Performance Analysis of TCM With Generalized Selection Combining on Rayleigh Fading Channels

A. Ramesh, A. Chockalingam, *Senior Member, IEEE*, and L. B. Milstein, *Fellow, IEEE*

***Abstract*—We derive the computational cutoff rate, R_0, for coherent trellis-coded modulation (TCM) schemes on independent indentically distributed (i.i.d.) Rayleigh fading channels with (K, L) generalized selection combining (GSC) diversity, which combines the K paths with the largest instantaneous signal-to-noise ratios (SNRs) among the L available diversity paths. The cutoff rate is shown to be a simple function of the moment generating function (MGF) of the SNR at the output of the (K, L) GSC receiver. We also derive the union bound on the bit error probability of TCM schemes with (K, L) GSC in the form of a simple, finite integral. The effectiveness of this bound is verified through simulations.**

***Index Terms*—Cutoff rate, diversity, fading channels, generalized selection combining (GSC), trellis-coded modulation (TCM).**

I. Introduction

TRELLIS-CODED modulation (TCM) schemes, with moderate encoder/decoder complexity, are capable of providing coding gains without expanding bandwidth. Several studies have considered the performance of TCM schemes on fading channels [1]–[3]. Recently, generalized selection combining (GSC), as a means of diversity reception, has become of interest [4], [5]. In a (K, L) GSC scheme, the receiver chooses and combines the best K out of the L available diversity branches. While [3] considers coded bit error performance of TCM with maximal ratio combining (MRC), equal gain combining (EGC), and selection combining (SC) diversity, it does not consider the performance of GSC schemes. The performance analyses in [4], [5], on the other hand, consider GSC schemes, but only for uncoded transmissions. Our contribution in this letter is the analysis of the coded bit error performance of TCM with (K, L) GSC on independent indentically distributed (i.i.d.) Rayleigh fading channels. The GSC scheme selects and combines the K paths with the largest instantaneous SNRs among the L available diversity paths.

We derive the performance limits of coherent TCM schemes, expressed in terms of computational cutoff rate [6], on i.i.d. Rayleigh fading channels with (K, L) GSC diversity. The cutoff rate, R_0, is shown to be an easy-to-compute logarithmic function of the moment generating function (MGF) of the SNR at the output of the GSC receiver. We illustrate that, to achieve a desired performance, the modulation complexity (alphabet size, M) and the GSC receiver complexity (number of paths combined, K) can be traded off. For example, at a cutoff rate of 1 bit/s/Hz, 8-PSK with (1, 3) GSC provides about the same power efficiency as obtained with QPSK with (2, 3) GSC. We also derive the pairwise error probability in closed form, and the union bound on the bit error probability of TCM with (K, L) GSC. The effectiveness of this bound is verified through simulations.

II. System Model

The TCM scheme with (K, L) GSC diversity is described as follows. The information bit stream u_k is encoded by a convolutional encoder of rate $n/(n+1)$. The encoded bit stream is interleaved and mapped onto an M-ary signal set $(M = 2^{n+1})$, whose symbols are transmitted over the fading channel. The receiver has L diversity antennas. We assume that the interleaver is ideal with infinite depth and the receiver has perfect knowledge of the complex fades on all the L diversity branches. The GSC combiner orders the L complex random fades in decreasing order of their modulus and retains the first K complex fades and the corresponding received signals, which are then combined in MRC fashion. This GSC combiner output, along with the first K sorted complex fades, are supplied to a Viterbi decoder for maximum likelihood sequence estimation. The complex lowpass equivalent received signal at the output of the channel, on the lth antenna path at time k, is given by

$$r_k^l = \alpha_k^l x_k + n_k^l, \qquad l = 1, 2, \ldots, L \tag{1}$$

where x_k belongs to the M-ary signal set $\{s_m\}_{m=0}^{M-1}$, $s_m = \sqrt{E_s}\exp(j2\pi m/M)$, with average energy $E[|x_k|^2] = E_s$, and n_k^l is a complex Gaussian random variable with independent real and imaginary components, each one distributed as $\mathcal{N}(0, N_0/2)$, where $N_0 = 2\sigma^2$ is the two sided power spectral density of the noise process. The amplitudes $|\alpha_k^l|$, $l = 1, \ldots, L$, of the complex fade random variables, α_k^l, are i.i.d. Rayleigh distributed. The Viterbi decoder performs maximum-likelihood decoding by processing $\mathbf{r} = (r_1^1, \ldots, r_1^L, \ldots, r_N^1, \ldots, r_N^L)$, $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ and $\mathbf{A}_\alpha = (\alpha_1^1, \ldots, \alpha_1^L, \ldots, \alpha_N^1, \ldots, \alpha_N^L)$, where N is the codeword length. The decoder chooses the sequence which minimizes the decoding metric

$$m(\mathbf{x}, \mathbf{r}; \mathbf{A}_\alpha) = \sum_{i=1}^{N} m(x_i, \underline{r}_i; \underline{\alpha}_i) = \sum_{i=1}^{N}\sum_{j=1}^{K}\left|r_i^{l_j} - \alpha_i^{l_j} x_i\right|^2 \tag{2}$$

where $\underline{\alpha}_i = (\alpha_i^1, \ldots, \alpha_i^L)$, $\underline{r}_i = (r_i^1, \ldots, r_i^L)$, and the indices $l_1, \ldots, l_K$ are such that $|\alpha_k^{l_1}| \geq |\alpha_k^{l_2}| \geq \cdots \geq |\alpha_k^{l_K}|$, $\forall\, k = 1, \ldots, N$.

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A. Ramesh and L. B. Milstein are with the Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093 USA.

A. Chockalingam is with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560012, India (e-mail: achockal@ece.iisc.ernet.in).

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III. Cutoff Rate for TCM With (K, L) GSC

For a discrete M-ary input and continuous output channel, with perfect knowledge of the channel state information (CSI) at the receiver, the cutoff rate, R_0, is defined as [6] (3), shown at the bottom of the page, where N is the length of the codeword $\mathbf{x}$ whose code symbols x belong to the complex field $\mathcal{C}$, and $q(\cdot)$ is the input probability distribution of the codewords. The conditional pdf of the received sequence $\mathbf{r}$ and fading sequence $\mathbf{A}_\alpha$, when $\mathbf{x}$ is transmitted, $p(\mathbf{r}, \mathbf{A}_\alpha|\mathbf{x})$ is given by $p(\mathbf{r}, \mathbf{A}_\alpha|\mathbf{x}) = p(\mathbf{r}|\mathbf{x}, \mathbf{A}_\alpha)p(\mathbf{A}_\alpha)$. For a symmetric channel, the expression in (3) can be maximized with the equiprobable input distribution $q(\mathbf{x}) = 1/M^N$. Consequently, (3) can be written as (4), shown at the bottom of the page.The quantity $\sum_{\mathbf{x}} 1/M^N \sqrt{p(\mathbf{r}|\mathbf{x}, \mathbf{A}_\alpha)}$ can be simplified as

$$\begin{aligned}\sum_{\mathbf{x}} \frac{1}{M^N}\sqrt{p(\mathbf{r}|\mathbf{x}, \mathbf{A}_\alpha)} &= \sum_{\mathbf{x}} \left[\prod_{k=1}^{N} \frac{1}{M}\sqrt{p(\underline{r}_k|x_k, \underline{\alpha}_k)}\right] \\ &= \prod_{k=1}^{N} \left[\sum_{m=0}^{M-1} \frac{1}{M}\sqrt{p(\underline{r}_k|s_m, \underline{\alpha}_k)}\right] \\ &= \prod_{k=1}^{N} \mathcal{T}(k), \end{aligned} \tag{5}$$

where $\mathcal{T}(k) = \sum_{m=0}^{M-1} 1/M\sqrt{p(\underline{r}_k|s_m, \underline{\alpha}_k)}$. Substituting (5) in (4), we obtain

$$R_0 = -\lim_{N\to\infty} \frac{1}{N}\log_2\left(\int_{\mathcal{C}^{NL}}\int_{\mathcal{C}^{NL}} \prod_{k=1}^{N} \mathcal{T}^2(k) d\mathbf{r} p(\mathbf{A}_\alpha) d\mathbf{A}_\alpha\right). \tag{6}$$

Noting that $p(\mathbf{A}_\alpha) = \prod_{k=1}^{N} p(\underline{\alpha}_k)$, and $\mathcal{T}(1), \dots, \mathcal{T}(N)$ are i.i.d., (6) can be further simplified as

$$\begin{aligned}R_0 = &-\lim_{N\to\infty}\frac{1}{N}\log_2\left(\left[\int_{\mathcal{C}^L}\int_{\mathcal{C}^L} \mathcal{T}^2(k)\, d\underline{r}_k\, p(\underline{\alpha}_k)\, d\underline{\alpha}_k\right]^N\right) \\ =&2\log_2(M) \\ &-\log_2\left(\sum_{m=0}^{M-1}\sum_{n=0}^{M-1}\int_{\mathcal{C}^L}\int_{\mathcal{C}^L}\sqrt{p(\underline{r}_k|s_m, \underline{\alpha}_k)p(\underline{r}_k|s_n, \underline{\alpha}_k)}\right. \\ &\left.\times\, d\underline{r}_k p(\underline{\alpha}_k) d\underline{\alpha}_k\right). \end{aligned} \tag{7}$$

When s_m is the transmitted symbol, the received symbol, r_k, at time k, at the output of the (K, L) GSC is given by

$$r_k = \sum_{j=1}^{K} r_k^{l_j}\left(\alpha_k^{l_j}\right)^* = x_k\sum_{j=1}^{K}\left|\alpha_k^{l_j}\right|^2 + \eta_k. \tag{8}$$

Conditioned on $\alpha_k^{l_1}, \dots, \alpha_k^{l_K}$, η_k is a complex Gaussian random variable with zero mean and variance $N_0\sum_{j=1}^{K}|\alpha_k^{l_j}|^2$. With this, it is not difficult to show that [7]

$$\int_{\mathcal{C}^L}\sqrt{p(\underline{r}_k|s_m, \underline{\alpha}_k)p(\underline{r}_k|s_n, \underline{\alpha}_k)}\, d\underline{r}_k = \exp\left(-\frac{\beta_k|s_m - s_n|^2}{8\sigma^2}\right) \tag{9}$$

where $\beta_k = \sum_{j=1}^{K}|\alpha_k^{l_j}|^2$. Upon substituting (9) in (7), the cutoff rate R_0 is given by

$$R_0 = 2\log_2(M) - \log_2\left(\sum_{m=0}^{M-1}\sum_{n=0}^{M-1}\mathrm{MGF}_{\beta_k}\left(-\frac{|s_m - s_n|^2}{8\sigma^2}\right)\right) \tag{10}$$

where the MGF of the random variable β_k is defined as $\mathrm{MGF}_{\beta_k}(s) = E[e^{s\beta_k}]$. From [5], $\mathrm{MGF}_{\beta_k}(s)$ is given by

$$\mathrm{MGF}_{\beta_k}(s) = \left(\frac{1}{1-s}\right)^K \prod_{l=K+1}^{L}\frac{1}{1-\frac{sK}{l}}. \tag{11}$$

Upon substituting (11) in (10), we obtain the final expression for the cutoff rate, R_0.

IV. Error Probability Analysis

The pairwise error probability, $P(\mathbf{x} \to \mathbf{x}')$, is the probability that the transmitted sequence $\mathbf{x}$ is incorrectly decoded as $\mathbf{x}'$. That is

$$\begin{aligned}P(\mathbf{x}\to\mathbf{x}') &= E_{\mathbf{A}_\alpha}\left[E_{\mathbf{r}}\left[\mathrm{Prob}\left(m(\mathbf{x}, \mathbf{r}; \mathbf{A}_\alpha) \geq m(\mathbf{x}', \mathbf{r}; \mathbf{A}_\alpha)\right)\right]\right] \\ &= E_{\mathbf{A}_\alpha}\left[Q\left(\sqrt{\frac{E_s}{2N_0}\sum_{i\in\mathcal{I}}\sum_{j=1}^{K}|\alpha_i^{l_j}|^2|x_i - x_i'|^2}\right)\right] \end{aligned} \tag{12}$$

where $\mathcal{I}$ is the set of all i such that $x_i \neq x_i'$. Define $\beta_i = \sum_{j=1}^{K}|\alpha_i^{l_j}|^2$ and observe that $\beta_1, \beta_2, \dots, \beta_{\mathcal{I}}$ are i.i.d. random variables. Now, by using $Q(x) = 1/\pi\int_0^{\pi/2} e^{-(x^2/(2\sin^2\theta))}d\theta$, for $x \geq 0$, and the MGF approach of [8] to calculate the bit error probability, and by defining $D(\theta) = e^{-(E_s/(4N_0\sin^2\theta))}$, we obtain

$$\begin{aligned}P(\mathbf{x}\to\mathbf{x}') =& E\left[\frac{1}{\pi}\int_{\theta=0}^{\pi/2}\prod_{i\in\mathcal{I}}[D(\theta)]^{\beta_i|x_i - x_i'|^2}\, d\theta\right] \\ =&\frac{1}{\pi}\int_{\theta=0}^{\pi/2}\prod_{i\in\mathcal{I}}\mathrm{MGF}_{\beta_i}\left(-\frac{E_s|x_i - x_i'|^2}{2N_0\sin^2\theta}\right)d\theta. \end{aligned} \tag{13}$$

The above expression can be computed easily, as the integrand is a simple rational polynomial of $\sin^2\theta$ and the integration

$$R_0 = \lim_{N\to\infty}\max_{q(\mathbf{x})}\left\{-\frac{1}{N}\log_2\left(\int_{\mathcal{C}^{NL}}\int_{\mathcal{C}^{NL}}\left[\sum_{\mathbf{x}} q(\mathbf{x})\sqrt{p(\mathbf{r}, \mathbf{A}_\alpha|\mathbf{x})}\right]^2 d\mathbf{r}\, d\mathbf{A}_\alpha\right)\right\} \tag{3}$$

$$R_0 = -\lim_{N\to\infty}\frac{1}{N}\left\{\log_2\left(\int_{\mathcal{C}^{NL}}\int_{\mathcal{C}^{NL}}\left[\frac{1}{M^N}\sum_{\mathbf{x}}\sqrt{p(\mathbf{r}|\mathbf{x}, \mathbf{A}_\alpha)}\right]^2 d\mathbf{r} p(\mathbf{A}_\alpha) d\mathbf{A}_\alpha\right)\right\}. \tag{4}$$

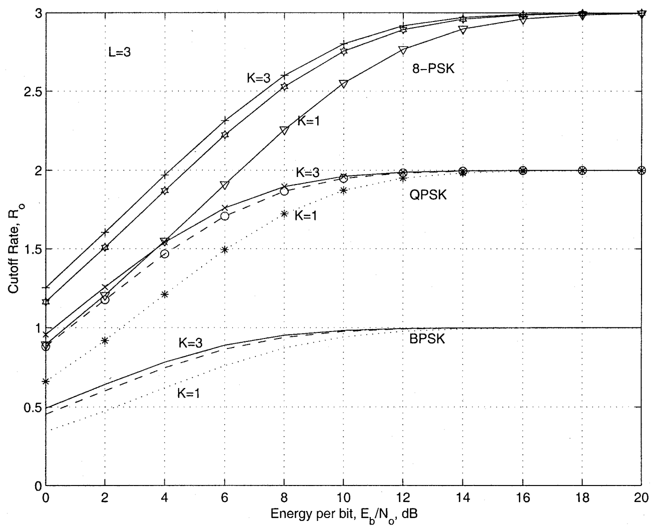

Fig. 1. Computational cutoff rate, R_0, on i.i.d Rayleigh fading for TCM with (K, L) GSC. $L = 3$, $K = 1$, 2, and 3. BPSK, QPSK, and 8-PSK.

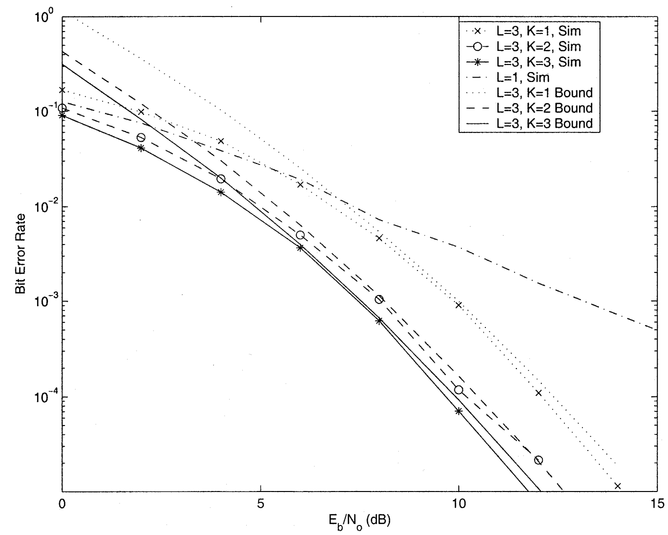

Fig. 2. Bit error probability performance of TCM with (K, L) GSC on i.i.d. Rayleigh fading. $L = 3$, $K = 1, 2,$ and 3. Number of states in the TCM encoder is 2.

limits are finite. The bit error probability, P_b, can then be upper bounded by

$$\begin{aligned} P_b \leq & \frac{1}{n}\sum_{\mathbf{x}} P(\mathbf{x}) \sum_{\mathbf{x}'} d(\mathbf{x}, \mathbf{x}') P(\mathbf{x} \rightarrow \mathbf{x}') \\ \leq & \frac{1}{n\pi} \int_{\theta=0}^{\pi/2} \left\{ \sum_{\mathbf{x}} P(\mathbf{x}) \sum_{\mathbf{x}'} d(\mathbf{x}, \mathbf{x}') \right. \\ & \left. \times \prod_{i \in \mathcal{I}} E\left[D(\theta)^{\beta_i |x_i - x_i'|^2} \right] \right\} d\theta \\ \leq & \frac{1}{n\pi} \int_{\theta=0}^{\pi/2} \frac{d}{dI} T(\overline{D(\theta)}, I)|_{I=1} d\theta. \end{aligned} \tag{14}$$

In (14), n is the number of information bits input to the TCM encoder, $P(\mathbf{x})$ is the probability that the codeword $\mathbf{x}$ was transmitted, $d(\mathbf{x}, \mathbf{x}')$ is the number of information bit errors that occurred by choosing $\mathbf{x}'$ instead of $\mathbf{x}$, $\overline{D(\theta)} = E[D^{\beta|x_m - x_m'|^2}(\theta)]$, and $T(\overline{D(\theta)}, I)$ is the transfer function of the underlying trellis code with each branch gain replaced by $E[D^{\beta|x_m - x_m'|^2}(\theta)]$.

V. Results and Discussion

In Fig. 1, the cutoff rate, R_0, is plotted for BPSK, QPSK, and 8-PSK for $L = 3$. The number of diversity paths to combine, K, is varied from 1 to 3. The E_b/N_0 is set to nE_s/LN_0. From Fig. 1, we observe that, for a given L, as expected, the cutoff rate increases with the number of paths to combine, K, and with the modulation alphabet size, M. It can be observed that, at a cutoff rate of 1 bit/s/Hz, 8-PSK with (1,3) GSC requires about the same E_b/N_0 as QPSK with (2,3) GSC. Also, at 1.5 bits/s/Hz, 8-PSK with (1,3) GSC and QPSK with (3,3) GSC require about the same E_b/N_0. Further, by expanding the signal set by a factor of two from BPSK to QPSK, gains of about 9 and 11 dB, respectively, can be obtained by using QPSK with (1,3) and (2,3) GSC, respectively, compared to BPSK with (3,3) GSC. These observations illustrate that, in TCM schemes with GSC diversity, the modulation complexity (i.e., alphabet size, M) and the GSC receiver complexity (i.e., the number of paths combined, K) can be traded off to achieve a desired performance.

We simulated a rate-1/2 TCM with two states, with the encoder and the signal set mapping of [2]. Fig. 2 shows the bit error probability bound as well as the simulation results for the rate-1/2 TCM scheme with 2 states for $L = 3$ and $K = 1$, 2 and 3. It is observed that for moderate to high SNRs, the bound is accurate within about 0.5 dB of the true value of the BER. It can be observed that, at a bit error rate of 10^{-3} with $L = 3$, selection combining, i.e., (1,3) GSC, achieves a diversity gain of more than 6 dB over the no diversity scheme (i.e., $L = 1$), whereas an additional 2 dB diversity gain can be obtained by combining one more diversity path, i.e., by using (2,3) GSC.

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