Analysis of Link-Layer Backoff Schemes on Point-to-Point Markov Fading Links

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***Abstract*—Backoff algorithms are typically employed in multiple-access networks (e.g., Ethernet) to recover from packet collisions. In this letter, we propose and carry out the analysis for three types of link-layer backoff schemes, namely, linear backoff, exponential backoff, and geometric backoff, on point-to-point wireless fading links where packet errors occur nonindependently. In such a scenario, the backoff schemes are shown to achieve better energy efficiency without compromising much on the link layer throughput performance.**

***Index Terms*—Backoff algorithms, energy efficiency, fading channels.**

I. Introduction

BACKOFF algorithms are typically used in multiple-access networks to recover from packet collisions. For example, a truncated binary exponential backoff scheme is employed in Ethernet [1]. The backoff delay is increased by larger and larger amounts on each successive collision, up to a finite number of retransmission attempts. In this letter, we propose that backoff schemes could be applied beneficially on point-to-point wireless links, as well. The motivation arises from the potential for substantial energy savings when the link experiences deep fades and bursty errors.

During channel fades, it is likely that a large number of consecutive packets are received in error due to memory in the multipath fading process [2]. A backoff scheme at the link layer (LL), applying an appropriate backoff rule upon each LL packet error event, can leave the channel idle for some specified number of slots, thereby reducing the energy wastage due to packet transmissions in error. Such a backoff is done at a cost of possible reduction in the throughput. However, in mobile stations, saving battery power can outweigh a slight loss in throughput [3].

Nonindependent errors on wireless channels are, with reasonable accuracy, modeled by a first-order Markov chain in most analyses in the literature [4], [5]. LL automatic repeat-request (ARQ) protocols, like Go-Back-N and Selective Repeat, have been analyzed in the presence of nonindependent errors in [6]–[8]. However, all these studies focus only on throughput and delay performance and do not consider energy savings through LL backoff strategies. Our new contribution in this paper is the proposal and throughput-energy efficiency analysis of three easily implementable LL backoff strategies, namely, linear backoff (LBO), binary exponential backoff (BEBO), and geometric backoff (GBO). We adopt a first-order Markov chain representation of the wireless channel. Numerical results show that the proposed backoff schemes achieve better energy efficiency without compromising much on the LL throughput performance.

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II. LL Backoff Algorithms

The proposed LL backoff algorithms are defined as follows.

Linear Backoff (LBO). In an LBO scheme, on the ith successive failure of a packet, the LL leaves the channel idle for i number of subsequent slots, i.e., the backoff delay grows linearly on each successive failure.

Binary Exponential Backoff (BEBO). In this scheme, the LL leaves the channel idle for $2^i - 1$ number of slots on the ith successive failure.

Geometric Backoff (GBO). In this scheme, there is a parameter g, $0 < g \leq 1$. Following an idle or packet failure, the LL leaves the channel idle in the next slot with probability g (or, equivalently, transmits a packet with probability $1 - g$). In other words, the expected number of backoff slots following a failure is given by $g/(1 - g)$.

III. Performance Analysis

Among the LL ARQ protocols, Selective Repeat (SR) gives the best throughput performance. The maximum throughput achieved by the ideal SR scheme is bounded by $(1 - \epsilon)$, where ϵ is the average packet error rate on the link [7]. In this section, we will derive the LL throughput and energy efficiency performance of the proposed backoff schemes and compare them with the performance of the ideal SR scheme without backoff.

Consider bulk data to be transmitted over a point-to-point wireless link. LL packets are of same size. The time axis is split into slots of duration equal to one LL packet duration. Instantaneous and perfect acknowledgement/negative acknowledgement (ACK/NACK) feedback from the receiver is assumed. We employ a first-order Markov chain representation of the wireless channel with Markov parameters p and $(1 - q)$ being the probabilities that the packet transmitted in the kth slot is a success, given the packet transmitted in $(k - 1)$th slot is a success and a failure, respectively.

Using the above Markov channel model, the following probabilities are defined. Define δ_i and γ_i as the probabilities that success and failure occur, respectively, at the $(k+i)$th slot, given the kth slot is a failure for any $k \geq 1$ and $i \geq 1$. Observe that,

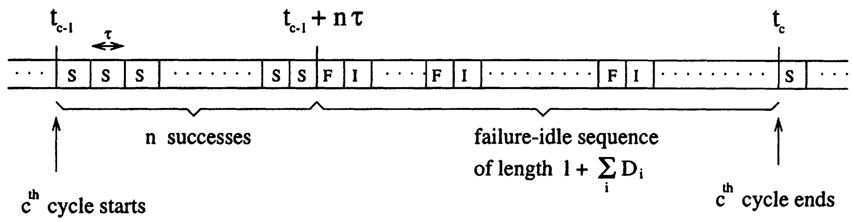

Fig. 1. Transmission cycle in LL backoff schemes.

for any $i \geq 1$, $\delta_i + \gamma_i = 1$. With boundary conditions $\delta_1 = 1 - q$ and $\gamma_1 = q$, we can write recursive relations on δ_i and γ_i as

$$\delta_i = \gamma_{i-1}(1-q) + \delta_{i-1}p, \qquad i \geq 2 \tag{1}$$

$$\gamma_i = \gamma_{i-1}q + \delta_{i-1}(1-p), \qquad i \geq 2 \tag{2}$$

where p and q are the Markov channel parameters defined above.

We calculate the throughput and energy efficiency as the reward rates in a renewal process [9]. Specifically, we count the success, failure, and idle events as rewards in a renewal cycle defined in the following. Assume that the first packet was transmitted at time $t = t_0$. This first transmitted packet may be a success or a failure, depending on the channel condition. Now define a cycle for the case of first packet success (*Case a*) and failure (*Case b*) as follows.

Case a: Consider the case when the first packet transmitted at $t = t_0$ is a success. Let τ be the LL packet duration, assumed constant. Let the first failure occur at $t = t_0 + n\tau$, i.e., first n packets are in success. Define D_i, $i \geq 1$, as the number of slots kept idle after the ith failure of a packet. For example, for the ith failure, D_i is equal to i for LBO and $2^i - 1$for BEBO. Note that i should be reset whenever a new packet is transmitted. Since the $(n+1)$th packet is a failure, the next D_1 slots are kept idle and a packet is transmitted again at $t = t_0 + (n + 1 + D_1)\tau$. If this packet again fails, next D_2 number of slots are kept idle and a packet is transmitted at $t = t_0 + (n+1+D_1+1+D_2)\tau$, and so on. Finally a packet gets through successfully, say, after l number of attempts (i.e., l failures) which was transmitted at $t = t_0 + [n + l + \sum_{i=1}^{l} D_i]\tau = t_1$. Since the packet transmitted at t_1 is a success and we are assuming a first-order Markov channel model, the future evolution of the success–failure–idle sequence is same as that started at t_0. The duration $t_1 - t_0$ is defined as the first cycle. In general, t_i is defined as the epoch at which the $(i+1)$th cycle begins (see Fig. 1).

Case b: Now consider the case when the first packet transmitted at $t = t_0$ is a failure. In this case, the transmitter keeps idle for the next D_1 slots and makes an attempt at the (D_1+1)th slot. Observe that the evolution of the success–failure–idle sequence in this case is identical to the evolution in *Case a* that started at $t = t_0 + n\tau$.

Now define $T_c = t_c - t_{c-1}, c \geq 1$, as the duration of the cth cycle. Since τ is a constant, we normalize T_c with τ (i.e., T_c is the number of slots in a cycle). Define the process $T = \{T_c, c \geq 1\}$. Observe that T is a renewal process [9]. *Case a* explained above corresponds to an ordinary renewal process, whereas *Case b* corresponds to a delayed renewal process. The renewal reward theorem [9], which we apply later in this section, is applicable to both cases.

Let S_c, F_c, and I_c be the random variables representing the number of successes, failures, and idles in cth cycle, respectively. Let n_c be the duration of a cycle, and n_f, n_i, and n_s be the number of failures, idles, and successes, respectively, in a cycle. For BEBO and LBO, the minimum value n_c can take is three, because a cycle should consist of at least one success, one failure, and one idle. Let $f(k)$ be the probability that the length of an effective fade is k, where the effective fade in a cycle is the total duration of the failure-idle sequence (as depicted in Fig. 1) in that cycle, i.e., $k = n_f + n_i$. Observe that, in BEBO and LBO schemes, we have constraint on n_f, say $1 \leq n_f \leq r$, where r is a number dependent on n_c. Also, n_i is dependent on n_f, and n_s is dependent on both n_c and n_f. In other words, for a given cycle length n_c, if the number of failures n_f is fixed, n_i and n_s are also fixed. Now, the expected renewal life time (in number of slots), for $j \geq 2$, is given by

$$E[T_j] = \sum_{n_c=3}^{\infty} n_c \sum_{n_f=1}^{r} p^{(n_s-1)}(1-p)f(n_f+n_i). \tag{3}$$

In the above equation, the factor inside the inner summation is the probability that the cycle length is n_c and the effective fade length is $(n_f + n_i)$. Because of the dependencies of the variables n_c, n_s, n_f, and n_i explained in the previous paragraph, this factor is also equal to $P\{T_c = n_c, S_c = n_s\}$, $P\{T_c = n_c, F_c = n_f\}$, and $P\{T_c = n_c, I_c = n_i\}$. The inner summation gives the probability that a cycle length equals n_c, and the summation of n_c weighted by this probability over all possible values of n_c gives the expected value of the cycle length. Since the probability of the cycle length being large is very small (p and q raised to large numbers tend to zero), the infinite summation can be truncated at a large enough finite value of n_c. Also, the expression for $f(\cdot)$ in the above equation is derived for LBO and BEBO in Section III-A and III-B, respectively.

Expressions for $E[S_j]$, $E[F_j]$, and $E[I_j]$, $j \geq 2$, can be derived in a similar manner, as

$$E[S_j] = \sum_{n_c=3}^{\infty} \sum_{n_f=1}^{r} n_s p^{(n_s-1)}(1-p)f(n_f+n_i) \tag{4}$$

$$E[F_j] = \sum_{n_c=3}^{\infty} \sum_{n_f=1}^{r} n_f p^{(n_s-1)}(1-p)f(n_f+n_i) \tag{5}$$

$$E[I_j] = \sum_{n_c=3}^{\infty} \sum_{n_f=1}^{r} n_i p^{(n_s-1)}(1-p)f(n_f+n_i). \tag{6}$$

Let π_s, π_f, and π_i be the stationary probabilities of success, failure, and idle, respectively. We calculate these stationary probabilities as reward rates of the above renewal process. Take the reward as the total duration of the failure slots. Then, τF_c is the reward in the cth cycle. The expected reward accrued till time t, $c(t)$, is the expected duration of failure slots in $[0, t]$. Now, applying renewal reward theorem [9]

$$\lim_{t\to\infty} \frac{c(t)}{t} = \frac{\tau E[F_j]}{\tau E[T_j]} = \frac{E[F_j]}{E[T_j]}, \qquad j \geq 2. \tag{7}$$

The left-hand side of the above equation gives the stationary probability of failure. In other words, $\pi_f = E[F_j]/E[T_j]$. Similarly, by taking the total success duration and idle duration as the reward, we get the stationary probabilities of success and idle, respectively, as $\pi_s = E[S_j]/E[T_j]$ and $\pi_i = E[I_j]/E[T_j]$.

A. LBO

In LBO, we have $n_i = (n_f(n_f+1))/2$ and $n_s = (n_c - (n_f(n_f+3))/2)$. Since there should be at least one success in a cycle, we have the constraint $(n_c - (n_f(n_f+3))/2) > 0$, which gives $n_f \leq r$, where $r = \lceil (\sqrt{9+8n_c} - 3)/2 \rceil - 1$. Then

$$f\left(\frac{n_f(n_f+3)}{2}\right) = \begin{cases} \delta_2, & n_f = 1 \\ \delta_{n_f+1} \prod_{m=2}^{n_f} \gamma_m, & 1 < n_f \leq r. \end{cases}$$

We can obtain the stationary probabilities π_s, π_f, and π_i for LBO by using these values of n_s, n_i, r, and $f(\cdot)$ in (3)–(6).

B. BEBO

In this backoff scheme, the number of idle slots in a cycle equals $n_i = 2(2^{n_f} - 1) - n_f$, and the number of successes equals $n_s = n_c - 2(2^{n_f} - 1)$. Using the same arguments as in LBO, we have the constraint $n_c - 2(2^{n_f} - 1) > 0$, which gives $n_f \leq r$, where $r = \lceil \log(n_c/2 + 1) \rceil - 1$. Then

$$f\left(2(2^{n_f} - 1)\right) = \begin{cases} \delta_2, & n_f = 1 \\ \delta_{2^{n_f}} \prod_{m=1}^{n_f - 1} \gamma_{2^m}, & 1 < n_f \leq r. \end{cases}$$

Again, using these values of n_s, n_i, r, and $f(\cdot)$ in (3)–(6), we get the stationary probabilities π_s, π_f, and π_i for BEBO.

Truncated BEBO: A variant of BEBO, namely, truncated BEBO (T-BEBO), is also of interest. In T-BEBO, we have a parameter $m > 0$. T-BEBO functions exactly same as the BEBO until m consecutive failures, after which the transmitter gives up the backoff and continues with the transmission of the next LL packet. The analysis for the T-BEBO is similar to BEBO except that the cases in which the number of failures is less than or equal to m and greater than m need to be considered separately. When the number of failures is greater than m (say, $m + k$, $k > 0$), the first m of them will be followed by a certain number (according to the BEBO rule) of idle slots, and the remaining k of them will be consecutive. Then, we can write $n_c - 2(2^m - 1) - k > 0$ which gives $k \leq n_c - 2^{m+1} + 1$. Thus, r takes a value which is the maximum of $(m + n_c - 2^{m+1} + 1)$ and $\lceil \log_2(n_c/2 + 1) \rceil - 1$. When n_f is less than m, n_s and n_i take the same values as that for BEBO without truncation, but when n_f is greater than m, we get $n_i = 2(2^m - 1) - m$ and $n_s = n_c - n_f - n_i$. With these, the expected renewal lifetime for $j \geq 2$ can be written as

$$E[T_j] = \sum_{n_c=3}^{\infty} \sum_{n_f=1}^{r} n_c\, p^{n_s - 1}(1-p) \left(I_{\{n_f \leq m\}} f(2(2^{n_f} - 1)) + I_{\{n_f > m\}} \alpha(m) q^{n_f - m - 1}(1-q) \right) \tag{8}$$

where $\alpha(m) = \prod_{l=1}^{m} \gamma_{2^l}$ and $I_{\{A\}}$ is the indicator function which is one when the event A happens, and zero, otherwise. $E[S_j]$, $E[F_j]$, and $E[I_j]$ for T-BEBO can be obtained by replacing n_c by n_s, n_f, and n_i, respectively, in (8). Note that $m = \infty$ corresponds to BEBO without truncation.

C. GBO

We adopt a different approach to analyze the GBO as follows. Due to the memoryless property of the GBO, the system can be modeled as a finite-state Markov chain with the system status in a slot being any one of success, failure, idle with possible success, and idle with possible failure. Denote these states by s, f, i_s, and i_f, and the stationary probabilities by π_s, π_f, π_{i_s}, and π_{i_f}, respectively. Then, for the Markov chain, the stationary probability vector $\underline{\pi} = [\pi_s\ \pi_f\ \pi_{i_s}\ \pi_{i_f}]$ and the transition probability matrix $\mathbf{P}$ are related by $\underline{\pi} = \underline{\pi}\,\mathbf{P}$, where

$$\mathbf{P} = \begin{bmatrix} p & (1-p) & 0 & 0 \\ (1-g)(1-q) & (1-g)q & g(1-q) & gq \\ (1-g)p & (1-g)(1-p) & gp & g(1-p) \\ (1-g)(1-q) & (1-g)q & g(1-q) & gq \end{bmatrix}. \tag{9}$$

Solving for $\underline{\pi} = \underline{\pi}\,\mathbf{P}$, we get

$$\pi_s = \frac{(1-g)(1-q)}{(2-p-q)(1-gp)} \tag{10}$$

$$\pi_f = \frac{(1-p)}{(2-p-q)(1-gp)} \cdot M \tag{11}$$

and $\pi_i = \pi_{i_s} + \pi_{i_f} = 1 - \pi_s - \pi_f$, where $M = g^2(p+q-1) - g(p+q) + 1$.

For all three backoff schemes, the throughput is given by π_s and the energy efficiency is given by $((\pi_s)/(\pi_s + \pi_f))$, normalized by the fade margin (F) of the link. We define one energy unit as corresponding to the transmission of an LL packet with a fade margin of 0 dB. For the first-order Markov representation of Rayleigh fading, the relation between average packet error rate (ϵ), fade margin (F), and parameters p and q are given by [5] $\epsilon = 1 - e^{-1/F} = (1-p)/(2-p-q)$ and $(1-q) = [Q(\theta, \rho\theta) - Q(\rho\theta, \theta)]/(e^{1/F} - 1)$, where $\theta = \sqrt{2/F(1-\rho^2)}$, $\rho = J_0(2\pi f_d \tau)$ is the correlation coefficient of two samples of the complex amplitude of the Rayleigh fading process[1] taken τ seconds apart, $J_0(\cdot)$ is the Bessel function of the first kind and zeroth order, and $Q(\cdot,\cdot)$ is the Marcum Q function.

For ideal SR with no backoff, energy efficiency is given by $(1-\epsilon)/F$. Note that maximum theoretical energy efficiency is achieved when there are no failed packets at all, i.e., when $\pi_f = 0$, which gives $1/F$ as the theoretical upper bound on the achievable energy efficiency.

[1] f_d is the Doppler bandwidth given by v/λ, where v is the user velocity and λ is the carrier wavelength [2].

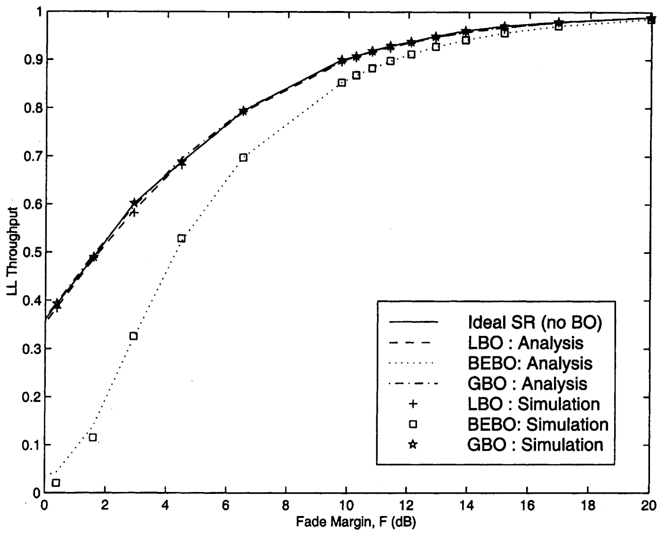

Fig. 2. LL throughput versus fade margin performance of LL backoff schemes at $f_d\tau = 0.001$.

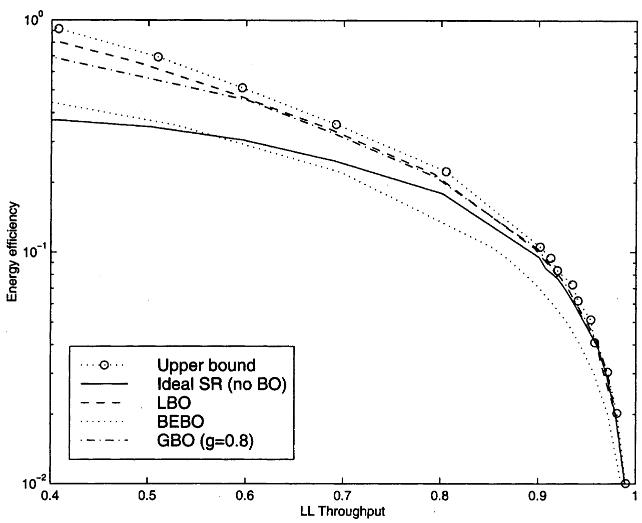

Fig. 3. Energy efficiency versus LL throughput performance of LL backoff schemes at $f_d\tau = 0.001$.

IV. Results

We computed the throughput and energy efficiency for all three backoff schemes using the channel parameters p and q for different fade margins, and a fixed normalized Doppler bandwidth $f_d\tau = 0.001$. The same was found using simulations as well. The parameter g for the GBO is chosen to be 0.8. At 900-MHz carrier frequency, $f_d\tau = 0.001$ corresponds to user speed of 1.2 Km/h, link speed of 1 Mb/s, and LL packet size of 1000 bits. The plots in Fig. 2 show close agreement between analytical and simulation results, thus validating the analysis. Figs. 2 and 3 show that the LBO and GBO schemes show good improvement in terms of energy efficiency without noticeable fall in the throughput, compared to the ideal SR scheme without backoff. At a throughput of 0.4, LBO gives an improvement

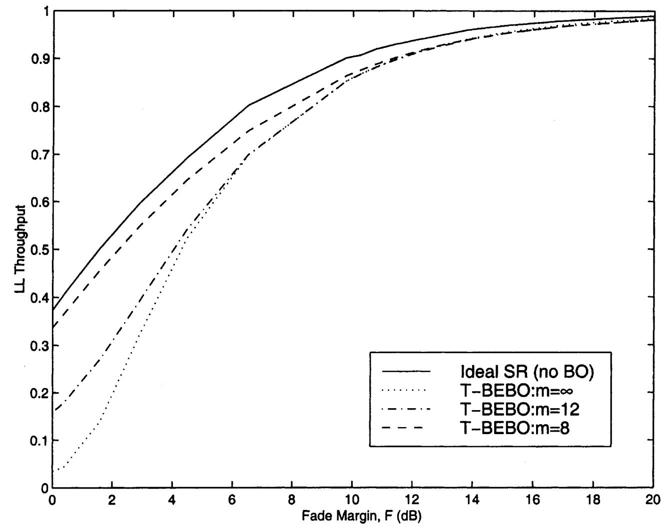

Fig. 4. Performance of T-BEBO for different values of m at $f_d\tau = 0.001$.

of 3.4 dB of energy savings, while GBO gives nearly 2.8 dB. Among all the three schemes, LBO is seen to perform best, achieving energy efficiency close to the theoretical upper bound. The BEBO scheme is seen to perform more poorly. This is due to the rapid growth of exponentiation in backoff delay, which misses possible successful transmission attempts during good state of the channel. This is bound to improve if we use a truncated backoff, in which the idle length should grow only until a maximum value. Such improvement is evident from Fig. 4, which shows the T-BEBO performance for different values of m. The performance analysis of the proposed LL backoff schemes, when used along with transport protocols like TCP and UDP, can be found in [10].

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