\begin{bmatrix} x_{1I}+jx_{2Q} & x_{2I}+jx_{1Q} \\ -x_{2I}+jx_{1Q} & x_{1I}-jx_{2Q} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 \\ -\tilde{x}_2^* & \tilde{x}_1^* \end{bmatrix}$. Also, for the code in (20) which is SSD for PCRC, if we choose the permutation matrix $\tilde{\mathbf{I}}$ as

$$\tilde{\mathbf{I}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

the resulting code is $CIOD_4$. Hence, $CIOD_4$ is also SSD for PCRC. In general, if we have a code with K complex information symbols which is SSD for PCRC, then we can generate $(2K)!$ codes which are SSD for PCRC by coordinate interleaving.

B. A class of rate-$\frac{1}{2}$ SSD DSTBCs

All the classes of codes discussed so far are STBCs from square designs. It is well known that the rate of square SSD codes for co-located MIMO systems falls exponentially as the number of antennas increases. In this subsection, it is shown that if non-square designs are used then SSD codes for PCRCs can be achieved with rate-$\frac{1}{2}$ for any number of antennas.

It is well known [1] that real orthogonal designs (ROD) with rate one exist for any number of antennas and these are non-square designs for more than 2 antennas and the delay increases exponentially with the number of antennas. Using these RODs, in [1], a class of rate-$\frac{1}{2}$ complex orthogonal designs for any number of antennas is obtained as follows: If $\mathbf{G}$ is a $p \times N$ rate-1 ROD, where p denotes the delay and N denotes the number of antennas with complex variables $x_1, x_2, \cdots, x_p$, (the real variables in the ROD replaced by complex variables) then, denoting by $\mathbf{G}^*$ the complex design obtained by replacing x_i with $x_i^*,\ i = 1, 2, \cdots, p$, the design $\begin{bmatrix} \mathbf{G} \\ \mathbf{G}^* \end{bmatrix}$ is a $2p \times N$ rate-$\frac{1}{2}$ COD. We refer to this construction as stacking construction. The following theorem asserts that the rate-$\frac{1}{2}$ CODs by stacking construction are SSD for PCRC.

Theorem 5: The rate-1/2 CODs, constructed from rate-1 RODs by stacking construction [1] are SSD-DSTBC-PCRC.

Proof: Let $\mathbf{G}_c$ be the rate-1/2 COD obtained from a $p \times N$ ROD $\mathbf{G}$ by stacking construction, i.e., $\mathbf{G}_c = \begin{bmatrix} \mathbf{G} \\ \mathbf{G}^* \end{bmatrix}$. Let the $p \times p$ real matrices $\hat{\mathbf{A}}_j\ j = 1, \cdots, N$ generate the columns of $\mathbf{G}$, i.e., $\mathbf{G} = \left[\hat{\mathbf{A}}_1\mathbf{x}, \hat{\mathbf{A}}_2\mathbf{x}, \cdots, \hat{\mathbf{A}}_N\mathbf{x}\right]$, where $\mathbf{x}$ is the $p \times 1$ real data vector and the matrices $\hat{\mathbf{A}}_j$ denote the column vector representation matrices used in [5]. By the definition of RODs, $\mathbf{G}^T\mathbf{G} = \left(\mathbf{x}^T\mathbf{x}\right)\mathbf{I}$. This implies that

$$\begin{aligned} \hat{\mathbf{A}}_j^T\hat{\mathbf{A}}_j &= \mathbf{I},\ j = 1, \cdots, N \text{ and} \\ \hat{\mathbf{A}}_j^T\hat{\mathbf{A}}_i &= -\hat{\mathbf{A}}_i^T\hat{\mathbf{A}}_j,\ i, j = 1, \cdots, N, i \neq j. \end{aligned}$$

It is noted that the Hurwitz-Radon family of matrices satisfy the above equations and explicit construction for any N is given in [1]. It is noted that the representation in [1] is different from the column vector representation used in this paper. An important consequence is that the Hurwitz-Radon family of matrices satisfy the conditions

$$\begin{aligned} \hat{\mathbf{A}}_j^T\hat{\mathbf{A}}_j &= \mathbf{I}, \text{ and } \hat{\mathbf{A}}_j^T = -\hat{\mathbf{A}}_j,\ j = 1, \cdots, N; \\ \hat{\mathbf{A}}_j\hat{\mathbf{A}}_i &= -\hat{\mathbf{A}}_i\hat{\mathbf{A}}_j,\ i, j = 1, \cdots, N, i \neq j, \end{aligned}$$

and hence $\hat{\mathbf{A}}_j\hat{\mathbf{A}}_j^T = \mathbf{I}\ \forall j$, which we will use in our proof. Viewing $\mathbf{G}_c$ as a $T_2 \times N$ distributed STBC with $T_1 = p$ and $T_2 = 2p$, the $T_2 \times 2T_1$ relay matrices $\mathbf{A}_j$ of $\mathbf{G}_c$ have the structure $\mathbf{A}_{jI} = \begin{pmatrix} \mathbf{U}_j \\ \mathbf{U}_j \end{pmatrix}$ and $\mathbf{A}_{jQ} = \begin{pmatrix} \mathbf{V}_j \\ -\mathbf{V}_j \end{pmatrix}$. Since $\mathbf{G}_c$ is constructed from a ROD, the coefficients of real and imaginary components are same, i.e., the matrices $\mathbf{U}_j$ and $\mathbf{V}_j$ have the form $\mathbf{U}_j = [\gamma_{1,j}, \mathbf{0}, \gamma_{2,j}, \mathbf{0}, \cdots, \gamma_{T_1,j}, \mathbf{0}]$, $\mathbf{V}_j = \left[\mathbf{0}, \gamma_{1,j}, \mathbf{0}, \gamma_{2,j}, \cdots, \mathbf{0}, \gamma_{T_1,j}\right]$, with $\gamma_{i,j}$ are column vectors of $\hat{\mathbf{A}}_j$. Since $\hat{\mathbf{A}}_j\hat{\mathbf{A}}_j^T = \mathbf{I}\ \forall j$, it is easily verified that $\mathbf{U}_j\mathbf{U}_j^T = \mathbf{I}$ and $\mathbf{V}_j\mathbf{V}_j^T = \mathbf{I}\ \forall j$. It is also easily seen that $\mathbf{U}_j\mathbf{V}_j^T = \mathbf{0}$ and $\mathbf{V}_j\mathbf{U}_j^T = \mathbf{0}$. Hence, we have $\mathbf{A}_{jI}{\mathbf{A}_{jI}}^T + \mathbf{A}_{jQ}{\mathbf{A}_{jQ}}^T = 2\mathbf{I}$ and $\mathbf{A}_{jI}{\mathbf{A}_{jQ}}^T + \mathbf{A}_{jQ}{\mathbf{A}_{jI}}^T = \mathbf{0}$, substituting which in (15), we get the left hand side of (15) to be $2\left({\mathbf{A}_{j_1I}}^T\mathbf{A}_{j_3I} + {\mathbf{A}_{j_3}}^T\mathbf{A}_{j_1I} + {\mathbf{A}_{j_1Q}}^T\mathbf{A}_{j_3Q} + {\mathbf{A}_{j_3Q}}^T\mathbf{A}_{j_3Q}\right)$, which, by (12), is always a block diagonal matrix of the form (13). Also the left hand side of (16) is $\mathbf{0}$. Hence, $\mathbf{G}_c$ is SSD for PCRC. □

In [17], it is shown that if the N relays do not have any CSI and the destination has all the CSI, then an upper bound on the rate of distributed SSD codes is $\frac{2}{N}$, which decreases rapidly as the number of relays increases. However, Theorem 5 shows that, if the relay knows only the phase information of the source-relay channels then the lower bound on the rate of the distributed SSD codes is $\frac{1}{2}$, which is independent of the number of relays.

C. Full-diversity, single-symbol non-ML detection

Theorem 6: The PCRC system given by (10) achieves full diversity irrespective of whether the total noise ($\tilde{\mathbf{z}}_d$) is correlated or not, if the STBC achieves full diversity in the co-located case and condition (12) is satisfied.

Proof: Since the noise $\tilde{\mathbf{z}}_d$ is not assumed to be uncorrelated, the optimal detection of $\mathbf{x}$ in the maximum likelihood sense is given by $\widehat{\mathbf{x}} = \arg\min\ (\mathbf{y} - \mathbf{H}_{eq}^{(pc)}\mathbf{x})^{\mathcal{H}}\mathbf{\Omega}^{-1}(\mathbf{y} - \mathbf{H}_{eq}^{(pc)}\mathbf{x})$, where $\mathbf{\Omega}$ is co-variance matrix of the noise, given by $\mathbf{\Omega} = E\{\tilde{\mathbf{z}}_d\tilde{\mathbf{z}}_d^{\mathcal{H}}\}$. We consider the sub-optimal metric (ignoring $\mathbf{\Omega}^{-1}$) $\widehat{\mathbf{x}} = \arg\min\ (\mathbf{y} - \mathbf{H}_{eq}^{(pc)}\mathbf{x})^{\mathcal{H}}(\mathbf{y} - \mathbf{H}_{eq}^{(pc)}\mathbf{x})$, and show that this decision metric achieves full diversity. Proceeding on the similar lines for the proof for the co-located case, the pair-wise error probability is upper bounded by

$$P\left(\mathbf{x}_1 \rightarrow \mathbf{x}_2\right) \leq E\left\{e^{-d^2(\mathbf{x}_1, \mathbf{x}_2)E_t/4}\right\}, \tag{21}$$

where $d^2(\mathbf{x}_1, \mathbf{x}_2) = (\mathbf{x}_2 - \mathbf{x}_1)^T\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\mathbf{H}_{eq}^{(pc)}\right)(\mathbf{x}_2 - \mathbf{x}_1)$. Since (12) is satisfied, this can be written as sum of T_1 terms as

$$d^2(\mathbf{x}_1, \mathbf{x}_2) = \sum_{i=1}^{T_1}\Delta\mathbf{x}^{(i)T}\left(\sum_{j=1}^{N}|h_{sj}|^2|h_{jd}|^2\mathbf{D}_{j,i}^{(1)}\right)\Delta\mathbf{x}^{(i)} \tag{22}$$

$$= \sum_{j=1}^{N}|h_{sj}|^2|h_{jd}|^2\left(\sum_{i=1}^{T_1}\Delta\mathbf{x}^{(i)T}\mathbf{D}_{j,i}^{(1)}\Delta\mathbf{x}^{(i)}\right). \tag{23}$$

Substituting (23) in (21) and evaluating the expectation with respect to $|h_{jd}|^2$, we get

$$P\left(\mathbf{x}_1 \rightarrow \mathbf{x}_2 |\, h_{sj}\right) \leq \prod_{j=1}^{N}\left(\frac{1}{1 + |h_{sj}|^2\sum_{i=1}^{T_1}\Delta\mathbf{x}^{(i)T}\mathbf{D}_{j,i}^{(1)}\Delta\mathbf{x}^{(i)}E_t/4,}\right),$$

which, for high SNRs, could be approximated as

$$P\left(\mathbf{x}_1 \rightarrow \mathbf{x}_2 | h_{sj}\right) \leq \prod_{j=1}^{N} \left(\frac{1}{\sum_{i=1}^{T_1} \Delta \mathbf{x}^{(i)T} \mathbf{D}_{j,i}^{(1)} \Delta \mathbf{x}^{(i)} E_t/4} \right) \prod_{j=1}^{N} \left(\frac{1}{|h_{sj}|^2} \right).$$

Now, evaluating the expectation with respect to $|h_{sj}|$, we get

$$P\left(\mathbf{x}_1 \rightarrow \mathbf{x}_2\right) \leq \prod_{j=1}^{N} \left(\frac{1}{\sum_{i=1}^{T_1} \Delta \mathbf{x}^{(i)T} \mathbf{D}_{j,i}^{(1)} \Delta \mathbf{x}^{(i)} E_t/4} \right) \left(\mathbf{Ei}(0)\right)^N,$$

where $\mathbf{Ei}(x)$ is the exponential integral $\int_x^\infty \frac{e^{-t}}{t} dt$. From the above inequality, it is clear that the condition for achieving maximum diversity is identical to that of co-located MIMO. □

Theorem 6 means that by using any STBC which satisfies the conditions (12) and achieves full diversity in co-located MIMO system, it is possible to do decoding of one symbol at a time and achieve full diversity, though not optimal in the ML sense, in a distributed setup with phase compensation done at the relay, *even if (15) and (16) are not satisfied*. For example, the $CIOD_8$ is SSD and gives full-diversity in a co-located 8-transmit antenna system for any signal set with coordinate product distance (CPD) not equal to zero, and is not SSD for PCRC since it does not satisfy (15) and (16). However, according to Theorem 6 a SSD decoder for $CIOD_8$ in a PCRC will result in full-diversity of order 8.

V. Discussion and Simulation Results

The results of our necessary and sufficient conditions (12), (15) and (16) as well as the sufficient condition in [18], evaluated for various classes of codes for PCRC are shown in Table I. As can be seen from the last column of Table I, the sufficient condition in [18] identifies only COD_2 (Alamouti) and CUW_4 as SSDs for PCRC. However, our conditions (12), (15) and (16) identify $CIOD_4$, RR_8, and CODs from RODs, in addition to COD_2 and CUW_4, as SSDs for PCRC (4th column of Table I). It is noted that, $CIOD_4$ being a construction by using $\mathbf{G} = COD_2$ in (19) and coordinate interleaving, it is SSD for PCRC from *Lemma 2* and *Theorem 4*. Similarly, since RR_8 code is constructed by using $\mathbf{G} = CUW_4$ in (19), it follows from *Lemma 2* that RR_8 is also SSD for PCRC. Also, CODs from RODs are SSD for PCRC from *Theorem 5*. Since COD_4, COD_8, and $CIOD_8$ do not satisfy our conditions, they are not SSD for PCRC.

Next, we present the bit error rate (BER) performance of various classes of codes without and with phase compensation at the relays (i.e., PCRC). For the purposes of the simulation results and discussions in this section, we classify the decoding of codes for PCRC into two categories: i) codes for which single symbol decoding is ML-optimal; we refer to this decoding as ML-SSD; we consider ML-SSD of COD_2 and $CIOD_4$, and ii) codes which when decoded using single symbol decoding are not ML-optimal, but achieve full diversity; we refer to this decoding as non-ML-SSD; we consider non-ML-SSD of COD_4, COD_8, and $CIOD_8$. When no phase compensation is done at the relays, we consider ML decoding.

In Fig. 3, we plot the BER performance for COD_2, COD_4, and COD_8 without and with phase compensation at the relays (i.e., PCRC) for 16-QAM. Note that COD_2 is SSD for PCRC whereas COD_4 and COD_8 are not SSD for PCRC. So decoding of COD_2 with PCRC is ML-SSD, whereas decoding of COD_4 and COD_8 with PCRC is non-ML-SSD. When no phase compensation is done at the relays, we do ML decoding for all COD_2, COD_4, and COD_8. The following observations can be made from Fig. 3: i) COD_2 without and with phase compensation at the relays (PCRC) achieve the full diversity order of 2, ii) COD_2 with PCRC and ML-SSD achieves better performance by about 3 dB at a BER of 10^{-2} compared to ML decoding of COD_2 without phase compensation, and iii) even the non-ML-SSD of COD_4 and COD_8 with PCRC achieves full diversity of 4 and 8, respectively (but not the ML performance corresponding to PCRC), and even with this suboptimal decoding, PCRC achieves about 1 dB and 0.5 dB better performance at a BER of 10^{-2}, respectively, compared to ML decoding of COD_4 and COD_8 without phase compensation at the relays.

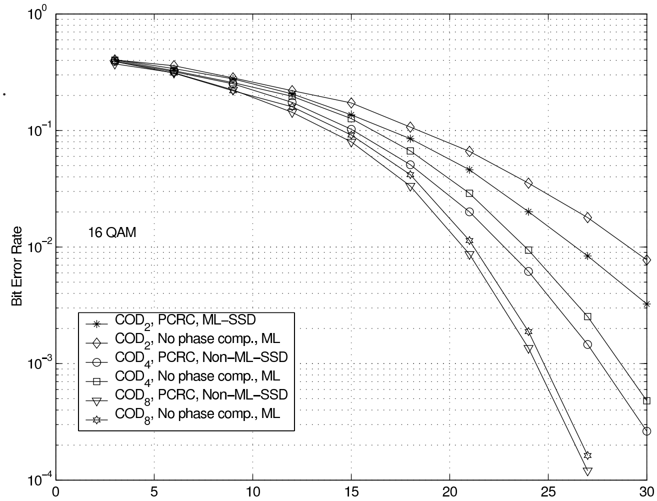

Fig. 3. Comparison of BER performance of COD_2, COD_4, and COD_8 without and with phase compensation at the relays, using 16-QAM.

In Fig. 4, we present a similar BER performance comparison for CIODs without and with phase compensation at the relays. QPSK modulation with 30° rotation of the constellation is used. Here again, both $CIOD_4$ and $CIOD_8$ achieve their full diversities of 4 and 8, respectively. We further observe that $CIOD_4$ (which is SSD for PCRC) with PCRC and ML-SSD achieves better performance by about 3 dB at a BER of 10^{-3} compared to ML decoding of $CIOD_4$ without phase compensation. Likewise, $CIOD_8$ (which is not SSD for PCRC) with PCRC and non-ML-SSD achieves better performance by about 1 dB at a BER of 10^{-3} compared to ML decoding of $CIOD_8$ without phase compensation.

Finally, a performance comparison between CODs and CIODs with PCRC for a given spectral efficiency is presented in Fig. 5. A comparison at a spectral efficiency of 3 bps/Hz is made between i) COD_4 with rate-3/4 and 16-PSK (spectral efficiency = $\frac{3}{4} \times \log_2 16 = 3$ bps/Hz), and ii) $CIOD_4$ with rate-1 and 8-PSK with 10° rotation (spectral efficiency = $1 \times \log_2 8 = 3$ bps/Hz). Likewise, a comparison is made at a spectral efficiency of 1.5 bps/Hz between COD_8 and $CIOD_8$.

TABLE I
TEST FOR NECESSARY AND SUFFICIENT CONDITIONS FOR VARIOUS CLASSES OF CODES FOR PCRC.

Code	Number of Relays	Rate	Necessary and sufficient Conditions (12), (15) & (16)	Sufficient Condition in [18]
COD_2 (Alamouti)	$N = 2$	1	True	True
COD_4	$N = 4$	3/4	False	False
$CIOD_4$	$N = 4$	1	True	False
CUW_4	$N = 4$	1	True	True
COD_8	$N = 8$	1/2	False	False
$CIOD_8$	$N = 8$	3/4	False	False
RR_8	$N = 8$	1	True	False
CODs from RODs	$N \geq 4$	1/2	True	False

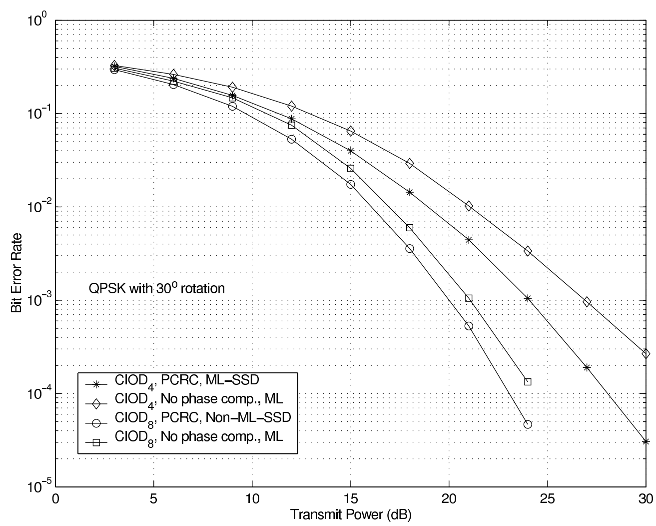

Fig. 4. Comparison of BER performance of $CIOD_4$ and $CIOD_8$ without and with with phase compensation at the relays, using QPSK with 30^o rotation.

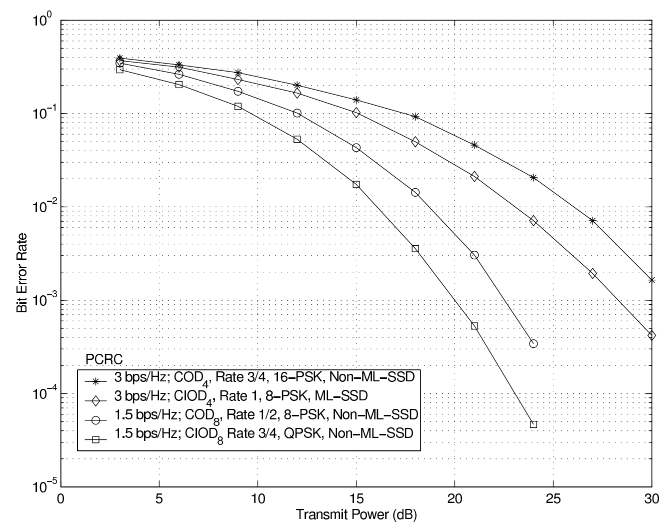

Fig. 5. Comparison of BER performance of CODs and CIODs with phase compensation at the relays (i.e., PCRC) for a given spectral efficiency: $i)$ 3 bps/Hz; rate-3/4 COD_4 with 16-PSK versus rate-1 $CIOD_4$ with 8-PSK (10° rotation), and $ii)$ 1.5 bps/Hz; rate-1/2 COD_8 with 8-PSK versus rate-3/4 $CIOD_8$ with QPSK (30° rotation).

It can be observed that, as in the case of co-located MIMO [8], in distributed STBCs with PCRC also, CIODs perform better than COD, i.e., coordinate interleaving improves performance. All these simulation results reinforce the claims made in the paper in Sec. I.

VI. CONCLUSIONS

We summarize the conclusions in this paper and future work as follows. Amplify-and-forward (AF) schemes in cooperative communications are attractive because of their simplicity. Full diversity (FD), linear-complexity single symbol decoding (SSD), and high rates of DSTBCs are three important attributes to work towards AF cooperative communications. Earlier work in [17] has shown that, without assuming phase knowledge at the relays, FD and SSD can be achieved in AF distributed orthogonal STBC schemes; however, the rate achieved decreases linearly with the number of relays N. Our work in this paper established that if phase knowledge is exploited at the relays in the way we have proposed, then FD, SSD, and high rate can be achieved simultaneously; in particular, the rate achieved in our scheme can be $\frac{1}{2}$, which is independent of the number of relays N. We proved the SSD for our scheme in Theorem 2. FD was proved in Theorem 6. Rate-1/2 construction for any N was presented in Theorem 5. In addition to these results, we also established other results regarding $i)$ invariance of SSD under coordinate interleaving (Theorem 4), and $ii)$ retention of FD even with single-symbol non-ML decoding. Simulation results confirming the claims were presented. All these important results have not been shown in the literature so far. These results offer useful insights and knowledge for the designers of future cooperative communication based systems (e.g., cooperative communication ideas are being considered in future evolution of standards like IEEE 802.16).

In this work, we have assumed only phase knowledge at the relays. One can assume that both amplitude as well as the phase of source-to-relay are known at the relay. A natural question that can arise then is 'what can amplitude knowledge at the relay (in addition to phase knowledge) buy?' Since we have shown that phase knowledge alone is adequate to achieve FD, some extra coding gain may be possible with amplitude knowledge. This aspect of the problem is beyond the scope of this paper; but it is a valid topic for future work.

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