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Robust Tomlinson-Harashima Precoders for Multiuser MISO Downlink with Imperfect CSI

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Abstract In this paper, we consider robust non-linear precoding for the downlink of a multiuser multiple-input single-output (MISO) communication system in the presence of imperfect channel state information (CSI). The base station (BS) is equipped with multiple transmit antennas and each user terminal is equipped with a single receive antenna. We propose two robust Tomlinson-Harashima precoder (THP) designs. The first design is based on the minimization of the total BS transmit power under constraints on the mean square error (MSE) at the individual user receivers. We show that this problem can be solved by an iterative procedure, where each iteration involves the solution of a pair of convex optimization problems that can be solved efficiently. A robust linear precoder with MSE constraints can be obtained as a special case of this robust THP. The second design is based on the minimization of a stochastic function of the sum MSE under a constraint on the total BS transmit power. We formulate this design problem as an optimization problem that can be solved by the method of alternating optimization, the application of which results in a second-order cone program that can be numerically solved efficiently. Simulation results illustrate the improvement in performance of the proposed precoders compared to other robust linear and non-linear precoders in the literature.

Keywords Multiuser MISO downlink · Tomlinson-Harashima precoder · Imperfect CSI

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1 Introduction

There has been considerable interest in multiuser multiple-input multiple-output (MIMO) wireless communications in view of their potential for transmit diversity and increased channel capacity [1,2]. Since it is difficult to provide mobile user terminals with large number of antennas due to space constraints, multiuser multiple-input single-output (MISO) wireless communications on the downlink, where the base station (BS) is equipped with multiple transmit antennas and each user terminal is equipped with a single receive antenna has generated significant interest. In such multiuser MISO systems, multiuser interference at the receiver is a crucial issue. One approach to deal with this interference issue is to use multiuser detection at the receivers, which increases the receiver complexity. As an alternate approach, transmit side processing in the form of precoding has been studied widely [2–4]. Several linear precoders such as transmit zero-forcing (ZF) and minimum mean square error (MMSE) filters, and non-linear precoders including Tomlinson-Harashima precoder (THP) have been proposed and widely investigated in the literature [5,6]. Precoding based on QoS criteria are considered in [7,8]. Non-linear precoding strategies, though more complex than the linear strategies, result in improved performance compared to linear pre-processing. Transmit side precoding techniques, linear or non-linear, can render the receiver side processing at the user terminal simpler. However, transmit side precoding techniques require channel state information (CSI) at the transmitter.

Several studies on transmit precoding assume perfect knowledge of CSI at the transmitter. However, in practice, CSI at the transmitter suffers from inaccuracies caused by errors in channel estimation and/or limited, delayed or erroneous feedback. The performance of precoding schemes is sensitive to such inaccuracies [9]. Hence, in practical systems, it becomes important to adopt precoding schemes which are robust in the presence of inaccuracies in CSI.

Several papers in the literature have proposed precoder designs, both linear as well as non-linear, which are robust in the presence of imperfect CSI [10–12]. Linear robust precoding for MISO downlink with signal-to-interference-plus-noise ratio (SINR) constraints with imperfect CSI at the transmitter is considered in [13]. In [14], Payaro et al. consider robust power allocation for fixed beamformers with mean square error (MSE) constraints and formulate the problem as a convex optimization problem. Robust power control for fixed beamformers with SINR constraints is considered in [15]. The approaches adopted in these works are either minimax [16,17] or stochastic. In the former case, the design is conservative but ensures a minimum performance for all values of the uncertain parameter belonging to a predefined uncertainty set. This approach is applicable when the parameter uncertainties are characterized by a predefined uncertainty set. In the latter case, robustness is achieved by optimizing an average or some other appropriate stochastic measure of the performance metric. This approach is possible if the distribution of the parameter uncertainty is available.

In this paper, we propose two robust THP designs. First, we propose a robust THP design based on the minimization of the total BS transmit power under constraints on the MSE at the individual user receivers. The proposed solution to this problem involves an iterative procedure, where each iteration involves the solution of a pair of convex optimization problems that can be solved efficiently. A robust linear precoder with MSE constraints can be obtained as a special case of this robust THP. We point out that this proposed linear precoder with MSE constraints differs from that in [14] in that the proposed precoder performs a *joint* design of the beamforming vector and power allocation, whereas the precoder in [14] computes only the power allocation for different users for given beamforming vectors. Next, we propose a robust THP design based on the minimization of a stochastic function of the sum MSE under a constraint on the total BS transmit power. We formulate this design problem as an optimization problem that can be solved efficiently by the method of alternating optimization (AO) [18]. In this method of optimization, the entire set of optimization parameters is partitioned into non-overlapping subsets, and an iterative sequence of optimizations on these subsets is carried out, which is often simpler compared to simultaneous optimization over all parameters. In our problem, the application of the AO method results in a second-order cone program (SOCP) which can be numerically solved efficiently. The proposed non-linear precoders are shown to be robust to imperfect CSI. Simulation results illustrate the improvement in performance of the proposed precoders compared to other robust linear and non-linear precoders in the literature.

The rest of the paper is organized as follows. In Sect. 2, we present the system model. The proposed MSE-constrained robust THP design is presented in Sect. 3. The proposed total transmit power-constrained robust THP design is presented in Sect. 4. Performance results and comparisons are presented in Sect. 5. Conclusions are presented in Sect. 6.

2 System Model

We consider a multiuser MISO system, where a BS communicates with N_u users on the downlink. A block diagram of the system considered is shown in Fig. 1. The BS employs N_t transmit antennas and each user is equipped with one receive antenna. Let $\mathbf{u} \in \mathbb{C}^{N_\mathrm{u}\times 1}$ denote[1] the $N_\mathrm{u} \times 1$ data symbol vector, where u_i, $i = 1, 2, \ldots, N_\mathrm{u}$, denotes the data symbol meant for user i. The output of the precoding operation is represented by vector $\mathbf{x} \in \mathbb{C}^{N_\mathrm{t}\times 1}$, where x_j, $j = 1, 2, \ldots, N_\mathrm{t}$, denotes the symbol transmitted on the jth transmit antenna. The received signal at user i, denoted by y_i, can be written as

$$y_i = \mathbf{h}_i^H \mathbf{x} + n_i, \tag{1}$$

where $\mathbf{h}_i \in \mathbb{C}^{N_\mathrm{t}\times 1}$ denotes channel gain vector between the transmitter and the ith user receiver, and n_i is an independent and identically distributed (i.i.d) complex Gaussian random variable with zero mean and variance σ_n^2, representing the additive noise at the ith user receiver. The components of $\mathbf{h}_i$ are assumed to be i.i.d zero mean and unit variance complex Gaussian random variables. The received signals at all the user nodes can be represented in vector form as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_{N_\mathrm{u}}]^H$, $\mathbf{y} = [y_1, y_2, \ldots, y_{N_\mathrm{u}}]^T$, $\mathbf{x} = \left[x_1, x_2, \ldots, x_{N_\mathrm{t}}\right]^T$, and $\mathbf{n} = \left[n_1, n_2, \ldots, n_{N_\mathrm{u}}\right]^T$.

2.1 CSI Error Model

We consider the following models for the CSI error. Consider that the transmitter CSI $\widehat{\mathbf{H}}$ is related to the true channel $\mathbf{H}$ as

$$\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}, \tag{3}$$

[1] We use the following notation: Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[\cdot]^T$, $[\cdot]^H$, and $[\cdot]^\dagger$ denote transpose, Hermitian, and pseudo-inverse operations, respectively. $[\mathbf{A}]_{ij}$ denotes the element on the ith row and jth column of the matrix $\mathbf{A}$. vec$(\cdot)$ operator stacks the columns of the input matrix into one column-vector. $\mathbf{A} \succeq \mathbf{B}$ implies $\mathbf{A} - \mathbf{B}$ is positive semi-definite. $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

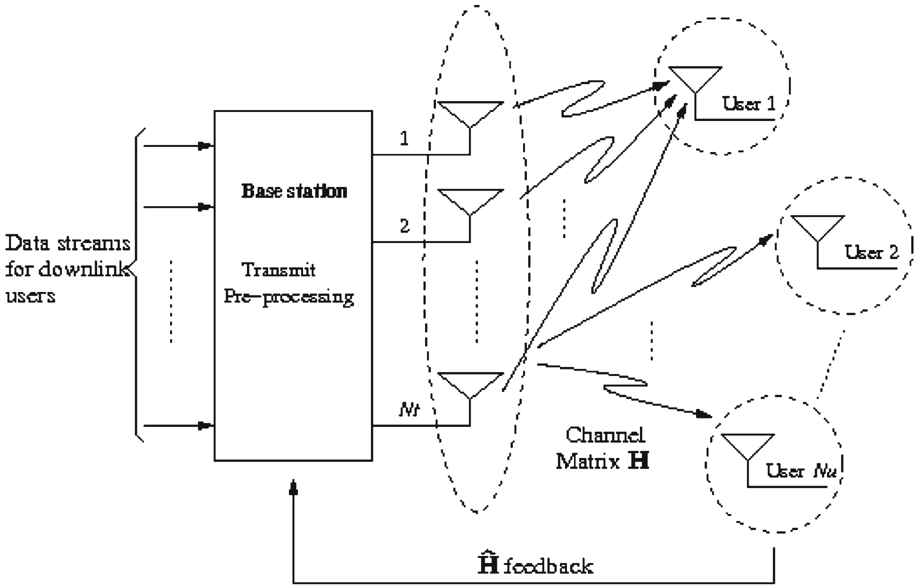

Fig. 1 Multiuser MISO downlink with imperfect CSI at the transmitter

where $\mathbf{E} = [\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_{N_\mathrm{u}}]^H$ denotes CSI error matrix. In a norm-bounded error (NBE) model,

$$\|\mathbf{e}_k\| \leq \delta_k, \quad 1 \leq k \leq N_\mathrm{u}, \tag{4}$$

or equivalently, the true channel $\mathbf{h}_k$ belongs to the uncertainty set $\mathcal{R}_k$ given by

$$\mathcal{R}_k = \{\boldsymbol{\zeta} | \boldsymbol{\zeta} = \widehat{\mathbf{h}}_k + \mathbf{e}_k, \|\mathbf{e}_k\| \leq \delta_k\}, \quad 1 \leq k \leq N_\mathrm{u}. \tag{5}$$

An alternate CSI error model is a stochastic error (SE) model, where $\widehat{\mathbf{H}}$ is an imperfect estimate of the true channel matrix $\mathbf{H}$, and $\mathbf{E}$ is an error matrix of i.i.d complex Gaussian random variables with zero mean and $\mathbb{E}\{\mathbf{E}\mathbf{E}^H\} = \sigma_\mathbf{E}^2 \mathbf{I}$. When the transmitter performs the channel estimation in systems where channel reciprocity holds (e.g., as in TDD systems), it is suitable to adopt the SE model for CSI error. But, when the transmitter obtains the CSI through a feedback channel from the receiver (e.g., as in FDD systems), the CSI error is mainly due to quantization. In this case, the NBE model represented by (3)–(5) is suitable. Both models have been employed in robust precoder designs reported in the literature [11,14,15]. In this paper, we use the NBE model in Sect. 3 and the SE model in Sect. 4.

2.2 Tomlinson-Harashima Precoder

We consider the well known Tomlinson-Harashima precoder as the transmit side pre-processor [19]. The block diagram of a THP is shown in Fig. 2. $\mathbf{B} \in \mathbb{C}^{N_\mathrm{t} \times N_\mathrm{u}}$ is the precoding matrix. The matrix $\mathbf{G} \in \mathbb{C}^{N_\mathrm{u} \times N_\mathrm{u}}$ essentially performs successive interference cancellation at the transmitter and the modulo device $M(\cdot)$ ensures that the resulting values are within an acceptable range. The matrix $\mathbf{G}$ is strictly lower triangular. As the receivers are decentralized, the receiver matrix $\mathbf{A}$ is diagonal with $[\mathbf{A}]_{ii} = \alpha_i$, $1 \leq i \leq N_\mathrm{u}$. The modulo operation is performed at the receivers in order to cancel the effect of modulo operation at

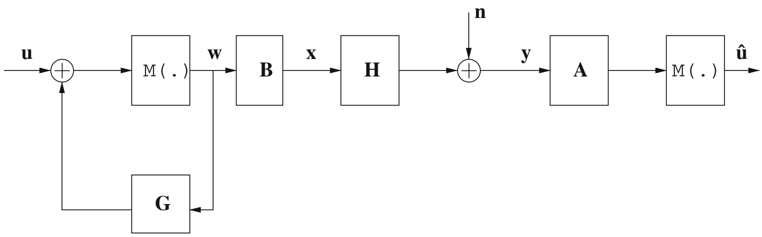

Fig. 2 Matrix representation of Tomlinson-Harashima precoder

the transmitter. The estimate of u_i at user i can be written as [20]

$$\begin{aligned}\widehat{u}_i &= \big(\alpha_i \mathbf{h}_i^H \mathbf{B}\mathbf{w} + \alpha_i n_i\big) \bmod M \\ &= \big(\alpha_i \mathbf{h}_i^H \mathbf{B}\mathbf{w} + \mathbf{g}_i \mathbf{w} - \mathbf{g}_i \mathbf{w} + \alpha_i n_i\big) \bmod M \\ &= \big(\alpha_i \mathbf{h}_i^H \mathbf{B}\mathbf{w} - \mathbf{i}_i \mathbf{w} - \mathbf{g}_i \mathbf{w} + u_i + \alpha_i n_i\big) \bmod M, \end{aligned} \tag{6}$$

where $\mathbf{w}$ is the input vector to the precoder, $\mathbf{g}_i$ is the ith row of $\mathbf{G}$, $\mathbf{i}_i$ is the ith row of $\mathbf{I}_{N_u \times N_u}$ identity matrix, and the third equality follows from the fact that $(\mathbf{g}_i \mathbf{w}) \bmod M = (u_i - w_i) \bmod M$. Neglecting the loss due to the modulo operation, the error between the transmitted symbol and its estimate can be written as

$$\widehat{u}_i - u_i = \big(\alpha_i \mathbf{h}_i^H \mathbf{B} - \mathbf{i}_i - \mathbf{g}_i\big)\mathbf{w} + \alpha_i n_i. \tag{7}$$

Assuming $\mathbb{E}\{\mathbf{w}\mathbf{w}^H\} = \mathbf{I}_{N_u N_u}$ [19], and $\mathbf{w}$ and n_i are independent, the MSE μ_i can be written as

$$\mu_i = \mathbb{E}\{|\widehat{u}_i - u_i|^2\} = \big\|[\alpha_i \mathbf{h}_i^H \mathbf{B} - \mathbf{i}_i - \mathbf{g}_i \quad \alpha_i \sigma_i]\big\|^2. \tag{8}$$

3 Proposed Robust THP with MSE Constraints

3.1 THP Design with Perfect CSI

In this subsection, we consider the design of a THP which transmits the minimum power required to meet the MSE constraints at the user terminals. The MSE constraints are of the form $\mu_i \le \gamma_i$, $1 \le i \le N_u$, where γ_i is the maximum allowed MSE at the ith user terminal. The total transmit power is

$$P_T = \mathbb{E}\big\{\mathbf{x}^H \mathbf{x}\big\} \tag{9}$$

$$= \mathbb{E}\big\{\mathbf{w}^H \mathbf{B}^H \mathbf{B}\mathbf{w}\big\} \tag{10}$$

$$= \text{trace}\big(\mathbf{B}^H \mathbf{B}\big) \tag{11}$$

$$= ||\mathbf{b}||^2, \tag{12}$$

where $\mathbf{b} = \text{vec}(\mathbf{B})$. The problem of designing a precoder which transmits minimum power in order to meet the MSE constraints at the user terminals can be written as

$$\begin{aligned} \min_{\mathbf{B},\alpha_i} \quad & \text{trace}\{\mathbf{B}^H\mathbf{B}\} \\ \text{subject to} \quad & \mu_i(\mathbf{h}_i, \mathbf{B}, \alpha_i) \leq \gamma_i, \\ & 1 \leq i \leq N_\text{u}. \end{aligned} \tag{13}$$

Based on (8), the constraint in (13) can be equivalently written as the following second-order cone (SOC) constraint:

$$\left\| \left[\mathbf{h}^H\mathbf{B} - \beta_i \mathbf{i}_i - \mathbf{q}_i \quad \sigma_i \right] \right\| \leq \beta_i \sqrt{\gamma_i}, \tag{14}$$

where $\beta_i = 1/\alpha_i$, $\mathbf{q}_i = \beta_i \mathbf{g}_i$. Here, without loss of generality, we have assumed that α_i is a positive number, as any arbitrary phase factor of α_i can be absorbed into $\mathbf{B}$. Introducing a slack variable τ, the precoder design problem (13) can now be written as the following SOCP:

$$\begin{aligned} \min_{\mathbf{B},\mathbf{q}_i,\beta_i} \quad & \tau \\ \text{subject to} \quad & \left\| \mathbf{b} \right\| \leq \tau, \\ & \left\| \left[\mathbf{h}_i^H\mathbf{B} - \beta_i \mathbf{i}_i - \mathbf{q}_i \quad \sigma_i \right] \right\| \leq \beta_i \sqrt{\gamma_i} \\ & 1 \leq i \leq N_\text{u}. \end{aligned} \tag{15}$$

This is a convex optimization problem which can be efficiently solved. When the CSI available at transmitter is imperfect, this precoder design results in degraded performance. The robust precoder design proposed in the following section takes the CSI imperfections into account.

3.2 Proposed THP Design with Imperfect CSI

In this subsection, we consider THP design with MSE constraints in the presence of imperfect CSI. Here, we use NBE model to characterize the CSI error. The transmitter has knowledge of only the estimate of the channel and the size of the uncertainty region. The true channel, unknown to the transmitter, may lie anywhere in the uncertainty region. In order to ensure, a priori, that the MSE constraints are met for the actual channel, the robust precoder should be so designed that the constraints are met for all members of the uncertainty set. This problem can be stated as

$$\begin{aligned} \min_{\mathbf{B},\alpha_i} \quad & \text{trace}\{\mathbf{B}^H\mathbf{B}\} \\ \text{subject to} \quad & \max_{\mathbf{h}_i \in \mathcal{R}_i} \mu_i(\mathbf{h}_i, \mathbf{B}, \alpha_i) \leq \gamma_i, \\ & 1 \leq i \leq N_\text{u}. \end{aligned} \tag{16}$$

As a tractable approach to solve this problem, we consider iterating over pessimization and optimization steps till we ensure that the MSE constraints are met for all channels belonging to the uncertainty set. In the pessimization step, we compute the worst case channel which violates the constraint in (16) for fixed values of $\mathbf{B}$ and α_i. In the optimization step, we solve (16) for $\mathbf{B}$ and α with $\mathbf{h}_i$, $1 \leq i \leq N_\text{u}$ belonging to a set with a finite number of elements. This approach was introduced in [21] in the context of robust linear beamforming with SINR constraints. Here, we adopt this approach for the robust THP design with MSE constraints. The pessimization or worst case analysis step involves finding those channels in uncertainty region which violates the MSE constraints for a given precoder and the optimization step involves computing the precoder which meets the MSE constraints for all channels found in the pessimization step. Both these steps can be formulated as convex optimization problems which can be solved efficiently.

3.2.1 Pessimization

In the pessimization step, we want to find a $\mathbf{h}_i \in \mathcal{R}_i$, for a given $\mathbf{B}$ and α_i, which will violate the MSE requirement. For each user i, $1 \leq i \leq N_\mathrm{u}$, this can be found by solving the following optimization problem:

$$\begin{aligned} \max_{\mathbf{h}_i} \quad & \mu_i(\mathbf{h}_i, \mathbf{B}, \alpha_i) \\ \text{subject to} \quad & \mathbf{h}_i \in \mathcal{R}_i. \end{aligned} \tag{17}$$

If $\mathbf{h}_i^{\mathrm{opt}}$ is a solution to the optimization problem (17), and $\mu_i(\mathbf{h}_i^{\mathrm{opt}}, \mathbf{B}, \alpha_i) > \gamma_i$, then $\mathbf{h}_i^{\mathrm{opt}}$ violates the MSE constraint for a given $\mathbf{B}$ and α_i. If $\mu_i(\mathbf{h}_i^{\mathrm{opt}}, \mathbf{B}, \alpha_i) \leq \gamma_i$, then all the channel vectors in $\mathcal{R}_i$ satisfy the MSE constraints.

Equation 8 can be expanded to express the MSE as a quadratic function of $\mathbf{h}_i$, as

$$\mu_i = \alpha_i^2 \mathbf{h}_i^H \widetilde{\mathbf{B}} \mathbf{h}_i - 2\Re\left(\mathbf{h}_i^H \mathbf{B}(\alpha_i \mathbf{i}_i + \mathbf{g}_i)\right) + \|\mathbf{g}_i\|^2 + \alpha_i^2 + \alpha_i^2 \sigma_i^2, \tag{18}$$

where $\widetilde{\mathbf{B}} = \mathbf{B}\mathbf{B}^H$ and $\Re(\cdot)$ represents the real part of the argument. The optimization problem in (17) is equivalent to the following problem:

$$\begin{aligned} \min_{\mathbf{h}_i} \quad & \widetilde{\mu}_i \\ \text{subject to} \quad & \mathbf{h}_i^H \mathbf{h}_i - 2\Re\{\mathbf{h}_i^H \widehat{\mathbf{h}}_i\} + \widehat{\mathbf{h}}_i^H \widehat{\mathbf{h}}_i \leq \delta_i^2, \end{aligned} \tag{19}$$

where $\widetilde{\mu}_i = 2\Re\left(\mathbf{h}_i^H \mathbf{B}(\beta_i \mathbf{i}_i + \mathbf{q}_i)\right) - \mathbf{h}_i^H \widetilde{\mathbf{B}} \mathbf{h}_i - \|\mathbf{q}_i\|^2 - \beta_i^2 - \sigma_i^2$. The optimization problems in (17) and (19) are equivalent as the maximization of μ_i and minimization of $\widetilde{\mu}_i = -\mu_i/\alpha_i^2$ over $\mathbf{h}_i$ under the given constraints have the same optimal solution. The Lagrangian associated with (19) is

$$\begin{aligned} \mathcal{L}_i(\mathbf{h}_i, \lambda_i) &= \widetilde{\mu}_i(\mathbf{h}_i, \mathbf{B}, \alpha_i) + \lambda_i(||\mathbf{h}_i - \widehat{\mathbf{h}}_i||^2 - \delta_i^2), \\ &= \mathbf{h}_i^H \mathbf{Z}_i \mathbf{h}_i + 2\Re\{\mathbf{h}_i^H \mathbf{p}_i\} + \lambda_i ||\widehat{\mathbf{h}}_i||^2 - \|\mathbf{q}_i\|^2 - \beta_i^2 - \sigma_i^2 - \lambda_i \delta_i, \end{aligned} \tag{20}$$

where $\mathbf{Z}_i = \lambda_i \mathbf{I} - \widetilde{\mathbf{B}}$, $\mathbf{p}_i = \mathbf{B}(\beta_i \mathbf{i}_i + \mathbf{q}_i) - \lambda_i \widehat{\mathbf{h}}_i$, and λ_i is the dual variable. The dual function associated with (19) is

$$\begin{aligned} \phi_i(\lambda_i) &= \inf_{\mathbf{h}_i} L_i(\mathbf{h}_i, \lambda_i) \\ &= \begin{cases} -\mathbf{p}_i^H \mathbf{Z}_i^{\dagger} \mathbf{p}_i + \lambda_i(\|\mathbf{h}\|^2 - \delta_k) - \|\mathbf{q}_i\|^2 - \beta_i^2 - \sigma_i^2, \\ \qquad\qquad \text{if } \mathbf{Z}_i \succeq 0 \text{ and } \mathbf{p}_i \in \text{ Range}(\mathbf{Z}_i) \\ -\infty \qquad\qquad\qquad \text{otherwise.} \end{cases} \end{aligned} \tag{21}$$

The dual problem of (19) can be written as the following semi-definite program (SDP) [22]

$$\begin{aligned} \max \quad & \tau \\ \text{subject to} \quad & \lambda \geq 0 \\ & \mathbf{\Phi}_i \succeq 0, \end{aligned} \tag{22}$$

where

$$\mathbf{\Phi}_i = \begin{bmatrix} \mathbf{Z}_i & \mathbf{p}_i \\ \mathbf{p}_i^H & \lambda_i(\|\mathbf{h}\|^2 - \delta_k) - \|\mathbf{q}_i\|^2 - \beta_i^2 - \sigma_i^2 - \tau \end{bmatrix}.$$

As strong duality holds in the present problem, both primal and dual problems have the same optimal value. The worst case channel is

$$\mathbf{h}_i^{\text{opt}} = -\mathbf{Z}^{\dagger}\left(\mathbf{B}(\beta_i \mathbf{i}_i + \mathbf{q}_i) - \lambda_i^{\text{opt}} \widehat{\mathbf{h}}_i\right), \tag{23}$$

where λ_i^{opt} is the optimal solution of the dual problem (22).

3.2.2 Iterative Procedure to Solve (16)

The proposed robust precoder design involves iterating over a sequence of optimization and pessimization steps till a stopping criterion is met. Consider the set $\mathcal{H}$ of channels which violate the MSE constraints. Initially, this set contains only the channel estimates available at the transmitter. The first step involves the solution of the optimization problem in (15) for all elements of the set $\mathcal{H}$. This step provides $\mathbf{B}$, $\mathbf{G}$, and α. The second step is the pessimization (22). If the resulting optimal solution $\mathbf{h}^{\text{opt}}$ violates the MSE constraint this channel vector is added to $\mathcal{H}$. These two steps are iterated till the worst case analysis produces no more channels violating the MSE constraints. This iterative procedure has been shown to converge [21]. From our simulations, it is found that the procedure converges in about 10 iterations, each iteration having an optimization and a pessimization step. In the kth iteration, pessimization step involves an SDP with N_u constraints, each of size $(N_\text{t}+1) \times (N_\text{t}+1)$. The optimization step involves an SOCP with N_k constraints, each of size $(N_\text{u}+2) \times 1$, where N_k is of the cardinality of $\mathcal{H}$ in the kth iteration. The performance results for this proposed THP design are presented in Sect. 5. We note that a robust linear precoder with MSE constraints can be obtained as a special case of this proposed THP by setting $\mathbf{G}=\mathbf{0}$. In Sect. 5, we will compare the performance of this linear precoder with that in [14].

4 Robust THP Design with Total Transmit Power Constraint

In this section, we present a robust THP design under a constraint on the total BS transmit power. In this case, we use the SE model to characterize the CSI error, and adopt a stochastic approach to the robustification. In this case, the matrices $\mathbf{B}$ and $\mathbf{G}$ are chosen to minimize a stochastic function of the sum-MSE (SMSE), taking into account the known distribution of the CSI error. We consider the SMSE averaged over the CSI error as the objective function to be minimized. In order to simplify the analysis, we consider a receiver which decides in favor of u_k when the distance between the scaled version of the received symbol $c\widehat{y}$ and u_k is less than those to other constellation points [10]. The scaling factor c depends on the channel and the power constraint.

Consider the linear representation of the modulo device [19], as shown in Fig. 3. The signal vectors $\mathbf{a}$ and $\mathbf{d}$ in Fig. 3 are introduced to satisfy the requirement that $\mathbf{w}$ has the same value as in the case of modulo operation. The modulo operation at the transmitter alters the statistics of the precoded symbols. Let $\mathbf{\Phi_w} = E\{\mathbf{w}\mathbf{w}^H\}$ and $\mathbf{\Phi_u} = E\{\mathbf{u}\mathbf{u}^H\}$. $\mathbf{\Phi_w}$ is a diagonal matrix, and $[\mathbf{\Phi_w}]_{ii} = \frac{M}{(M-1)}[\mathbf{\Phi_u}]_{ii}$, $i = 1, \ldots, N_\text{u}$. Based on the above development, the SMSE, μ, between the scaled value of the symbol vector $\mathbf{d}$ and the received vector $\mathbf{y}$, is given by

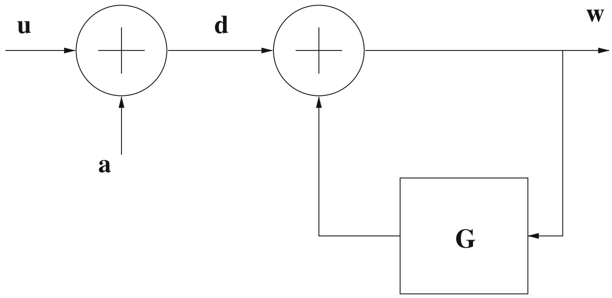

Fig. 3 Linear representation of the modulo device

$$\begin{aligned}\mu &= \mathbb{E}\left\{ \parallel c\mathbf{d} - \mathbf{y} \parallel^2 \right\} \\ &= \mathbb{E}_{\mathbf{w},\mathbf{n}}\left\{ \parallel c\mathbf{d} - \mathbf{HBw} + \mathbf{n} \parallel^2 \right\},\end{aligned} \tag{24}$$

where the 2nd step in (24) follows from $\mathbf{x} = \mathbf{Bw}$ (as seen from Fig. 2), and the expectation is over the noise vector, $\mathbf{n}$, and the modified symbol vector, $\mathbf{w}$. Also, from Fig. 3, it can be seen that $\mathbf{d} = (\mathbf{I} + \mathbf{G})\mathbf{w}$. Substituting $(\mathbf{I} + \mathbf{G})\mathbf{w}$ for $\mathbf{d}$ in (24), μ can be written as

$$\begin{aligned}\mu &= \mathbb{E}_{\mathbf{w}}\left\{\mathbf{w}^H (c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})^H (c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})\mathbf{w}\right\} + N_{\mathrm{u}}\sigma_n^2 \\ &= \mathrm{trace}((c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})^H (c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})) + N_{\mathrm{u}}\sigma_n^2,\end{aligned} \tag{25}$$

where we have assumed $\Phi_{\mathbf{w}} = \mathbf{I}_{N_{\mathrm{u}}}$.

Under imperfect CSI, since $\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}$, in order to make the design robust, we consider the SMSE in (25) averaged over the error matrix $\mathbf{E}$ (in addition to averaging over $\mathbf{w}$ and $\mathbf{n}$) as the performance metric for the optimization. This optimization is performed subject to a total transmit power constraint. As the last term in (25) is a constant, it can be dropped from the objective function. Now, taking the expectation of (25) over $\mathbf{E}$, we get

$$\begin{aligned}&\mathbb{E}_{\mathbf{E},\mathbf{w}}\left\{\mathbf{w}^H (c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})^H (c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})\mathbf{w}\right\} \\ &\quad = \mathbb{E}_{\mathbf{E}}\{\mathrm{trace}\big((c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})^H (c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})\Phi_{\mathbf{w}}\big)\} \\ &\quad = \mathbb{E}_{\mathbf{E}}\{\mathrm{trace}\big((c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})^H (c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})\big)\}.\end{aligned} \tag{26}$$

Based on the above, we can formulate the proposed robust THP design problem as the following constrained optimization program:

$$\begin{aligned}\min_{\mathbf{B},\mathbf{G},c}\quad & \mathbb{E}_{\mathbf{E}}\left\{\mathrm{trace}\big((c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})^H (c(\mathbf{I} + \mathbf{G}) - \mathbf{HB})\big)\right\} \\ \text{subject to}\quad & \mathrm{trace}\big(\mathbf{B}^H\mathbf{B}\big) \le P_{\mathrm{th}}, \\ & c \ge c_{\mathrm{th}},\end{aligned} \tag{27}$$

where P_{th} is the maximum allowed total BS transmit power. The scaling factor c is lower-bounded in order to avoid the trivial solution of $c = 0$ and $\mathbf{B} = 0$. Since c effectively represents the magnitude of the received signal [10], c_{th} can be taken to be the average received signal magnitude resulting from the use of the THP in [6]. Let $\mathbf{g} = \mathrm{vec}((\mathbf{I} + \mathbf{G}))$, $\mathbf{b} = \mathrm{vec}(\mathbf{B})$, and $\mathbf{A} = \mathbf{I} \otimes (\widehat{\mathbf{H}} + \mathbf{E}) = \widehat{\mathbf{A}} + \tilde{\mathbf{A}}$. Then,

$$\mathrm{vec}(c(\mathbf{I} + \mathbf{G}) - (\widehat{\mathbf{H}} + \mathbf{E})\mathbf{B}) = c\mathbf{g} - \mathbf{Ab}. \tag{28}$$

Substituting $\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}$ and using (28) in (26), the objective in (27) can be rewritten as

$$\begin{aligned}&\mathbb{E}_{\mathbf{E}}\big\{\text{trace}\big((c(\mathbf{I}+\mathbf{G}) - (\widehat{\mathbf{H}}+\mathbf{E})\mathbf{B})^H (c(\mathbf{I}+\mathbf{G}) - (\widehat{\mathbf{H}}+\mathbf{E})\mathbf{B})\big)\big\}\\ &\quad = \mathbb{E}_{\mathbf{E}}\big\{(c\mathbf{g} - \mathbf{A}\mathbf{b})^H (c\mathbf{g} - \mathbf{A}\mathbf{b})\big\}\\ &\quad = \| \widehat{\mathbf{A}}\mathbf{b} - c\mathbf{g} \|^2 + \mathbf{b}^H E_{\mathbf{E}}\big\{\tilde{\mathbf{A}}^H \tilde{\mathbf{A}}\big\}\mathbf{b}\\ &\quad = \| \widehat{\mathbf{A}}\mathbf{b} - c\mathbf{g} \|^2 + \sigma_{\mathbf{E}}^2 \| \mathbf{b} \|^2 . \end{aligned} \tag{29}$$

In the above, both $\mathbf{b}$ and $\mathbf{g}$ are optimization parameters, whereas in a linear precoder only $\mathbf{b}$ is the optimization parameter. Using (29), and noting that the second constraint in (27) is active at optimality, the optimization problem in (27) can be rewritten as

$$\begin{aligned}&\min_{\mathbf{b},\mathbf{g}} J(\mathbf{b},\mathbf{g}) \stackrel{\triangle}{=} \| \widehat{\mathbf{A}}\mathbf{b} - c_{\text{th}}\mathbf{g} \|^2 + \sigma_{\mathbf{E}}^2 \| \mathbf{b} \|^2 \\ &\text{subject to } \| \mathbf{b} \|^2 \leq P_{\text{th}}.\end{aligned} \tag{30}$$

4.1 Alternating Optimization

As the optimization has to be performed with respect to both $\mathbf{b}$ and $\mathbf{g}$ in (30), we can use the method of alternating optimization [18], wherein the optimization over an entire set of variables is replaced by a sequence of easier optimizations involving grouped subsets of the variables. In the present problem, we partition the optimization set $\{\mathbf{b}, \mathbf{g}\}$ into the non-overlapping subsets $\{\mathbf{b}\}$ and $\{\mathbf{g}\}$ and perform the optimization with respect to these subsets in an alternating fashion. We note that a semi-closed form solution to (30) is presented in [11] which involves a numerical root-finding.

The algorithmic form of the alternating optimization for the computation of the matrices $\mathbf{G}$ and $\mathbf{B}$ is shown in Table 1. At the $(n + 1)$th iteration, the value of $\mathbf{b}$ is the solution to the following problem

$$\mathbf{b}^{n+1} = \underset{\mathbf{b}}{\text{argmin}}\ J(\mathbf{b}, \mathbf{g}^n), \tag{31}$$

where $\mathbf{b}$ satisfies the constraint in (30). This problem can be efficiently solved as a second order cone program (SOCP) [23]. Having computed $\mathbf{b}^{n+1}$, $\mathbf{g}^{n+1}$ is the solution to the following problem:

$$\mathbf{g}^{n+1} = \underset{\mathbf{g}}{\text{argmin}}\ J(\mathbf{b}^{n+1}, \mathbf{g}). \tag{32}$$

This problem has the following solution:

$$\mathbf{g}^{n+1} = \Big(\frac{1}{c_{\text{th}}}\Big)\text{vec}\big(\text{tril}\big(\text{mat}\big(\mathbf{A}\mathbf{b}^{n+1}\big)\big) + \mathbf{I}\big), \tag{33}$$

where mat$(\cdot)$ operator constructs a square matrix from the input vector and tril$(\cdot)$ operator extracts the lower triangular part of the input matrix. This alternating optimization over $\{\mathbf{b}\}$ and $\{\mathbf{g}\}$ can be repeated till convergence of the optimization variables. From (31) and (32), we have

$$J(\mathbf{b}^{n+1}, \mathbf{g}^{n+1}) \leq J(\mathbf{b}^{n+1}, \mathbf{g}^n) \leq J(\mathbf{b}^n, \mathbf{g}^n). \tag{34}$$

Coupled with the fact that the SMSE is lower bounded, (34) implies that the proposed algorithm is guaranteed to converge to a limit as $n \to \infty$. The iteration is terminated when the norm of the difference in the results of consecutive iterations are below a threshold or when the maximum number of iterations is reached.

Table 1 Algorithm for computation of precoding matrices **B** and **G**

Select N_{max} (maximum number of iterations), T_{th}(convergence threshold),
Initialize $\mathbf{X}^0 = [\mathbf{b}^0\mathbf{g}^0]$
1) $n = 0$
2) **while** $n \le N_{max}$
3) Compute $\mathbf{b}^{n+1}$ using $\mathbf{g}^n$ Eqn.(31)
4) Compute $\mathbf{g}^{n+1}$ using $\mathbf{b}^{n+1}$ Eqn.(33)
5) $\mathbf{x}^{n+1}$=$[\mathbf{b}^{n+1}\mathbf{g}^{n+1}]$
6) **if** $\left\|\mathbf{x}^{n+1} - \mathbf{x}^n\right\| \le T_{th}$ **then**
7) **break**
8) **endif**
9) $n \leftarrow n + 1$
10) **endwhile**

5 Simulation Results

In this section, we present the performance of the proposed robust THP designs evaluated through simulations. First, we present the performance results of the proposed robust precoder designs with MSE constraints presented in Sect. 3. The performance of the proposed robust THP and linear precoder are compared with the robust power control algorithm in [14]. We compare with [14] as other robust designs reported in the literature use QoS metrics other than MSE. In the first experiment, we fix the uncertainty size of transmit CSI for all users as $\delta = 0.05$ and compute the transmit power for achieving different effective SINRs (SINR^e) required at each user, where $\mathrm{SINR}^e = (\mathrm{MSE})^{-1}$. Figure 4 shows the results of this experiment for $N_\mathrm{t} = 3$ and $N_\mathrm{u} = 3$. From Fig. 4, it can be seen that the proposed robust linear precoder and the robust power allocation algorithm in [14] have similar performance. The proposed THP design, on the other hand, transmits much less power compared to the linear precoders. In the second experiment, we fix the effective SINR requirement of each user as $\mathrm{SINR}^e = 10\,\mathrm{dB}$ and compute the transmit power required for different sizes of the channel uncertainty region. Figure 5 shows the results of this experiment for $N_\mathrm{t} = 3$ and $N_\mathrm{u} = 3$. The proposed robust THP transmits less power compared to the proposed robust linear precoder and the robust precoder in [14]. The proposed linear precoder is also found to perform better than the power allocation algorithm in low SINR^e region. While we used the NBE model in Figs. 4 and 5, in the next experiment, we study the performance of the precoders in the presence of Gaussian distributed CSI error matrix $\mathbf{E}$, i.e., the SE model. We fix the uncertainty size of each user $\delta = 0.05$ and the target effective SINR as $\mathrm{SINR}^e = 10\,\mathrm{dB}$. Distributions of the achieved SINR^e is evaluated for $\sigma_{\mathbf{E}}^2 = 0.01$ and $\sigma_{\mathbf{E}}^2 = 0.1$. Figure 6 shows the results of this third experiment. When the uncertainty size is fixed, the probability of the CSI error falling within the uncertainty region is higher for lower $\sigma_{\mathbf{E}}$. The results in Fig. 6 show that the proposed precoders provide the target QoS more often compared to the precoder in [14]. For example, when $\sigma_{\mathbf{E}}^2 = 0.01$, the proposed linear and THP precoders provide the target QoS with probability of 0.67, whereas the corresponding probability for the precoder in [14] is 0.29. Similar improved performance of the proposed precoders can be observed for the case of $\sigma_{\mathbf{E}}^2 = 0.1$ also.

Next, we evaluate the performance of the robust THP with a total BS transmit power constraint presented in Sect. 4. We compare this performance with other precoders in the

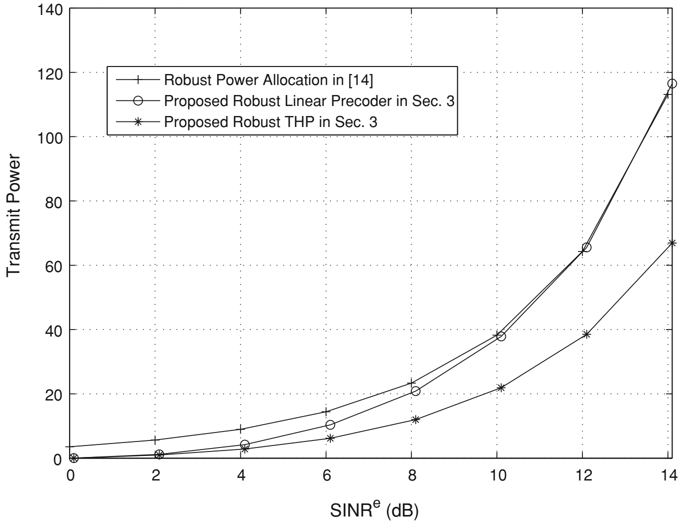

Fig. 4 Transmit power versus $\mathrm{SINR}^e = 1/\mathrm{MSE}$ requirement of the users. $N_\mathrm{t} = N_\mathrm{u} = 3$. Uncertainty size $\delta = \delta_1 = \delta_2 = \delta_3 = 0.05$. NBE model of CSI error

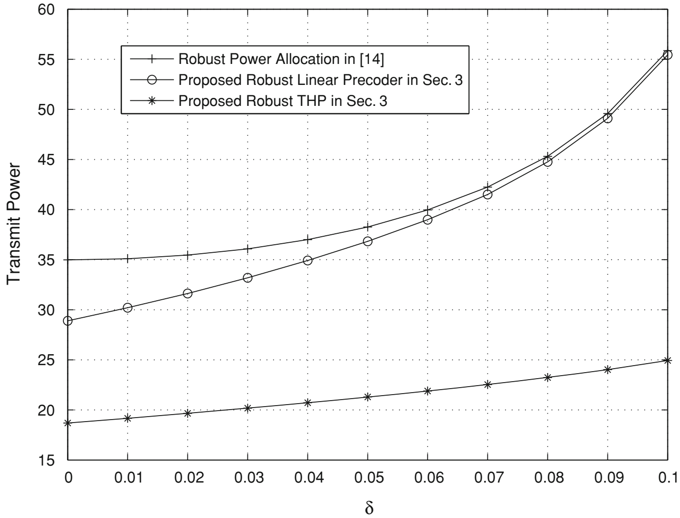

Fig. 5 Transmit power versus channel uncertainty size δ for $N_\mathrm{t} = N_\mathrm{u} = 3$ and $\mathrm{SINR}^e = 10\,\mathrm{dB}$ for each user. NBE model of CSI error

literature. The comparison is based on the average uncoded bit error rate (BER) versus the average signal-to-noise ratio (SNR), which is defined as $\frac{P_T}{N_\mathrm{u}\sigma_n^2}$ [10]. The modulation scheme used is QPSK. The elements of the estimation error matrix, $\mathbf{E}$, are generated independently from zero-mean Gaussian distribution of variance $\sigma_\mathbf{E}^2$ (i.e., SE model of CSI error). We compare the BER performance of the proposed robust THP with that of (i) the robust linear MMSE precoder in [10], and (ii) the robust ZF-THP in [12]. Figure 7 shows the BER performance of the various precoders in a system with four transmit antennas ($N_\mathrm{t} = 4$) at the BS,

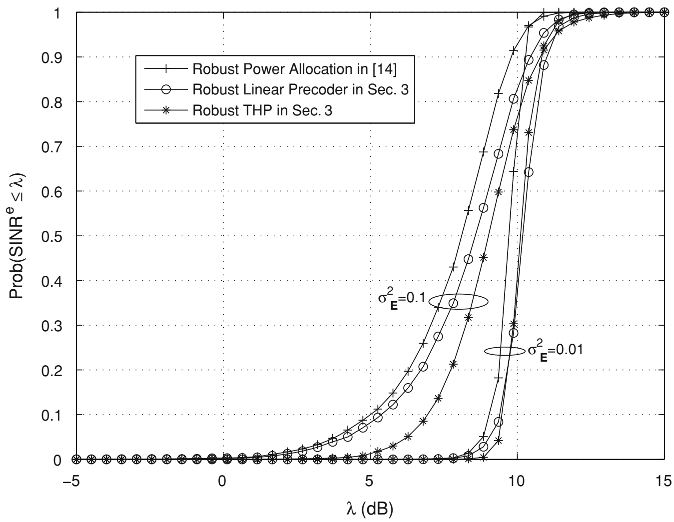

Fig. 6 CDF of the achieved SINR^e for different precoders with Gaussian distributed CSI error (i.e., SE model of CSI error). $N_\mathrm{t}=3$, $N_\mathrm{u}=3$, Channel uncertainty size $\delta = 0.05$

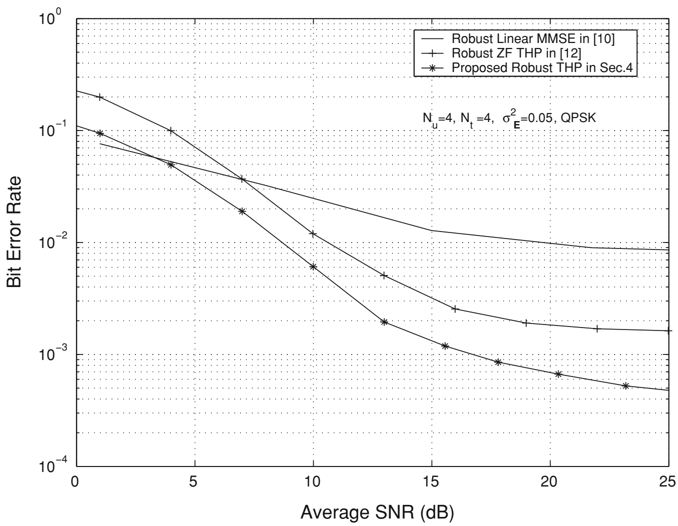

Fig. 7 Uncoded BER versus average SNR performance for different precoders with imperfect CSI at the transmitter: (i) robust linear MMSE precoder in [10], (ii) robust ZF THP in [12], and (iii) proposed robust THP in Sect. 4. $N_\mathrm{t}=4$, $N_\mathrm{u}=4$, QPSK, $\sigma_\mathbf{E}^2=0.05$

four users ($N_\mathrm{u}=4$) with one receive antenna each, and channel estimation error variance $\sigma_\mathbf{E}^2=0.05$. For the same system parameters setting, Fig. 8 presents the results for $\sigma_\mathbf{E}^2=0.2$. From Figs. 7 and 8, it can be observed that the proposed robust THP in Sect. 4 performs better than the robust linear MMSE precoder in [10] as well as the robust ZF-THP in [12]. The performance cross-overs between the THP precoders and the linear MMSE precoder at low SNRs are due to the power enhancement effect of the modulo operation in the THP.

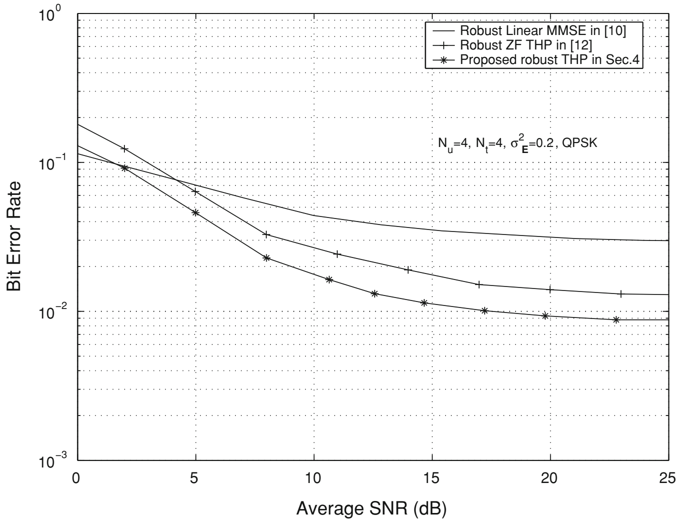

Fig. 8 Uncoded BER versus average SNR performance for different precoders with imperfect CSI at the transmitter: (i) robust linear MMSE precoder in [10], (ii) robust ZF THP in [12], and (iii) proposed robust THP in Sect. 4. $N_t = 4$, $N_u = 4$, QPSK, $\sigma_{\mathbf{E}}^2 = 0.2$

Thus, the results illustrate that the proposed robust THP design outperform other designs in the literature.

6 Conclusions

We investigated the problem of designing robust THPs for MISO systems with imperfect CSI under MSE and total BS transmit power constraints. The first design was based on the minimization of total BS transmit power under constraints on the MSE at the individual user receivers. We presented an iterative procedure to solve this problem, where each iteration involves the solution of a pair of convex optimization problems. The second design was based on the minimization of a stochastic function of the SMSE under a constraint on the total BS transmit power. We solved this problem efficiently by the method of alternating optimization. Through simulation results, we showed that the proposed robust THP designs outperform other robust THP designs in the literature.

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