A Low-Complexity Maximum-Likelihood Detector for Differential Media-Based Modulation

Y. Naresh and A. Chockalingam

***Abstract*—Media-based modulation (MBM) uses radio frequency (RF) mirrors at the transmit antenna in order to create different channel fade realizations based on their on/off status. These complex fade realizations constitute the channel modulation alphabet. This channel modulation alphabet has to be estimated a priori at the receiver for detection. In this letter, we present a differential MBM (DMBM) scheme which does not require estimation of channel modulation alphabet at the receiver for detection. Consecutive MBM blocks are differentially encoded. We propose a low-complexity maximum-likelihood (ML) detection algorithm for DMBM. Simulation results show that the DMBM has only about 2 to 4 dB performance loss compared to MBM with perfect knowledge of the channel alphabet.**

***Index Terms*—Media-based modulation, differential MBM, low-complexity ML detection.**

I. INTRODUCTION

MEDIA-based modulation (MBM) is a recently proposed modulation scheme for wireless communications in rich scattering environments [1]-[4]. In MBM, M_{rf} radio frequency (RF) mirrors (parasitic elements) are placed near the transmit antenna in order to create different channel fade realizations based on their on/off status. These mirrors act as controlled scatterers in the propagation environment close to the transmit antenna. The wave originating from the transmit antenna is reflected back or is passed through a mirror depending on whether the mirror is off or on, respectively. An M_{rf}-length pattern of on/off status of the M_{rf} mirrors is called the 'mirror activation pattern (MAP)'. The propagation environment close to the transmitter changes from one MAP to another MAP, which, in turn, creates independent fade realizations for different MAPs. Since each mirror can be either on or off, the number of possible MAPs is $2^{M_{rf}}$. Out of them, the transmitter uses $n_m \leq 2^{M_{rf}}$ MAPs. The complex fade realizations corresponding to these n_m MAPs constitute the channel modulation alphabet. This channel alphabet is used to convey information bits by turning the mirrors on/off based on information bits. In addition, symbols from a conventional modulation alphabet (e.g., QAM/PSK) transmitted by the antenna also convey information bits. Studies have revealed that MBM signal vectors possess very good distance properties and because of this MBM achieves much better performance compared to conventional modulation schemes [1]-[4].

The MBM channel modulation alphabet needs to be estimated a priori at the receiver for detection. This is typically achieved through pilot transmissions. The number of complex channel fades to be estimated is n_m (which grows exponentially in the number of mirrors). Therefore, channel sounding to learn the alphabet a priori becomes a limiting factor in MBM when the number of mirrors becomes large. Differential signaling is a commonly used approach to deal with the channel sounding problem. Transmission takes place block-wise and consecutive blocks are differentially encoded. The receiver does not require any channel state information for detection.

Despite its importance and need, a study of differentially encoded MBM is yet to appear. We, in this letter, investigate differential MBM (DMBM) and propose a low-complexity maximum-likelihood (ML) detection scheme for DMBM. In the related literature, a differential scheme for spatial modulation (SM) [5] has been presented in [6]. In SM, only one antenna is activated in a channel use and the index of the active antenna conveys information bits. In the differential SM (DSM) scheme in [6], transmission takes place in blocks and the antenna activation order in a block conveys information bits. Low-complexity detectors for DSM have been proposed in [7],[8]. The detector in [7] was based on Viterbi algorithm, where the number of states in the trellis grows exponentially with the block size, which makes it unfeasible for large block sizes. The detector in [8] works in two steps: 1) indices of the active transmit antennas in each channel use are detected independently, and 2) the indices in step-1 are used to detect the antenna activation order. The complexity highly depends on the number of distinct indices in step-1. Our new contributions in this letter can be summarized as follows.

- A differential scheme for MBM (referred to as DMBM) which does not require the knowledge of the channel alphabet at the receiver is presented. The order in which the MAPs are selected (referred as 'MAP permutation') in a transmission block conveys information bits.
- A novel low-complexity ML detector for DMBM based on a constrained linear programming (LP) is presented. Such an LP formulation has not been reported, and this approach can be applied for detection in DSM as well. Results show that DMBM achieves better performance compared to DSM, and that the loss compared to MBM with perfect channel alphabet knowledge is only 2-4 dB.

II. DIFFERENTIAL MBM SCHEME

The DMBM transmitter is shown in Fig. 1. It consists of a transmit antenna surrounded by M_{rf} RF mirrors. The transmission takes place block-wise over n_m channel uses, where n_m denotes the number of MAPs used for transmission, $n_m \leq 2^{M_{rf}}$. Let the tth block consist of channel uses $tn_m + 1$ to $(t+1)n_m$. The encoding for the tth block is done as follows. The transmitter conveys $n_m \log_2 M + \lfloor \log_2(n_m!) \rfloor$ bits in each block. The $n_m \log_2 M$ bits are mapped to a modulation symbol

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The authors are with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560012, India (e-mail: ynr0109434@gmail.com, achockal@ece.iisc.ernet.in).

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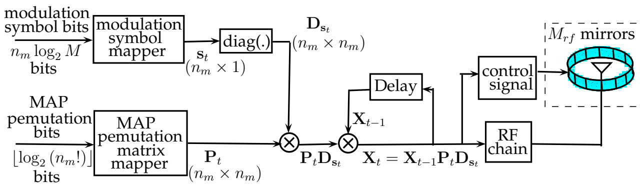

Fig. 1. Differential MBM transmitter.

vector $\mathbf{s}_t \in \mathbb{A}^{n_m}$, where $\mathbb{A}$ denotes an M-ary PSK alphabet. The $\lfloor \log_2(n_m!) \rfloor$ bits are mapped to a MAP permutation matrix $\mathbf{P}_t$ of size $n_m \times n_m$ such that the matrix $\mathbf{P}_t$ has only one non-zero element (= 1) in each row and in each column, i.e., $\mathbf{P}_t$ is of the form $\mathbf{P}_t = [\mathbf{e}_{l_1^t} \ \mathbf{e}_{l_2^t} \ \ldots \ \mathbf{e}_{l_{n_m}^t}]$, $1 \leq i, j, l_i^t, l_j^t \leq n_m$, $l_i^t \neq l_j^t \ \forall i \neq j$, $\mathbf{e}_{l_i^t}$ is an $n_m \times 1$ vector whose l_i^tth coordinate is 1 and all other coordinates are zeros, and l_i^t denotes the index of the selected MAP in the $(tn_m + i)$th channel use. Note that each MAP is selected only once in a block. The achieved rate in DMBM in bits per channel use (bpcu) is given by

$$\eta = \underbrace{\log_2 M}_{\text{Modulation symbol bits}} + \underbrace{\frac{1}{n_m} \lfloor \log_2(n_m!) \rfloor}_{\text{MAP permutation bits}} \text{ bpcu.} \quad (1)$$

Let $\mathbf{X}_t$ denote the $n_m \times n_m$ transmission matrix in the tth block, which is generated in a differential manner as

$$\mathbf{X}_t = \mathbf{X}_{t-1} \mathbf{P}_t \mathbf{D}_{\mathbf{s}_t}, \quad (2)$$

where $\mathbf{D}_{\mathbf{s}_t} = \text{diag}(\mathbf{s}_t)$, $\mathbf{X}_0 = \mathbf{P}_0 \mathbf{D}_{\mathbf{s}_0}$, $\mathbf{P}_0$ can be any arbitrary MAP permutation matrix, and $\mathbf{s}_0$ can be any arbitrary vector in $\mathbb{A}^{n_m}$. Without loss of generality, we assume $\mathbf{P}_0 = \mathbf{I}_{n_m}$ and $\mathbf{s}_0$ is a vector of all ones, where $\mathbf{I}_{n_m}$ is an identity matrix of size $n_m \times n_m$. Note that for any t, $\mathbf{X}_t$ can have only one non-zero element belonging to $\mathbb{A}$ in each row and in each column. Let $x_{l_i,i}^t$, the (l_i, i)th entry of $\mathbf{X}_t$, be the non-zero element belonging to $\mathbb{A}$. Then, in the $(tn_m + i)$th channel use, the transmit antenna transmits the symbol $x_{l_i,i}^t$ and the RF mirrors are activated as per the l_ith MAP.

A. DMBM signal set

A total of $n_m!$ MAP permutation matrices are possible. Out of them, $2^{\lfloor \log_2(n_m!) \rfloor}$ matrices are chosen to form a set of MAP permutation matrices, denoted by $\mathbb{P}$. The DMBM signal set, denoted by $\mathbb{S}_{\text{dmbm}}$, is given by

$$\mathbb{S}_{\text{dmbm}} = \{\{\mathbf{P}, \mathbf{s}\} : \mathbf{P} \in \mathbb{P}, \mathbf{s} \in \mathbb{A}^{n_m}\}. \quad (3)$$

Note that $\forall \ \{\mathbf{P}, \mathbf{s}\} \in \mathbb{S}_{\text{dmbm}}$, $\mathbf{P}\mathbf{D}_\mathbf{s}\mathbf{D}_\mathbf{s}^\dagger \mathbf{P}^\dagger = \mathbf{D}_\mathbf{s}^\dagger \mathbf{P}^\dagger \mathbf{P} \mathbf{D}_\mathbf{s} = \mathbf{I}_{n_m}$, where $(.)^\dagger$ denotes the conjugate transpose.

Example 1: For $n_m = 2$ and BPSK, the bpcu is 1.5, and the set $\mathbb{P}$ is given by

$$\mathbb{P} = \left\{ \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 \end{bmatrix}, \begin{bmatrix} \mathbf{e}_2 & \mathbf{e}_1 \end{bmatrix} \right\} = \Bigg\{ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\triangleq \mathbf{P}_1}, \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\triangleq \mathbf{P}_2} \Bigg\}. \quad (4)$$

The corresponding DMBM signal set $\mathbb{S}_{\text{dmbm}}$ is given by

$$\begin{aligned} \mathbb{S}_{\text{dmbm}} = \Big\{ & \{\mathbf{P}_1, [1 \ 1]^T\}, \{\mathbf{P}_1, [1 \ -1]^T\}, \{\mathbf{P}_1, [-1 \ 1]^T\}, \\ & \{\mathbf{P}_1, [-1 \ -1]^T\}, \{\mathbf{P}_2, [1 \ 1]^T\}, \{\mathbf{P}_2, [1 \ -1]^T\}, \\ & \{\mathbf{P}_2, [-1 \ 1]^T\}, \{\mathbf{P}_2, [-1 \ -1]^T\} \Big\}, \end{aligned} \quad (5)$$

where $(.)^T$ denotes the transpose. Three bits are taken at a time; the first bit is used to choose one among $\mathbf{P}_1$ and $\mathbf{P}_2$, and the remaining two bits are used to choose the $\mathbf{s}$ vector. The resulting $\mathbf{X}_t$ matrix of size 2×2 generated as per (2) is then sent in two consecutive channel uses $2t + 1$ and $2(t + 1)$.

The size of $\mathbb{P}$ grows exponentially with n_m, which makes the mapping from $\lfloor \log_2(n_m!) \rfloor$ bits to a $\mathbf{P} \in \mathbb{P}$ difficult for larger values of n_m (e.g., for $n_m = 16$, $|\mathbb{P}| = 2^{\lfloor \log_2(16!) \rfloor} = 2^{44}$). Lehmer code in factorial number system [9] can be used to facilitate this mapping[1] for large n_m as follows. *i*) First, $\lfloor \log_2(n_m!) \rfloor$ bits are converted to the corresponding decimal integer $a \in [0, |\mathbb{P}| - 1]$. *ii*) The integer a is converted into a factorial sequence vector $\mathbf{b}^a = [b_1^a \ b_2^a \ \ldots \ b_{n_m}^a]$, where b_i^a takes integer values in $[0, n_m - i], 1 \leq i \leq n_m$, such that $a = \sum_{i=1}^{n_m} b_i^a (n_m - i)!$. *iii*) The vector $\mathbf{b}^a$ is converted into a permutation vector $\mathbf{l}^a = [l_1^a \ l_2^a \ \cdots \ l_{n_m}^a]$, where l_i^a is $(b_i^a + 1)$th element in the ordered set Θ_i with $\Theta_1 = \{1, 2, \cdots, n_m\}$ and $\Theta_i = \Theta_{i-1} \setminus l_{i-1}^a, \forall \ i \geq 2$. *iv*) Finally, the MAP permutation matrix $\mathbf{P}^a$ corresponding to $\mathbf{l}^a$ is given by $\mathbf{P}^a = [\mathbf{e}_{l_1^a} \ \mathbf{e}_{l_2^a} \ \cdots \ \mathbf{e}_{l_{n_m}^a}]$.

Example 2: For $n_m = 4$, we have $\lfloor \log_2(n_m!) \rfloor = 4$ and $|\mathbb{P}| = 16$. The matrix $\mathbf{P}^a$ corresponding to the input bits 1101 is obtained as follows. *i*) $a = 13$. *ii*) $\mathbf{b}^a = [2 \ 0 \ 1 \ 0]$. We can see that $(2 \times 3!) + (0 \times 2!) + (1 \times 1!) + (0 \times 0!) = 13$. *iii*) l_1^a is $(b_1^a + 1 = 3)$rd element in the set $\Theta_1 = \{1, 2, 3, 4\}$, which is 3, i.e., $l_1^a = 3$. Next, l_2^a is $(b_2^a + 1 = 1)$st element in the set $\Theta_2 = \{1, 2, 3, 4\} \setminus \{l_1^a = 3\} = \{1, 2, 4\}$, which is 1, i.e., $l_2^a = 1$. Similarly, we can see that $l_3^a = 4$ and $l_4^a = 2$. *iv*) Finally, $\mathbf{P}^a = [\mathbf{e}_3 \ \mathbf{e}_1 \ \mathbf{e}_4 \ \mathbf{e}_2]$.

Note 1: Let $\mathbf{l}^{\max}$ denote the permutation vector corresponding to $|\mathbb{P}| - 1$. Note that for any $a \in [0, \ |\mathbb{P}| - 1]$, the relation $l_{q+1}^a \leq l_{q+1}^{\max}$ holds for $q \in \mathbb{L}^a$, where $\mathbb{L}^a = \{q : \forall j \leq q, l_j^a = l_j^{\max}\}$. This is illustrated in the following example.

Example 3: For $n_m = 4$, $|\mathbb{P}| = 16$ and $\mathbf{l}^{\max} = [3 \ 2 \ 4 \ 1]$. For $a = 14$, $\mathbf{l}^a = [3 \ 2 \ 1 \ 4]$. We can see that $\mathbb{L}^a = \{1, 2\}$. For $q \in \mathbb{L}^a$, the relation $l_{q+1}^a \leq l_{q+1}^{\max}$ is verified as follows. For $q = 1$, $l_2^a = l_2^{\max} = 2$. For $q = 2$, $l_3^a = 1 < l_3^{\max} = 4$.

B. DMBM received signal

Let n_r denotes the number of receive antennas. Let $\mathbf{h}_k = [h_{1,k} \ h_{2,k} \ldots \ h_{n_r,k}]^T$ denote the $n_r \times 1$-sized channel coefficient vector at the receiver for the kth MAP, where $h_{i,k}$ is the fading coefficient corresponding to the kth MAP to the ith receive antenna, $i = 1, 2, \cdots, n_r$, and $k = 1, 2, \cdots, n_m$. The $h_{i,k}$s are assumed to be independent and identically distributed (i.i.d.) and distributed as $\mathcal{CN}(0, 1)$. Assuming that channel remains constant over two adjacent block intervals, the received signal blocks $\mathbf{Y}_{t-1}$ and $\mathbf{Y}_t$ in the $(t-1)$th and tth block intervals, respectively, are given by

$$\mathbf{Y}_{t-1} = \mathbf{H}\mathbf{X}_{t-1} + \mathbf{N}_{t-1}, \quad \mathbf{Y}_t = \mathbf{H}\mathbf{X}_t + \mathbf{N}_t, \quad (6)$$

where $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \ldots \ \mathbf{h}_{n_m}]$ denote the $n_r \times n_m$ channel matrix, $\mathbf{N}_{t-1}$ and $\mathbf{N}_t$ are the additive noise matrices of size $n_r \times n_m$ in the $(t-1)$th and tth blocks, respectively, whose elements are i.i.d. and distributed as $\mathcal{CN}(0, \sigma^2)$. From (2) and (6), we have

[1]We note that several mapping codes are possible. Each mapping can result in a different performance. A study of the effect of different mapping codes on the performance of DMBM can be a topic for further study.

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$$\mathbf{Y}_t = \mathbf{Y}_{t-1}\mathbf{P}_t\mathbf{D}_{\mathbf{s}_t} + \tilde{\mathbf{N}}_t, \quad (7)$$

where $\tilde{\mathbf{N}}_t = \mathbf{N}_t - \mathbf{N}_{t-1}\mathbf{P}_t\mathbf{D}_{\mathbf{s}_t}$. We can see that the elements of $\tilde{\mathbf{N}}_t$ are i.i.d. and distributed as $\mathcal{CN}(0, 2\sigma^2)$. The ML decision rule is given by

$$\{\hat{\mathbf{P}}_t, \hat{\mathbf{s}}_t\} = \underset{\{\mathbf{P},\mathbf{s}\}\in\mathbb{S}_{\text{dmbm}}}{\operatorname{argmin}} \|\mathbf{Y}_t - \mathbf{Y}_{t-1}\mathbf{P}\mathbf{D}_\mathbf{s}\|^2. \quad (8)$$

The vector $\hat{\mathbf{s}}_t$ is demapped to get $n_m \log_2 M$ modulation symbol bits, and the matrix $\hat{\mathbf{P}}_t$ is demapped to get $\lfloor \log_2(n_m!) \rfloor$ MAP permutation bits. The computational complexity in (8) grows exponentially with n_m, i.e., $O(2^{\lfloor \log_2(n_m!) \rfloor} M^{n_m})$, which makes it unfeasible for large values of n_m. We propose a low-complexity implementation of (8) in the next section.

III. Low-Complexity ML Detector

In this section, we present a novel low-complexity ML detector for DMBM. Let $\mathbf{y}_t^i$, $\mathbf{y}_{t-1}^i$, and $\mathbf{p}^i$, $1 \le i \le n_m$, denote the ith columns of $\mathbf{Y}_t$, $\mathbf{Y}_{t-1}$, and $\mathbf{P}$, respectively. For a given $\mathbf{P}$, let $\mathbf{s}_\mathbf{P}$ denote the solution to the optimization problem $\mathbf{s}_\mathbf{P} = \operatorname{argmin}_{\mathbf{s}\in\mathbb{A}^{n_m}} \|\mathbf{Y}_t - \mathbf{Y}_{t-1}\mathbf{P}\mathbf{D}_\mathbf{s}\|^2$. Then, the ML decision rule in (8) can be written as

$$\{\hat{\mathbf{P}}_t, \hat{\mathbf{s}}_{\hat{\mathbf{P}}_t}\} = \underset{\mathbf{P}\in\mathbb{P}}{\operatorname{argmin}} \Big\{ \sum_{i=1}^{n_m} \|\mathbf{y}_t^i - \mathbf{Y}_{t-1}\mathbf{p}^i s_\mathbf{P}^i\|^2 \Big\}, \quad (9)$$

where $s_\mathbf{P}^i$ denotes the ith element of $\mathbf{s}_\mathbf{P}$. Let $\mathbf{c}^i = [c_1^i \ \cdots \ c_{n_m}^i]$ and $\mathbf{d}^i = [d_1^i \ \cdots \ d_{n_m}^i]$, $1 \le i \le n_m$ denote the vectors of size $1 \times n_m$, where

$$d_j^i \triangleq \underset{s\in\mathbb{A}}{\operatorname{argmin}} \|\mathbf{y}_t^i - \mathbf{y}_{t-1}^j s\|^2, \quad (10)$$

$$c_j^i \triangleq \|\mathbf{y}_t^i - \mathbf{y}_{t-1}^j d_j^i\|^2. \quad (11)$$

Since the entries of $\mathbf{s}_\mathbf{P}$ are independent, we have $\|\mathbf{y}_t^i - \mathbf{Y}_{t-1}\mathbf{p}^i s_\mathbf{P}^i\|^2 = \mathbf{c}^i\mathbf{p}^i$ and $s_\mathbf{P}^i = \mathbf{d}^i\mathbf{p}^i$. Now, defining $\mathbf{C} = [\mathbf{c}^{1^T} \ \mathbf{c}^{2^T} \ \ldots \ \mathbf{c}^{n_m^T}]^T$, (9) can be written as

$$\hat{\mathbf{P}}_t = \underset{\mathbf{P}\in\mathbb{P}}{\operatorname{argmin}} \Big\{ \sum_{i=1}^{n_m} \mathbf{c}^i\mathbf{p}^i \Big\} = \underset{\mathbf{P}\in\mathbb{P}}{\operatorname{argmin}} \operatorname{Tr}(\mathbf{C}\mathbf{P}), \quad (12)$$

$$\hat{s}_{\hat{\mathbf{P}}_t}^i = \mathbf{d}^i\hat{\mathbf{p}}_t^i, \ 1 \le i \le n_m. \quad (13)$$

where Tr(.) denotes the trace operator. We need to solve the minimization problems in (10) and (12). Low-complexity implementations that give exact solutions to these problems are obtained as follows.

Step 1: A low-complexity implementation of (10) can be obtained as follows. Since the elements of $\mathbb{A}$ are $\{e^{\imath\frac{2\pi}{M}k}\}_{k=0}^{M-1}$, where $\imath = \sqrt{-1}$, we can write d_j^i as

$$d_j^i = e^{\imath\frac{2\pi}{M}k_j^i}, \quad (14)$$

$$\begin{aligned} k_j^i &= \underset{0\le k\le M-1}{\operatorname{argmin}} \|\mathbf{y}_t^i\|^2 + \|\mathbf{y}_{t-1}^j\|^2 - 2r_j^i \cos\Big(\theta_j^i - \frac{2\pi}{M}k\Big) \\ &= \underset{0\le k\le M-1}{\operatorname{argmax}} \cos\Big(\frac{2\pi}{M}\Big(\frac{M}{2\pi}\theta_j^i - k\Big)\Big) \\ &= (rnd(\theta_j^i M/2\pi))_{mod}, \end{aligned} \quad (15)$$

where r_j^i and θ_j^i are the amplitude and phase of $(\mathbf{y}_{t-1}^j)^\dagger \mathbf{y}_t^i$ respectively, $rnd(.)$ denotes the rounding operator, and $(.)_{mod}$ denotes the modulo-M operator. Using the d_j^is computed using (14), the c_j^is are calculated using (11).

Step 2: In this step, we obtain a low-complexity implementation of (12). We can write the set $\mathbb{P}$ as a collection of subsets $\mathbb{P}_i$s, as $\mathbb{P} = \{\mathbb{P}_1, \mathbb{P}_2, \ldots, \mathbb{P}_{n_m}\}$, where $\mathbb{P}_i$ is defined as

$$\mathbb{P}_i \triangleq \{\mathbf{P}: \mathbf{P} \in \mathbb{P}, z_j^\mathbf{P} = l_j^{\max} \ \forall \ 1 \le j \le i-1, z_i^\mathbf{P} < l_i^{\max}\}, \quad (16)$$

where $z_j^\mathbf{P}$ is the jth element of the vector $\mathbf{z}^\mathbf{P} = [1 \ 2 \ \ldots \ n_m]\mathbf{P}$, $l_j^{\max}$ and $l_i^{\max}$ are the jth and ith elements of $\mathbf{l}^{\max}$, respectively. Note that $\mathbb{P}_i \cap \mathbb{P}_j = \emptyset$ for $i \ne j$. Also, for any i, the (j,i)th entry of $\mathbf{P} \in \mathbb{P}_i$ is equal to zero for all $j \ge l_i^{\max}$. Now, by using (16), the solution to (12) can be written as

$$\hat{\mathbf{P}}_t = \underset{\{\hat{\mathbf{P}}^i\}_{i=1}^{n_m}}{\operatorname{argmin}} \operatorname{Tr}(\mathbf{C}\hat{\mathbf{P}}^i), \quad (17)$$

$$\hat{\mathbf{P}}^i \triangleq \underset{\mathbf{P}\in\mathbb{P}_i}{\operatorname{argmin}} \operatorname{Tr}(\mathbf{C}\mathbf{P}) = \sum_{1\le j\le i-1} c_{l_j^{\max}}^j + \underset{\mathbf{P}\in\mathbb{P}_i}{\operatorname{argmin}} \sum_{j=i}^{n_m} \mathbf{c}^j\mathbf{p}^j. \quad (18)$$

Since $\mathbf{p}^j = \mathbf{e}_{l_j^{\max}}$ for $1 \le j \le i-1$ when $\mathbf{P} \in \mathbb{P}_i$, the $\{l_j^{\max}\}_{1\le j\le i-1}$ elements of $\mathbf{p}^k, i \le k \le n_m$ are zeros. Then,

$$\min_{\mathbf{P}\in\mathbb{P}_i} \sum_{j=i}^{n_m} \mathbf{c}^j\mathbf{p}^j = \min_{\mathbf{V}_\mathbf{P}\in\mathbb{V}_i} \operatorname{Tr}(\mathbf{C}_i\mathbf{V}_\mathbf{P}), \quad (19)$$

where $\mathbf{C}_i$ is the matrix of size $(n_m-i+1)\times(n_m-i+1)$ obtained by removing $\{l_j^{\max}\}_{1\le j\le i-1}$ columns in $[\mathbf{c}^{i^T} \ldots \mathbf{c}^{n_m T}]^T$, i.e., $\mathbf{C}_i = [\mathbf{c}^{i^T} \ldots \mathbf{c}^{n_m T}]^T\mathbf{W}_i$, where $\mathbf{W}_i = [\mathbf{e}_{g_1^i} \ \mathbf{e}_{g_2^i} \ \ldots \ \mathbf{e}_{g_{n_m-i+1}^i}]$, and $g_j^i, 1 \le j \le n_m - i + 1$ is jth element in the ordered set $\mathbb{G}_i \triangleq \{1, 2, \ldots, n_m\} \setminus \{l_j^{\max} : 1 \le j \le i-1\}$. The set $\mathbb{V}_i$ is the set of permutation matrices of size $(n_m - i + 1) \times (n_m - i + 1)$ defined as $\mathbb{V}_i \triangleq \{\mathbf{V}_\mathbf{P} : \mathbf{P} \in \mathbb{P}_i\}$, where for a given $\mathbf{P} \in \mathbb{P}_i$, the jth column of $\mathbf{V}_\mathbf{P} \in \mathbb{V}_i$ is given by $\mathbf{v}_\mathbf{P}^j = \mathbf{f}_{l_j^\mathbf{P}}$, where $\mathbf{f}_{l_j^\mathbf{P}}$ is an $(n_m - i + 1) \times 1$ vector whose $l_j^\mathbf{P}$th coordinate is 1 and all other coordinates are zeros, and $l_j^\mathbf{P}$ is the index of the element $z_{j+i-1}^\mathbf{P}$ in the set $\mathbb{G}_i$. Similarly, for a given $\mathbf{V} \in \mathbb{V}_i$, the jth column of the corresponding permutation matrix $\mathbf{P}_\mathbf{V} \in \mathbb{P}_i$ is given by $\mathbf{p}_\mathbf{V}^j = \mathbf{e}_{l_j^{\max}}$, for $1 \le j \le i-1$ and $\mathbf{p}_\mathbf{V}^j = \mathbf{e}_{l_j^\mathbf{V}}$, for $i \le j \le n_m$, where $l_j^\mathbf{V}$ is the $u_{j-i+1}^\mathbf{V}$th element in the ordered set $\mathbb{G}_i$, and $u_{j-i+1}^\mathbf{V}$ denotes the $(j - i + 1)$th element of the vector $\mathbf{u}^\mathbf{V} = [1 \ 2 \ \ldots \ n_m - i + 1]\mathbf{V}$.

Now, let $\hat{\mathbf{V}}^i$ be the argument of the solution to the minimization problem on the RHS of (19), i.e.,

$$\hat{\mathbf{V}}^i = \underset{\mathbf{V}\in\mathbb{V}_i}{\operatorname{argmin}} \operatorname{Tr}(\mathbf{C}_i\mathbf{V}). \quad (20)$$

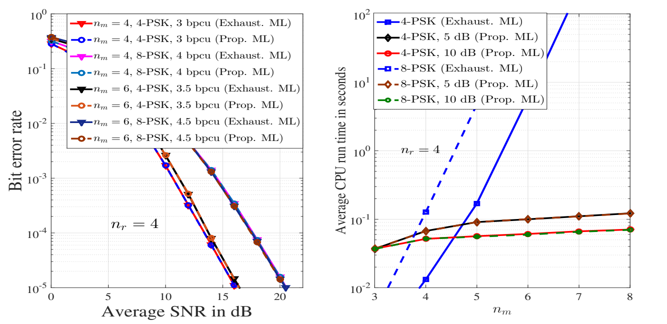
Note that (20) is a constrained integer linear program over $\{0, 1\}$, which is NP-hard. Hence, we convert this integer linear program into a constrained linear program over $[0, 1]$ which is solvable in polynomial time. Let us consider the set $\mathbb{Q}_i$ of matrices of size $(n_m - i + 1) \times (n_m - i + 1)$ such that

$$\mathbb{Q}_i \triangleq \Big\{\mathbf{Q} : \sum_{j=1}^{n_m-i+1} q_{j,k} = 1, \forall k; \sum_{k=1}^{n_m-i+1} q_{j,k} = 1, \forall j; \\ q_{j,k} \ge 0 \ \forall j,k; q_{j,1} = 0 \ \forall j \ge g_i^{\max}\Big\}, \quad (21)$$

where $q_{j,k}$ denotes the (j,k)th element of $\mathbf{Q}$ and $g_i^{\max}$ is the index of the element $l_i^{\max}$ in the set $\mathbb{G}_i$. Note that $\mathbb{Q}_i$ is a convex set over $[0, 1]$ for all i, and the set of vertices of $\mathbb{Q}_i = \mathbb{V}_i$. From the fundamental theorem of linear programming [10], any linear function has its minimum and maximum at the vertices of the function domain. So, we have

Algorithm 1 Listing of Step 2

```
Input :C, l^max, G_i ∀i
for i = 1 → n_m do
    W_i ← [e_{g_1^i} ... e_{g_{n_m-i+1}^i}],  C_i ← [c^{1 T} ... c^{n_m T}]^T W_i
    V̂^i ← argmin_{Q∈Q_i} Tr(C_i Q)
    P̂^i ← P_{V̂^i}
    if Tr(C P̂^i) ≤ Σ_{j=1}^{i} c^j_{l_j^max} then
        r = i; break
    end if
end for
Output : P̂_t as per (23) and ŝ^i_{P̂_t} as per (13) for all i
```



(a) Bit error rate (b) Average CPU run time

Fig. 2. BER and CPU run time comparison between the proposed low-complexity ML detector and the exhaustive search ML detector.

$$\hat{\mathbf{V}}^i = \underset{\mathbf{V}\in\mathbb{V}_i}{\operatorname{argmin}} \operatorname{Tr}(\mathbf{C}_i\mathbf{V}) = \underset{\mathbf{Q}\in\mathbb{Q}_i}{\operatorname{argmin}} \operatorname{Tr}(\mathbf{C}_i\mathbf{Q}). \quad (22)$$

Since the second optimization problem in (22) is a constrained linear program over $[0,1]$, it can be solved efficiently using the interior-point methods [10]. Since the solution of (19) is non-negative, if $\operatorname{Tr}(\mathbf{C}\hat{\mathbf{P}}^i) \leq \sum_{1\leq j\leq i} c^j_{l_j^{\max}}$, then $\operatorname{Tr}(\mathbf{C}\hat{\mathbf{P}}^i) \leq \operatorname{Tr}(\mathbf{C}\hat{\mathbf{P}}^k)$, $\forall k \geq i$. Therefore, (17) can be written as

$$\hat{\mathbf{P}}_t = \underset{\{\hat{\mathbf{P}}^i\}_{i=1}^r}{\operatorname{argmin}} \operatorname{Tr}(\mathbf{C}\hat{\mathbf{P}}^i), \quad (23)$$

where $r = \min\{i; \operatorname{Tr}(\mathbf{C}\hat{\mathbf{P}}^i) \leq \sum_{j=1}^{i} c^j_{l_j^{\max}}\}$. Finally, $\hat{s}^i_{\hat{\mathbf{P}}_t}$ is obtained as given in (13). The algorithm listing of Step 2 is given in **Algorithm 1**. The complexity of *Step 1* is $O(n_m^2)$. The complexities of (13), (22), and (23) in *Step 2* are $O(n_m), O((n_m - i + 1)^4)$, and $O(n_m)$, respectively. Hence, the total complexity of proposed detection algorithm is $O(n_m^4)$.

IV. Results and Discussions

In Fig. 2(a), we present the BER performance of DMBM with exhaustive search ML detection as per (8) and the proposed low-complexity ML detection for $n_m = 4, 6$, $M = 4, 8$, and $n_r = 4$. We also present the average CPU run time taken by both the ML detectors as function of n_m at fixed SNRs in Fig. 2(b). We can see in Fig. 2(a) that the proposed low-complexity ML detector has the performance same as that of the exhaustive search ML detector. In Fig. 2(b), the run time of exhaustive search ML detector is found to grow exponentially with n_m, whereas the run time can be significantly less in the proposed ML detector.

Figure 3 compares the ML detection performance of DMBM, DSM, and MBM (with perfect knowledge of $\mathbf{H}$ and MMSE estimate of $\mathbf{H}$) with $n_r = 4$. We also present the performance of the suboptimum detector presented in [8]. The following system parameters are considered. DMBM: *i*)

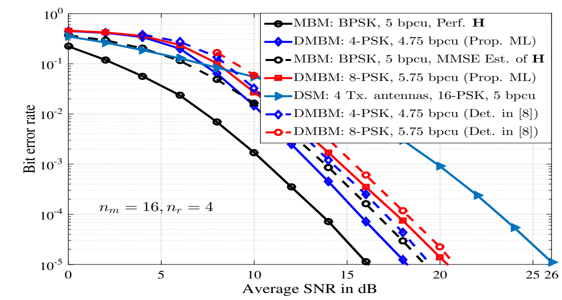


Fig. 3. BER comparison between DMBM, DSM, and MBM with $n_r = 4$.

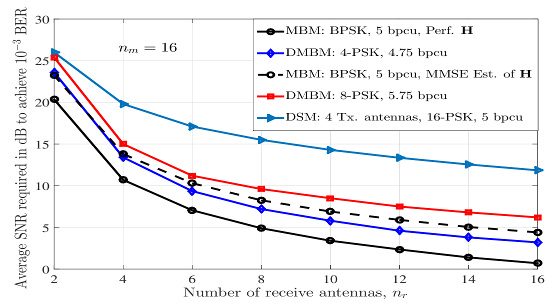


Fig. 4. Average SNR required to achieve 10^{-3} BER as a function of n_r for DMBM, DSM, and MBM.

$n_m = 16, M = 4$, 4.75 bpcu, *ii*) $n_m = 16, M = 8$, 5.75 bpcu. DSM: 4 Tx. antennas, $M = 16$, and 5 bpcu. MBM: $n_m = 16, M = 4$, 5 bpcu. It can be seen that both the DMBM schemes achieve better performance compared to DSM. Being suboptimum, the detector in [8] performs relatively poor; see thick lines (Prop. ML) versus dashed lines (det. in [8]) at 4.75 and 5.75 bpcu DMBM. Also, the performance loss in DMBM with 4.75 and 5.75 bpcu compared to MBM with 5 bpcu and perfect knowledge of $\mathbf{H}$ is only about 2 dB and 4 dB, respectively. It is further seen the performance of DMBM with 5.75 bpcu is just 1 dB inferior compared to MBM with 5 bpcu and MMSE estimation of $\mathbf{H}$. Figure 4 shows the average SNR required in DMBM, DSM, and MBM to achieve 10^{-3} BER as a function of n_r. This figure also shows that the performance loss of DMBM compared to MBM with perfect knowledge of $\mathbf{H}$ is not high and that DMBM outperforms DSM.

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