Throughput-Delay Analysis of a Multichannel Wireless Access Protocol in the Presence of Rayleigh Fading

A. Chockalingam, Senior Member, IEEE, *Weiping Xu, Michele Zorzi,* Senior Member, IEEE, *and Laurence B. Milstein,* Fellow, IEEE

Abstract

In this paper, we analyze the effect of bursty packet losses, caused by correlation in the multipath fading process, on the throughput and delay performance of a multichannel wireless access protocol. M equal-capacity, orthogonal, traffic channels are shared by N mobile users ($N \geq M$) on the uplink (mobile-to-base station link). Transmission attempts on the uplink are made based on the busy/idle status of the M receivers, which is broadcast by the base station, in every slot, on the downlink (base station-to-mobile link). Following a Markov chain analysis, we derive the analytical expressions for the average per channel throughput and the mean message transfer delay. Simulation results are shown to verify the analysis. We compare the effect of Doppler bandwidth on the performance of the protocol without link layer error recovery and a modified version of the protocol that employs a 'persist-until-success' link layer retransmission strategy to recover erroneous data packets.

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A. Chockalingam is with Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560012, INDIA.

W. Xu is with Nokia Mobile Phones, Inc., 9605 Scranton Road, San Diego, CA 92121, USA.

M. Zorzi is with Dipartimento di Ingegneria, Università di Ferrara, via Saragat 1, 44100 Ferrara, ITALY.

L. B. Milstein is with Department of Electrical and Computer Engineering, University of California at San Diego, La Jolla, CA 92093, USA.

1 Introduction

Next generation wireless networks are envisaged to support high data rates, packet oriented transport, and multimedia traffic. The design of efficient and robust wireless media access protocols, and the evaluation of their performance in the presence of physical layer impairments like channel fading, are key technical issues [1]-[6]. High capacity wireless networks may be realized either by assigning a single wideband channel or by using multiple narrowband channels that are orthogonal to each other. The latter approach, which we will consider in this paper, is particularly attractive when contiguous wide bandwidth spectrum is not available. Orthogonal multiple narrowband channels can be achieved, for example, in frequency, time, or code domains. In a multichannel system, there are several independent orthogonal channels, and a user can transmit on any of these channels based on a suitable access protocol. The performance of multichannel slotted ALOHA systems, where multiple equal-capacity channels are shared by many users, has been analyzed in [7], [8]. However, these studies assumed a deterministic channel model that did not consider the effect of multipath fading on the system performance. In fact, mobile radio channels are severely affected by time-varying losses due to distance, shadowing, and multipath fading. While the variation in the losses due to distance and shadowing is relatively slow, the variation due to multipath fading is quite rapid [9], [10]. The fading envelope due to multipath typically follows a Rayleigh distribution whose auto-correlation function depends on the normalized Doppler bandwidth, which in turn varies as a function of the mobile user velocity [10]. In this paper, we analyze the throughput and delay performance of a multichannel wireless access protocol taking physical layer impairments into account. Specifically, we analyze the effect of bursty packet losses, caused by correlation in the multipath fading process, on the protocol performance.

In [11], we presented and analyzed a single channel wireless access protocol that makes use of the uplink (mobile-to-base station link) channel status information, which is conveyed to the mobiles through a busy/idle flag broadcast on the downlink (base station-to-mobile link). This protocol can be viewed as a hybrid protocol employing the slotted ALOHA and reservation concepts [12]. A header packet is sent on a contention basis first, following which data packets are sent on a reservation basis. By this approach, packet losses due to collision are restricted to occur only among header packet transmissions. In this paper, we extend this protocol to utilize multiple, orthogonal, traffic channels on the uplink [13]. The multichannel wireless access protocol is described in Section 2. The analysis in [13] assumed the multipath Rayleigh fading to be independent and identically distributed (i.i.d.). However, the multipath fading process in mobile radio environments can typically be considered to be slowly varying, for the usual values of the carrier frequency (e.g., 900-1800 MHz) and for typical mobile speeds [10]. Particularly, the correlation in multipath fading behavior introduces burstiness (memory) in the packet error process, which can affect the protocol performance. In Section 3, we describe the fading channel model we use to analyze the protocol performance. In Section 4, we present the throughput-delay analysis of the multichannel protocol, taking into account the correlation in the fading process. We also analyze the performance of the multichannel protocol with a 'persist-until-success' link level (LL) *retransmission* strategy to recover erroneous data packets. Section 5 provides the analytical and simulation results. Conclusions and areas of future work are provided in Section 6.

2 Multichannel Wireless Access Protocol

Consider a packet communication wireless network where N mobile users share M equal-capacity, orthogonal, traffic channels ($N \geq M$) on the uplink to communicate with the base station. All the uplink channels are synchronized and slotted to one packet duration. All the mobiles in the network can use any of the M different channels following the access rules. The base station is provided with M receivers to demodulate all the uplink channels' traffic. Based on the busy/idle status of the M receivers, the base station broadcasts an M-bit *busy/idle word*, in every slot, on the downlink. The busy/idle flag corresponding to each channel is set or reset depending on whether or not the data packets are being transmitted on that channel.

Each message generated at the mobiles consists of two segments, namely the *header segment* and the *data segment*. The header segment is of one packet length. It carries control information, e.g., the destination address and the number of packets in the data segment. The data segment, which represents the actual traffic, consists of a random number of packets. Transmission attempts are made by the mobiles only at the slot boundaries by sending the header packet.

The mobile, once it receives a message to be sent to the base station, first checks the status of the busy/idle word which it periodically receives from the base station on the downlink. If all the M busy/idle flags indicate busy status, the mobile refrains from making a transmission attempt, and reschedules the attempt to a later time. If one or more flags indicate an idle status, then the mobile *randomly* picks one of these idle channels, and makes a transmission attempt by sending a header packet on the uplink slot of the chosen channel. If the header packet is received successfully, without packet loss due to fading or collision, the base station broadcasts the channel ID and the successful mobile ID (capturing mobile in the event of collision among header packets from different mobiles), and sets the corresponding channel's flag *busy* for the X subsequent slots, where X is the number of packets in the data segment of the successful mobile. This allows only the successful mobile to send its data packets in those X slots on that channel. During these X slots, other mobiles would receive a busy status flag for this channel and so they would not make transmission attempts on it. The base station resets the flag back to idle status after X slots. If the header packet is lost (due to collision or fading), then the base station will not respond with a busy status flag, but will continue to send the corresponding channel's flag as *idle*. This is an indication to the mobile that the header packet was lost, and so it has to reschedule its transmission attempt to a later time.

Note that, when the feedback is error-free, packet transmissions can experience fading, interference and noise during header slots, whereas data slots experience only fading and noise (no interference). Thus, in the case of error-free feedback to all the mobiles, collisions and hence capture are possible only during the header packet transmission and not during the transmission of data packets. However, errors in the busy/idle word reception will result in collisions, and hence packet losses, during the transmission of data packets as well.

2.1 Protocol with Link Level Retransmission

In the basic multichannel protocol described above, packets which get corrupted during the data segment transmission are lost and the recovery of such errors is left to the higher layer protocols. A simple way of recovering such data packet errors is through *retransmission* at the link level (LL) itself. Instead of ignoring packet errors, a data packet is retransmitted if it is received in error. In local wireless environments where the feedback delays are very small compared to the slot duration, a data packet in error can be retransmitted in the immediately following slot. Retransmission can persist until the packet is successfully received. In this case, the base station would need to send a non-binary feedback (busy/idle/retransmit) in order to avoid a collision among retransmission packets from a mobile with header packets from other mobiles. We call this modifi ed protocol as the protocol with 'persist-unitl-success' LL retransmission.

3 Markov channel model

In this section, we describe the fading channel model we use to analyze the performance of the protocols described above. We assume that all the mobiles' transmissions are power controlled so that the slowly varying distance and shadow losses are perfectly compensated, whereas the rapidly varying multipath fading remains uncompensated. We use a Markov channel model to characterize the correlated multipath fading, which is described in the following.

We consider the multipath fading to be frequency non-selective, which may be a reasonable assumption for data rates of the order of 100 kbps or less as considered in this paper. The effect of fading is then described as a multiplicative complex function, $\alpha(t)$, whose envelope is assumed to have a Rayleigh distribution [10]. In [14], the packet success/failure process was modeled as the outcome of a comparison of the instantaneous signal-to-noise ratio to a threshold value, SNR_t: if the received SNR is above the threshold, the packet is successfully decoded with probability 1; otherwise, it is lost with probability 1. If F is the value of the *fading margin* of the link, the instantaneous signal-to-noise ratio (taking into account the effect of fading) is given by $SNR_0|\alpha(t)|^2$, where $SNR_0 = SNR_t F$ is the average value of the SNR at the receiver. Hence, the binary process that describes packet successes/failures on the channel, β_i, can be obtained by comparison of the squared magnitude of $\alpha(t)$ with the threshold $1/F$. This gives rise to a binary packet error process which can be well approximated by a two-state Markov model with transition matrix

$$\boldsymbol{M_c} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}, \tag{1}$$

where p and $1-q$ are the probabilities that the packet transmission in slot j is successful, given that the packet transmission in slot $j-1$ was successful or unsuccessful, respectively*. The steady-state probability that a packet error occurs, P_E, is [14]

$$P_E = \frac{1-p}{2-p-q}. \tag{2}$$

For given values of the fading margin, F, and of the Doppler frequency, f_D, normalized by the packet transmission rate, $1/T$, we have [14]

$$P_E = 1 - e^{-1/F}, \tag{3}$$

$$q = 1 - \frac{Q(\theta, \rho\theta) - Q(\rho\theta, \theta)}{e^{1/F} - 1}, \tag{4}$$

where

$$\theta = \sqrt{\frac{2/F}{1-\rho^2}}, \tag{5}$$

*Following the established use for Markov chains, we arrange all transition probabilities from the same state in a row, i.e., M_{ij} is the probability of a transition from i to j. In other words, the probability vectors in Eqn. (2)) are row vectors.

$\rho = E[\alpha(t)\alpha^*(t+T)] = J_0(2\pi f_D T)$ is the correlation coefficient of two samples of the complex amplitude of the fading process taken T seconds apart [9],[10]. $J_0(.)$ is the Bessel function of the first kind and of zeroth order, and $Q(\cdot,\cdot)$ is the Marcum Q function, defined as

$$Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy. \tag{6}$$

Equations (3) and (4), together with (2), provide the complete statistical characterization of the packet error model in this case.

The derivation of this *threshold model* implicitly assumes that the fading envelope does not change during a packet and that the relationship between SNR and packet error probability is a step function. However, it has been shown in [15] that more realistic systems can still be modeled in this way, by defining an appropriate *equivalent* fading margin, which depends on the physical layer details (coding and modulation as well as the average signal-to-noise ratio, SNR_0). The parameters of the Markov model can in this case be evaluated by simulation, as explained in [15].

When considering different values of the Doppler frequency, we can follow two approaches. In the first approach, we let $f_D T$ vary while keeping constant all physical layer parameters, including SNR_0. The average packet error rate, P_E, which is relatively constant for small values of $f_D T$, will eventually show some dependence on $f_D T$ as $f_D T$ itself increases (for example, when coding is used, faster fading will eventually produce better packet error rate performance due to low correlation among bits in the same codeword). This comparison (for constant SNR_0) can be useful when assessing, for example, the effect of the mobile speed on the system performance without any control on the packet error rate.

In the second approach, we assume that for all values of $f_D T$ the packet error rate, P_E, is held constant. This means that, for some values of $f_D T$, we need to adjust the transmit power in order to achieve the target value of P_E. This approach is useful in accounting for correlation effects in the packet error modeling, since the case of large $f_D T$ corresponds to completely neglecting the second-order fading statistics. Also, this approach may correspond to a real system implementation where the physical layer includes a power control algorithm which tries to maintain the packet error rate at some target value, regardless of the fading bandwidth. For example, the overall power control strategy can be based on estimates of the packet error rate rather than on attenuation measurements.

In discussing the numerical results, we will consider both approaches. Note that in the latter approach, we can use the threshold model without worrying about the physical layer details, since we can use a constant value of the fading margin for all values of $f_D T$ (note from (2) that in the

f_DT	Model	Avg SNR (dB)	P_E	p	q	$(1-q)^{-1}$
0.01	Const. SNR_0	10.5	0.0963	0.9899	0.9056	10.59
	Const. P_E	10.5	0.0963	0.9899	0.9056	10.59
0.02	Const. SNR_0	10.5	0.0954	0.9834	0.8426	6.35
	Const. P_E	10.5	0.0954	0.9834	0.8426	6.35
0.04	Const. SNR_0	10.5	0.0958	0.9695	0.7122	3.47
	Const. P_E	10.5	0.0958	0.9695	0.7122	3.47
0.08	Const. SNR_0	10.5	0.1009	0.9433	0.4950	1.98
	Const. P_E	10.5	0.1009	0.9433	0.4950	.198
0.1	Const. SNR_0	10.5	0.1037	0.9324	0.4159	1.71
	Const. P_E	10.7	0.0994	0.9345	0.4068	1.68
0.2	Const. SNR_0	10.5	0.1149	0.8969	0.2066	1.26
	Const. P_E	11.2	0.0991	0.9101	0.1831	1.22
0.4	Const. SNR_0	10.5	0.1059	0.9022	0.1751	1.21
	Const. P_E	10.7	0.0998	0.9079	0.1698	1.20

Table 1: Markov parameters at different values of f_DT for a target error rate of 0.0952 (i.e, fading margin of 10 dB). BPSK modulation with (511,250,31) BCH code

threshold model a constant P_E is equivalent to a constant F). In the first approach, on the other hand, we need to specify the physical layer parameters of the scheme, in order to be able to track the variations of the Markov model as a function of f_DT. As a concrete example, in this paper we will consider the case in which each block is a codeword of a rate-1/2, (511,250,31) BCH code, which can correct up to 31 bits per block [16]. The value of SNR_0 will be chosen such that for slow fading the packet error rate is the same as for the corresponding threshold model. Table 1 shows the Markov parameters at different values of f_DT for a target error rae of 0.0952 (i.e., 10 dB fading margin), obtained through the technique outlined in [15] for the above BCH code with BPSK modulation, both in constant SNR_0 (first approch) and constant P_E (second approch) conditions.

4 Performance Analysis

In this section, we analyze the throughput and delay performance of the multichannel wireless access protocol described in Section 2. A single cell system with no inter-cell interference is considered. In order to analyze the system, we assume that the busy/idle word on the downlink is received instantaneously, and error-free by all the mobiles. In practice, the the busy/idle word might be corrupted by the channel (a busy status being flipped to idle status and vice versa). The reliability of the busy/idle word reception could be enhanced by providing adequate error protection. The instantaneous feedback assumption is appropriate where the delays due to propagation and processing are small compared to the slot duration.

A new message is assumed to arrive at each mobile with probability λ in each slot (Bernoulli arrival process). The mobile accepts a newly arriving message for transmission only when it has no message to send, and does not generate new messages when it already has a message to send. The length of the data segment of each message, X, measured in integer number of packets, is assumed to follow a geometric distribution with parameter g_d, and probability mass function

$$P[X = i] = \begin{cases} g_d(1 - g_d)^{i-1} & i = 1, 2, 3, ... \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

As per the multichannel protocol described in Section 2, in any given slot, each mobile can be in any one of the four states, namely, *idle/header_tx* state, *data_tx_success* state, *data_tx_failure* state, and *backlogged* state (see Figure 1). The data segment transmission is represented by separate *success* and *failure* states in order to take into account the one slot channel memory (defined by a first-order Markov chain with parameters p and q). In the *idle/header_tx* state, the mobile remains idle with probability $1 - \lambda$ or generates a new message with probability λ. In the latter case, the mobile randomly chooses an idle uplink channel (if available), and transmits the header packet in the uplink slot. If the header packet transmission is successful, the mobile moves from the *idle/header_tx* state to the transmission of data packets, i.e, to either *data_tx_success* state or *data_tx_failure* state. During the transmission of data packets, the mobile moves from *data_tx_success* state to *data_tx_success* state or *data_tx_failure* state with probability p and $1 - p$, respectively. Similarly, from *data_tx_failure* state, the mobile moves to *data_tx_success* state and *data_tx_failure* state with probability $1 - q$ and q, respectively. The mobile moves back to *idle/header_tx* state once the data packets transmission is complete. Note that the transition from *data_tx_failure* state to *idle/header_tx* state is shown in dashed lines to indicate that this transition is possible only in the protocol without LL retransmission, and is not possible in the modified protocol with 'persist-until-success' LL retransmission.

If the mobile finds all the uplink channels to be busy upon arrival of a message, it moves from the *idle/header_tx* state to the *backlogged* state. Similarly, if the header packet is lost due to collision or bad channel conditions, the mobile moves from the *idle/header_tx* state to the *backlogged* state. In the *backlogged* state, the mobile rechecks the status of the uplink channels after a random number of slots. This rescheduled transmission attempt delay is assumed to be geometrically distributed with parameter g_r. If a mobile in the *backlogged* state fails to transmit its header packet successfully, it stays in this state until its header packet transmission is successful, after which it moves to either *data_tx_success* state or *data_tx_failure* state.

4.1 Throughput Performance

Let x_t be the number of mobiles in the *data_tx_failure* state, y_t be the number of mobiles in the *data_tx_success* state, and z_t be the number of mobiles in the *backlogged* state at the beginning of slot t. The three dimensional random process $\{x_t, y_t, z_t\}$ can be modeled as a finite state Markov chain. For the protocol without LL retransmission, based on the conditional probability that n mobiles simultaneously transmit header packets and c_s of those packets are successfully received at the base station, the one step transition probability that the system moves from $(x_t = i_1, y_t = j_1, z_t = k_1)$ at time slot t to $(x_{t+1} = i_2, y_{t+1} = j_2, z_{t+l} = k_2)$ at time slot $t+1$ is given by

$$\begin{aligned} P_{i_1 j_1 k_1, i_2 j_2 k_2} = & \sum_{n=0}^{N-i_1-j_1} \sum_{c_s=0}^{min(M-i_1-j_1,n)} \sum_{s_j=0}^{c_s} \sum_{s_f=0}^{i_2-c_s+s_j} \sum_{f_s=0}^{j_2-s_j} \\ & \binom{N-i_1-j_1-k_1}{a} \lambda^a (1-\lambda)^{N-i_1-j_1-k_1-a} \\ & \cdot \binom{k_1}{n-a} g_r^{n-a} (1-g_r)^{k_1-n+a} \\ & \cdot \binom{j_1}{b_j} g_d^{j_1-b_j} (1-g_d)^{b_j} \binom{b_j}{s_f} (1-p)^{s_f} p^{b_j-s_f} \\ & \cdot \binom{i_1}{b_i} g_d^{i_1-b_i} (1-g_d)^{b_i} \binom{b_i}{f_s} (1-q)^{f_s} q^{b_i-f_s} \\ & \cdot \binom{c_s}{s_j} p^{s_j} (1-p)^{c_s-s_j} f(c_s|n, M-i_1-j_1), \end{aligned} \quad (8)$$

where M is the total number of uplink channels, N is the total number of mobiles, $N \geq M$, $0 \leq i_1 \leq M$, $0 \leq j_1 \leq M - i_1$, $0 \leq k_1 \leq N - i_1 - j_1$, $0 \leq i_2 \leq M$, $0 \leq j_2 \leq M - i_2$, $0 \leq k_2 \leq N - i_2 - j_2$, $a = k_2 - k_1 + c_s$, $b_j = j_2 - s_j - f_s + s_f$, $b_i = i_2 - c_s + s_j - s_f + f_s$, $f_s \leq b_i$, $s_f \leq b_j$, and the conditional probability $f(c_s|n, m_f)$ is defined as

$$\begin{aligned} f(c_s|n, m_f) = & \text{ Prob}(c_s \text{ header packets are successfully received } | \; n \text{ mobiles} \\ & \text{transmit header packets over } m_f \text{ channels}). \end{aligned} \quad (9)$$

Note that in (8), the term $\binom{N-i_1-j_1-k_1}{a}\lambda^a(1-\lambda)^{N-i_1-j_1-k_1-a}$ is the probability that a mobiles out of $N-i_1-j_1-k_1$ mobiles in the *idle/header_tx* state transmit new header packets, $\binom{k_1}{n-a}g_r^{n-a}(1-g_r)^{k_1-n+a}$ is the probability that $n-a$ mobiles out of k_1 mobiles in the *backlogged* state transmit old header packets, $\binom{j_1}{b_j}g_d^{j_1-b_j}(1-g_d)^{b_j}$ is the probability that b_j mobiles out of j_1 mobiles in the *data_tx_success* state do not end their message transmissions (since the length of the data segment of the message is geometrically distributed, g_d is the probability of ending the message transmission, and $1-g_d$ is the probability of continuing with the message transmission), and $\binom{b_j}{s_f}(1-p)^{s_f}p^{b_j-s_f}$ is the probability that s_f mobiles out of those b_j mobiles witness a data packet failure. Also, $\binom{i_1}{b_i}g_d^{i_1-b_i}(1-g_d)^{b_i}$ is the probability that b_i mobiles out of i_1 mobiles in the *data_tx_failure* state do not end their message transmissions, $\binom{b_i}{f_s}(1-q)^{f_s}q^{b_i-f_s}$ is the probability that f_s mobiles out of those b_i mobiles witness a data packet success, and $\binom{c_s}{s_j}p^{s_j}(1-p)^{c_s-s_j}$ is the probability that s_j mobiles out of c_s mobiles with successful header packets move to the *data_tx_success* state. Note that, in the above, we assumed that header packet transmissions do not have any correlation with their previous header or data packet transmissions. That is, header packet success probability is independent of the Markov parameters p and q. Also, the probability of moving from header success to *data_tx_success* is 'p' only when there is no capture involved in the header success. However, as an approximation, we use the same 'p' as the probability of moving from header capture to *data_tx_success*. Later we will see that the numerical results obtained from the analysis using the above approximations closely agree with results obtained from actual simulations.

The conditional probability $f(c_s|n,m_f)$ can be evaluated by a recursive expression as follows. By conditioning on the number of packets being simultaneously transmitted over an arbitrarily chosen channel (say, the first channel) and using total probability, we have

$$\begin{aligned} f(c_s|n,m_f) &= \sum_{i=0}^{n}\binom{n}{i}\left(1-\frac{1}{m_f}\right)^{n-i}\left(\frac{1}{m_f}\right)^{i} f(c_s|n-i,m_f-1)(1-p_s^{(i)}) \\ &\quad+\sum_{i=0}^{n}\binom{n}{i}\left(1-\frac{1}{m_f}\right)^{n-i}\left(\frac{1}{m_f}\right)^{i} f(c_s-1|n-i,m_f-1)p_s^{(i)} \end{aligned} \tag{10}$$

where $p_s^{(i)}$ is the probability of a header packet capture among i simultaneous transmissions, which is given by [17]

$$p_s^{(i)} = ie^{-1/F}\left(\frac{1}{1+B}\right)^{i-1}, \quad i \geq 0. \tag{11}$$

where B is the capture threshold. When there is no capture (i.e., $B\rightarrow\infty$),

$$p_s^{(i)} = \begin{cases} 1-P_E & i=1 \\ 0 & i>1. \end{cases} \tag{12}$$

The initial conditions for $f(c_s|n, m_f)$ are given by

$$f(c_s|n,0) = \begin{cases} 1 & \text{if } c_s = 0 \text{ and any } n \\ 0 & \text{if } c_s > 0 \text{ and any } n \end{cases}, \tag{13}$$

$$f(c_s|0,m_f) = \begin{cases} 1 & \text{if } c_s = 0 \text{ and any } m_f \\ 0 & \text{if } c_s > 0 \text{ and any } m_f \end{cases}, \tag{14}$$

$$f(c_s|1,m_f) = \begin{cases} 1 - p_s^{(1)} & \text{if } c_s = 0 \text{ and } m_f \geq 1 \\ p_s^{(1)} & \text{if } c_s = 1 \text{ and } m_f \geq 1 \\ 0 & \text{if } c_s > 1 \text{ and } m_f \geq 1 \end{cases}, \tag{15}$$

$$f(c_s|n,1) = \begin{cases} 1 - p_s^{(n)} & \text{if } c_s = 0 \text{ and } n > 1 \\ p_s^{(n)} & \text{if } c_s = 1 \text{ and } n > 1 \\ 0 & \text{if } c_s > 1 \text{ and } n > 1 \end{cases}, \tag{16}$$

and

$$f(c_s|n,m_f) = 0 \text{ if } c_s < 0. \tag{17}$$

If $\boldsymbol{P} = (P_{i_1 j_1 k_1, i_2 j_2 k_2})$ is the probability transition matrix and the row vector $\mathbf{\Pi} = \{\pi_{i_1 j_1 k_1}\}$, $0 \leq i_1 \leq M$, $0 \leq j_1 \leq M - i_1$, $0 \leq k_1 \leq N - i_1 - j_1$, contains the steady-state probabilities, then $\mathbf{\Pi}$ can be calculated by solving the linear equations $\mathbf{\Pi} = \mathbf{\Pi}\boldsymbol{P}$ and using the conservation relationship

$$\sum_{i_1=0}^{M} \sum_{j_1=0}^{M-i_1} \sum_{k_1=0}^{N-i_1-j_1} \pi_{i_1 j_1 k_1} = 1. \tag{18}$$

The number of successful data packets in a slot is, in this case, equal to the number of users in the *data_tx_success* state, so that the average number of successes per slot is given by

$$E\{S_d\} = \sum_{i_1=0}^{M} \sum_{j_1=0}^{M-i_1} \sum_{k_1=0}^{N-i_1-j_1} j_1 \pi_{i_1 j_1 k_1}, \tag{19}$$

and the average per channel throughput is obtained as

$$\eta_c = \frac{E\{S_d\}}{M}. \tag{20}$$

4.2 Delay Performance

Consider a system containing all mobiles which are either in the *backlogged* state or in the *data_tx_success* or *data_tx_failure* state. The number of users in that system is $\nu = i_1 + j_1 + k_1$, so that

$$E\{\nu\} = \sum_{i_1=0}^{M} \sum_{j_1=0}^{M-i_1} \sum_{k_1=0}^{N-i_1-j_1} (i_1 + j_1 + k_1) \pi_{i_1 j_1 k_1}, \tag{21}$$

On the other hand, the number of users which are not in the system is $N - \nu$, each generating a message in a slot with probability λ. Whenever a mobile generates a message, it joins the system one slot later. Therefore, the average arrival rate to that system is given by

$$\Lambda = \lambda(N - E\{\nu\}). \tag{22}$$

From Little's formula, the average time which each user spends in the system is given by the ratio between the average number of users in the system and the arrival rate. In our case, the average delay is one slot larger, since a users is assumed to join the system only after the slot in which the message is generated (which is not counted in the above calculation). Therefore, we have for the average delay suffered by a message:

$$E\{D\} = 1 + \frac{E\{\nu\}}{\Lambda}. \tag{23}$$

5 LL Retransmission Protocol Performance

With the 'persist-until-success' LL retransmission strategy described in Section 2.1, a geometric length message of X packets (with $E\left\{X\right\} = 1/g_d$) will take X' slots to finally get through, due to possible LL retransmissions. Therefore, we will have

$$X' = \sum_{i=1}^{X} Y_i, \tag{24}$$

where Y_i is an integer random variable equal to the number of transmissions it takes data packet i to be successfully received. If the case of i.i.d fading, each packet transmission can experience an error with probability P_E, the random variable Y_i will have a geometric distribution with parameter $(1 - P_E)$. In the case of correlated fading, Y_i will have a distribution given by

$$P[Y_i = y] = \begin{cases} p & y = 1 \\ (1-p)q^{y-2}(1-q) & y \geq 2 \end{cases}. \tag{25}$$

In both cases, it can be shown that $E\left\{Y_i\right\} = 1/(1 - P_E)$. Thus, the expected value of the effective length of the message (including the LL retransmission slots) is given by

$$E\{X'\} = \frac{1}{g_d(1 - P_E)}. \tag{26}$$

It can be seen that in the protocol with 'persist-until-success' LL retransmission, the mobile cannot directly move from the *data_tx_failure* state to the *idle/header_tx* state. Thus, the probability term $\binom{i_1}{b_i} g_d^{i_1 - b_i}(1 - g_d)^{b_i} \binom{b_i}{f_s}(1-q)^{f_s} q^{b_i - f_s}$ in (8) is modified to $\binom{i_1}{f_s}(1-q)^{f_s} q^{b_i - f_s}$, and $b_i = i_1 =$

$i_2 - c_s + s_j - s_f + f_s$, in order to account for the fact that, with retransmission of erroneous data packets at the link level, the message transmission cannot end in a *data_tx_failure* state. Also, the parameter g_d in the term $\binom{j_1}{b_j} g_d^{j_1-b_j}(1-g_d)^{b_j}$ gets replaced with $g_d(1-P_E)$.

6 Results

Numerical results for the average per channel throughput of the multichannel protocol without LL retransmission, obtained from (20) for $M = 3$, $N = 15$, $g_d = 0.1$, $g_r = 0.1$, and no capture, are plotted in Figure 2 for a normalized Doppler bandwidth of $f_D T = 0.02$. The $f_D T$ value of 0.02 represents slow fading (i.e., high correlation in fading), and may correspond to the user moving at a speed of 2.5 km/h for a carrier frequency of 900 MHz and a packet duration of 10 ms. The values of the fading margin considered are 5 and 10 dB. Note that at $f_D T = 0.02$, the fading is slow enough that the channel Markov parameters are the same for threshold, constant SNR_0, and constant P_E models. The performance in i.i.d. fading is also plotted for comparison. The g_d value of 0.1 corresponds to an average message length of 10 data packets. Likewise, the parameter $g_r = 0.1$ means that the average time between rescheduled transmission attempts is 10 slots. In addition to the analysis, the multichannel protocol was simulated as well, and the performance collected in over one million slots of simulation runs is also shown in Figure 2.

From Figure, 2 we observe the following. In i.i.d. fading, when the fading margin is 5 dB, the protocol offers a maximum throughput of about 0.53. This achievable throughput increases to 0.62 when $f_D T = 0.02$ (high correlation). Similarly, when $F = 10$ dB, the i.i.d. fading case yields a maximum throughput of 0.69, whereas the correlated fading case with $f_D T = 0.02$ results in a maximum throughput of 0.72. This shows that the i.i.d. fading model provides a pessimistic performance prediction when the fading is slow (i.e., when there is signifi cant burstiness in packet errors due to high correlation in fading). Also, in Figure 2, both the analytical and the simulation results for the correlated fading case are found to be in close agreement, thus validating the analysis. Figure 3 shows the mean message delay performance for the set of parameters in Figure 2. The curves represent the average delay values, in number of slots, obtained from (23). In Figure 3, a mean message delay of 11 slots (i.e., $1 + 1/g_d$) at low arrival rates implies that the messages get transmitted immediately on arrival without any waiting/rescheduling delays. Both increased message arrival rates and lower fading margins are seen to increase the mean delay beyond $1 + 1/g_d$. We also observed from numerical results that the header packet capture phenomenon due to multipath fading offers a per channel throughput improvement of about 10 to 15%, and a similar order of improvement in the delay performance.

The effect of the number of channels, M, on the throughput and delay characteristics of the multichannel protocol without LL retransmission is shown in Figures 4 and 5 for $g_d = 0.1$, $g_r = 0.05$, $f_D T = 0.02$, $F = 10$ dB, and no capture. The plots are parameterized by different values of M and N, namely, a) $M = 1$, $N = 5$, b) $M = 2$, $N = 10$, c) $M = 3$, $N = 15$, thus keeping the N/M ratio fixed at 5 in all three cases. For the chosen set of parameters, it is seen that the multichannel system performs better than a single channel system at moderate arrival rates ($0.01 < \lambda < 0.5$). This can be explained as follows. If x mobiles ($x > 1$) decide to transmit header packets in a slot, the probability of a success in that slot, assuming no capture, is zero for $M = 1$, whereas for $M > 1$, the probability of a success will be greater than zero because of the non-zero probability with which any of the n mobiles could pick an idle channel that is contention-free from other mobiles. However, at high arrival rates ($\lambda > 0.5$), the multichannel system performs a little poorer than a single channel system. This performance crossover occurs because, at high arrival rates, many of the mobiles could be in the backlogged state, and the number of backlogged mobiles which contend for a slot as soon as a channel becomes idle is higher in the case of $N = 15$ than in the case of $N = 5$, which increases the probability of a collision. We also observed that increasing the value of g_r (i.e., reducing the delay between rescheduled transmission attempts) shifts the crossover point to the left (i.e., towards lower arrival rates).

In Figures 6 and 7, the effect of the parameter g_r on the throughput and delay characteristics of the multichannel protocol without LL retransmission, at various values of new message arrival rate, λ, is shown as 3-D contour plots for $M = 3$, $N = 15$, $F = 10$ dB, and $g_d = 0.1$. The reason for the bell-shaped curve is that if g_r is too small (i.e., more waiting slots in the *backlogged* state), more slots will go unutilized during the waiting period, and if g_r is high, more slots will witness a collision resulting in decreased throughput and increased delays.

Figure 8 shows the throughput performance of the multichannel protocol, without LL retransmission, as a function of the normalized Doppler bandwidth, $f_D T$, for $\lambda = 1$, $M = 3$, $N = 15$, $g_d = 0.1$, $g_r = 0.1$, $F = 5, 10$ dB, and no capture. The range of $f_D T$ values shown corresponds to very slow fading at one end ($f_D T = 0.01$), and close to i.i.d. fading at the other ($f_D T = 1$). The performance achieved using threshold channel model, constant SNR_0 model and constant P_E model are compared. As expected, based on the explanation given in Section 3, the performance with constant SNR_0 and constant P_E models coincide for slow fading, as in this case the two models are essentially the same. In this slow fading regime, there is some difference between these results and those obtained from the threshold model, due to the inherent approximations in the latter. On the other hand, as the fading rate increases, the constant SNR_0 and constant P_E models show slightly different behavior, as the value of f_DT affects P_E differently in the two cases. It is interesting to see that the constant P_E model gives results which are very close to those for the threshold model. It is also interesting to see the behavior of the constant SNR_0 case: an increase of the fading rate always degrades the performance for $F = 5$ dB, whereas it eventually improves it for $F = 10$ dB. This is explained by noting that in the former case the channel is so bad that the code we are using does not help, and therefore in fast fading more packets are affected by errors which cannot be corrected by the code. On the contrary, as soon as the channel quality is sufficiently good, the joint effect of the error correction code and the time diversity induced by fast fading result in improved packet error rate (and therefore throughput) performance. In any event, it appears that both the the constant SNR_0 and constant P_E models are fairly well-approximated by the threshold model (with the possible exception of the cases in which performance is poor, where it gives an optimistic estimate), which in this case represents a good compromise between accuracy and complexity.

In Figures 9 and 10 the throughput and delay performance of the multichannel protocol both without LL retransmission and with LL retransmission are compared, as a function of the normalized Doppler bandwidth, f_DT, for $\lambda = 1$, $M = 3$, $N = 15$, $g_d = 0.1$, $g_r = 0.1$, $F = 5$ dB, and no capture. When $F = 5$ dB, the protocol without LL retransmission performs better at low values of f_DT compared to large values of f_DT. The throughput performance of the protocol with LL retransmission remains independent of the value of f_DT for the threshold and constant P_E models. At low values of f_DT (e.g., $f_DT < 0.02$), the protocol without LL retransmission gives better throughput performance than the protocol with LL retransmission, whereas at high values of f_DT ($f_DT > 0.02$) the protocol with retransmission LL performs better, consistent with the results presented in [11]. This performance variation over f_DT suggests that it is possible to devise more efficient versions of this protocol that could exploit the memory in the channel fading process for better performance. For example, if the base station detects a data packet error from a mobile, it can simply ask the mobile to terminate its ongoing data transmission and release the channel. Such a scheme is expected to give good results in the presence of significant channel burstiness (i.e., slow fading), as it avoids insisting on transmission in slots which are likely to be in error, and lets other mobiles (whose channel conditions might be good) to access and use the channel. On the other hand, the above strategy could be wasteful in fast fading conditions where packet errors could occur independently from slot to slot. In such fast fading conditions, error recovery by LL retransmission would be preferred [11].

For the protocol without retransmission, in the threshold and constant P_E models, the delay performance remains the same for both i.i.d. and correlated fading cases (see Figure 10), because 1) the average probability of header success $1 - P_E$ remains independent of the error correlation parameters, and 2) since there is no retransmission at the link level, the delay due to data segment transmission is $1/g_d$. We also see that the delay performance remains independent of $f_D T$ even in the case of the protocol with 'persist-until-success' retransmission. This is because with retransmission, the delay due to data segment transmission is just $1/g_d(1 - P_E)$. So both the protocols with and without retransmission will exihibit delay performance independent of $f_D T$. However, the protocol with retransmission results in larger delays than the protocol without retransmission. The above observations do not apply to the case of constant SNR_0, where the average probability of header success, $1 - P_E$, does depend on $f_D T$.

7 Conclusions

We analyzed the effect of bursty packet losses, caused by correlation in the multipath fading process, on the throughput and delay performance of a multichannel wireless access protocol. N mobile users shared M equal-capacity, orthogonal, traffi c channels ($N \geq M$) on the uplink. Transmission attempts were based on the busy/idle status of the M receivers at the base station. We modelled the fading channel memory using a fi rst-order Markov chain whose parameters were obtained as a function of the normalized Doppler bandwidth. Following a Markov chain analysis, analytical expressions for the average per channel throughput and the mean message delay were derived. Simulation results were shown to verify the analysis. A simple 'persist-until-success' retransmission strategy to recover erroneous data packets at the link level was also analyzed. It was shown that the protocol without LL retransmission benefi ted from highly correlated fading. It was further observed that the multichannel protocol without LL retransmission performed better on slow fading channels (e.g., pedestrian user speeds), whereas the protocol with retransmission performed better in fast fading channels (e.g., vehicular user speeds). It was pointed out, through an example, that more effi cient versions of this protocol could be devised which could exploit the memory in the channel fading behavior. The effect of non-zero propagation and processing delays and unreliable feedback on the protocol performance are topics for further investigation.

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A. Chockalingam (S'92–M'95–SM'98) was born in Rajapalayam, Tamilnadu State, India. He received the B.E. (Honours) degree in Electronics and Communication Engineering from the P. S. G. College of Technology, Coimbatore, India, in 1984, the M.Tech. degree with specialization in satellite communications from the Indian Institute of Technology, Kharagpur, India, in 1985, and the Ph.D. degree in Electrical Communication Engineering (ECE) from the Indian Institute of Science (IISc), Bangalore, India, in 1993.

During 1986 to 1993, he worked with the Transmission R & D division of Indian Telephone Industries Ltd., Bangalore. From December 1993 to May 1996, he was a Postdoctoral Fellow and an Assistant Project Scientist at the Department of Electrical and Computer Engineering, University of California, San Diego (UCSD), where he conducted research in DS-CDMA wireless communications. From May 1996 to December 1998, he served Qualcomm, Inc., San Diego, CA, as a Staff Engineer/Manager in the systems engineering group. In December 1998, he joined the faculty of Department of ECE, IISc, Bangalore, where he is an Assistant Professor, working in the area of wireless communications, and directing research at the Wireless Research Lab (WRL), IISc. He was a visiting faculty to UCSD during summer 1999.

Dr. Chockalingam is a recipient of the CDIL (Communication Devices India Ltd) award for a paper published in the IETE Journal. His research interests lie in the area of DS-CDMA systems and wireless networks and protocols.

Weiping Xu received the B.S. and M.S. degrees in radio engineering from Southeast University, China, in 1985 and 1988, respectively, and the Ph. D. degree in electrical engineering from the University of California, San Diego, in 1998. She is currently a member of the technical staff specializing in communication system design with the Research and Development Center at Nokia Mobile Phones in San Diego. From 1988 to 1995, she was on the faculty of the National Mobile Communication Research Laboratory at Southeast University. She had held a communication system engineer position with Hung Nien Electronics, Hong Kong during 1991-1993. Her research interests include digital communication theory with special emphasis on spread-spectrum communication systems and multimedia access protocols.

Michele Zorzi (S'89–M'95–SM'98) was born in Venice, Italy, in 1966. He received the Laurea Degree and the Ph.D. in Electrical Engineering from the University of Padova, Italy, in 1990 and 1994, respectively. During the Academic Year 1992/93, he was on leave at the University of California, San Diego (UCSD), attending graduate courses and doing research on multiple access in mobile radio networks. In 1993, he joined the faculty of the Dipartimento di Elettronica e Informazione, Politecnico di Milano, Italy. After spending three years with the Center for Wireless Communications at UCSD, in 1998 he joined the School of Engineering of the Università di Ferrara, Italy, where he is currently an Associate Professor. His present research interests include performance evaluation in mobile communications systems, random access in mobile radio networks, and energy constrained communications protocols.

Dr. Zorzi currently serves on the Editorial Boards of the *IEEE Personal Communications Magazine* and of the *ACM/URSI/Baltzer Journal of Wireless Networks*. He is also guest editor for special issues in the *IEEE Personal Communications Magazine* (Energy Management in Personal Communications Systems) and the *IEEE Journal on Selected Areas in Communications* (Multi-media Network Radios). E-mail: zorzi@ing.unife.it

Laurence B. Milstein (S'66–M'68–SM'75–F'85) received the B.E.E. degree from the City College of New York, New York, NY, in 1964, and the M.S. and Ph.D. degrees in electrical engineering from the Polytechnic Institute of Brooklyn, Brooklyn, NY, in 1966 and 1968, respectively.

From 1968 to 1974, he was employed by the Space and Communications Group of Hughes Aircraft Company, and from 1974 to 1976, he was a member of the Department of Electrical and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY. Since 1976, he has been with the Department of Electrical and Computer Engineering, University of California at San Diego, La Jolla, CA, where he is a Professor and former Department Chairman, working in the area of digital communication theory with special emphasis on spread-spectrum communication systems. He has also been a consultant to both government and industry in the areas of radar and communications.

Dr. Milstein was an Associate Editor for Communication Theory for the IEEE Transactions on Communications, an Associate Editor for Book Reviews for the IEEE Transactions on Information Theory, and an Associate Technical Editor for the IEEE Communications Magazine, and is currently the Editor-in-Chief of the IEEE Journal on Selected Areas in Communications. He was the Vice President for Technical Affairs in 1990 and 1991 of the IEEE Communications Society, and has been a member of the Board of Governors of both the IEEE Communications Society and the IEEE Information Theory Society. He is also a member of Eta Kappa Nu and Tau Beta Pi, and is a Fellow of the IEEE.

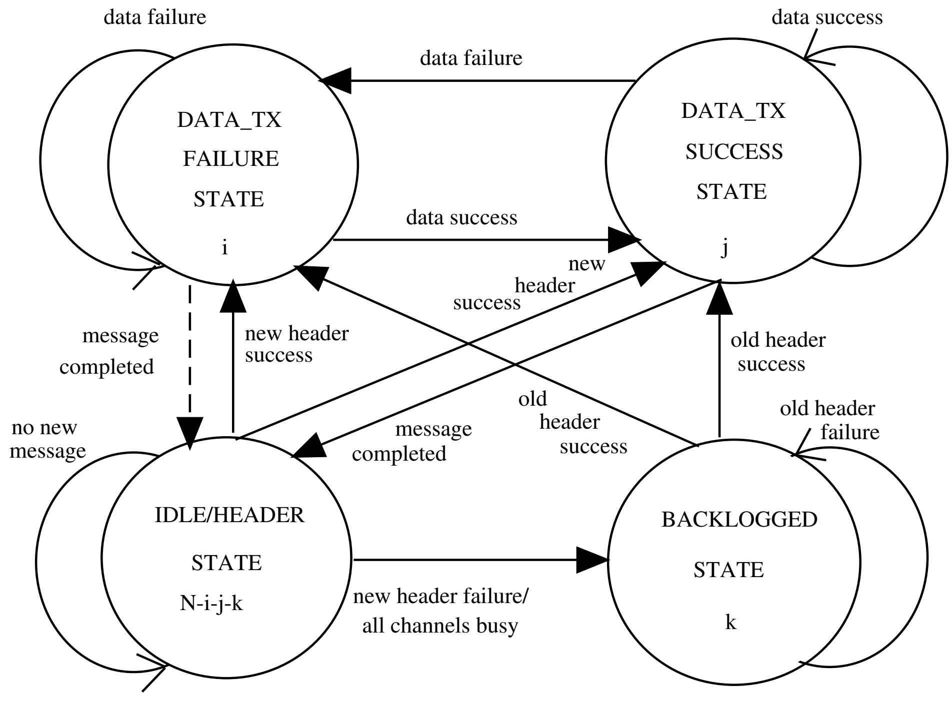

Figure 1: Mobile state transition diagram (in Markov fading)

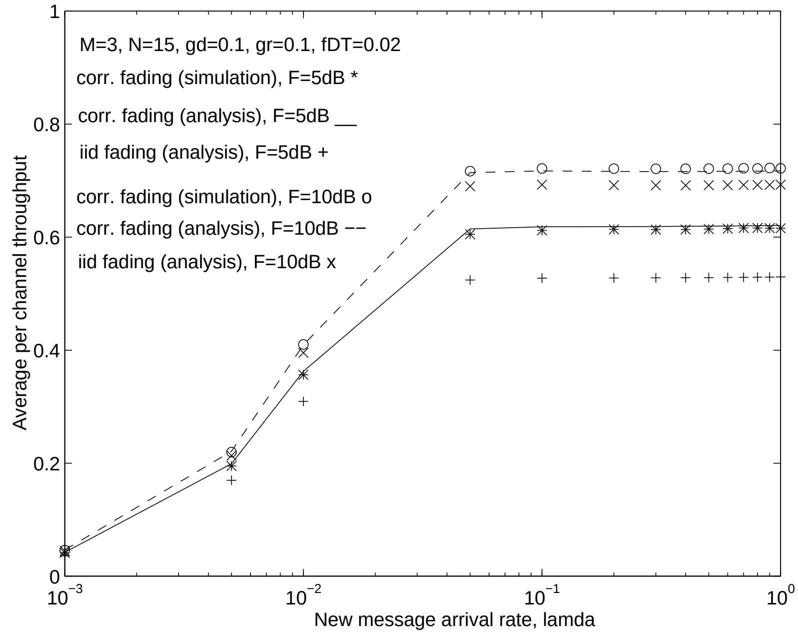

Figure 2: Average per channel throughput, η_c, vs new message arrival rate, λ, in correlated Rayleigh fading. No LL retransmission. $M = 3$, $N = 15$, $g_d = 0.1$, $g_r = 0.1$, $f_D T = 0.02$, F = 5, 10 dB

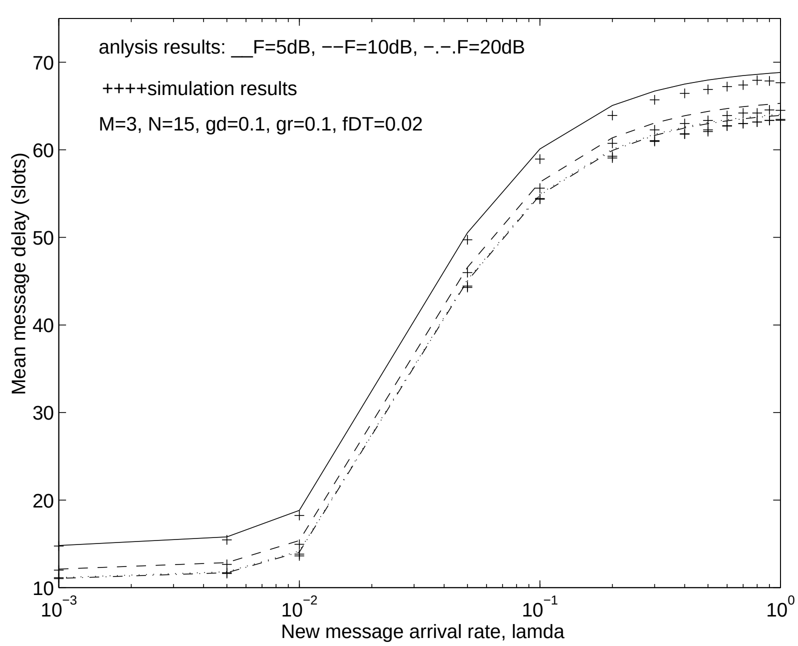

Figure 3: Mean message delay vs new message arrival rate, λ, in correlated Rayleigh fading. No LL retransmission. $M = 3$, $N = 15$, $g_d = 0.1$, $g_r = 0.1$, $f_D T = 0.02$, $F = 5, 10, 20$ dB

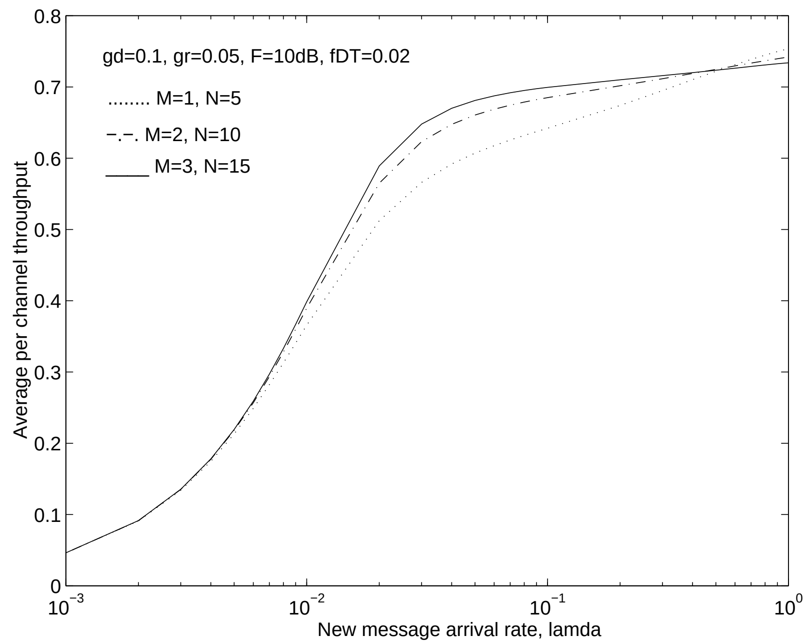

Figure 4: Effect of number of channels, M, on the average per channel throughput, η_c for a) $M = 1$, $N = 5$, b) $M = 2$, $N = 10$, and c) $M = 3$, $N = 15$. No LL retransmission. $g_d = 0.1$, $g_r = 0.05$, $f_D T = 0.02$, $F = 10$ dB

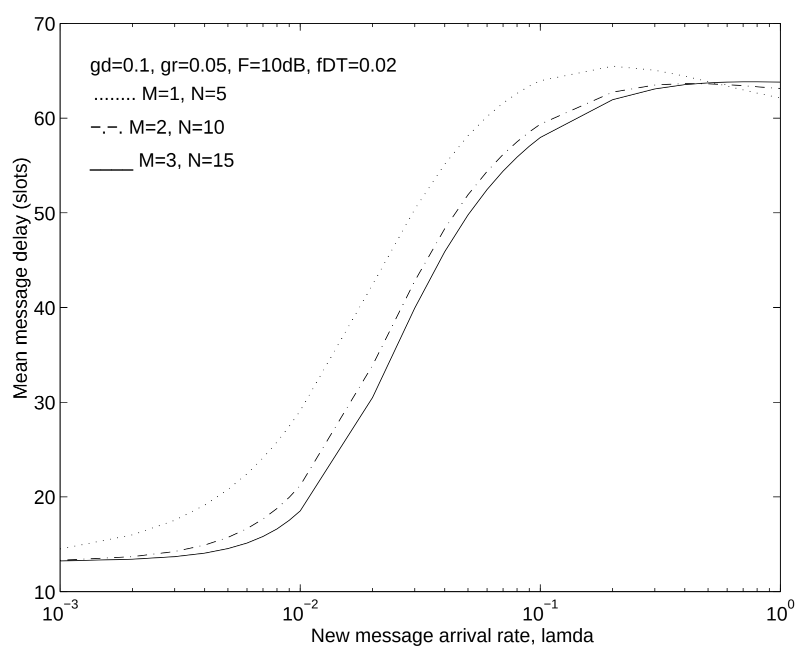

Figure 5: Effect of number of channels, M, on the mean message delay for a) $M = 1$, $N = 5$, b) $M = 2$, $N = 10$, and c) $M = 3$, $N = 15$. No LL retransmission. $g_d = 0.1$, $g_r = 0.05$, $f_D T = 0.02$, F = 10 dB

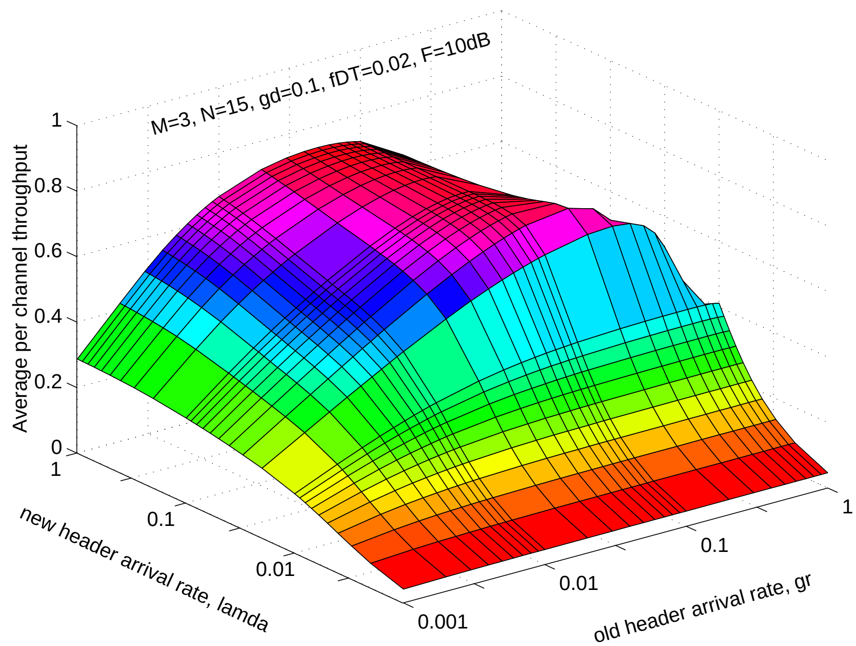

Figure 6: Effect of g_r and λ on the average per channel throughput, η_c. No LL retransmission. $M = 3$, $N = 15$, $g_d = 0.1$, $f_D T = 0.02$, $F = 10$ dB

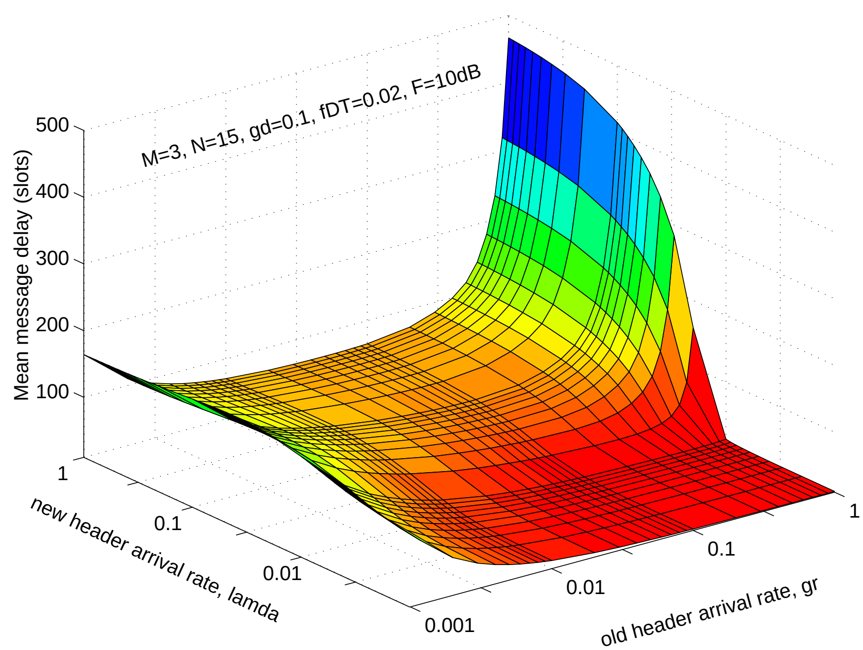

Figure 7: Effect of g_r and λ on the mean message delay. No LL retransmission. $M = 3$, $N = 15$, $g_d = 0.1$, $f_D T = 0.02$, $F = 10$ dB

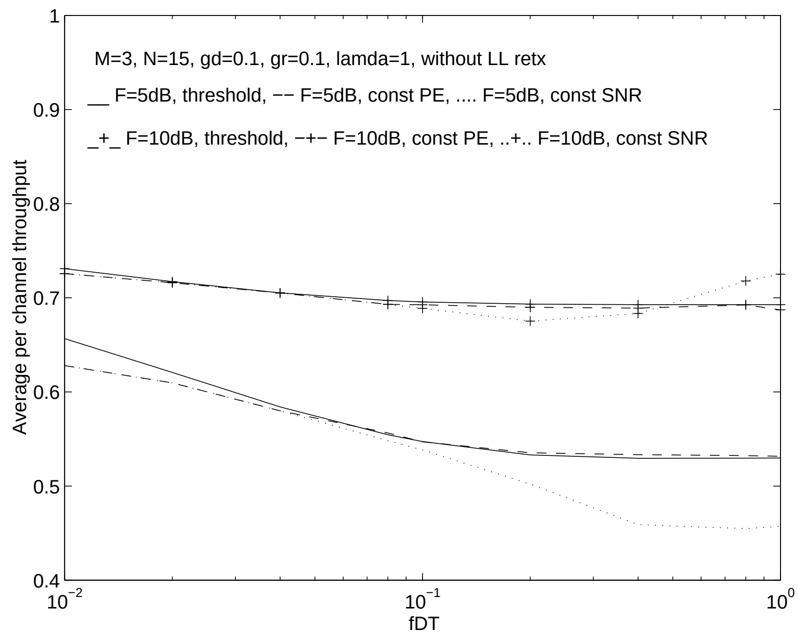

Figure 8: Average per channel throughput, η_c, versus $f_D T$. Protocol *without* LL retransmission. $M = 3$, $N = 15$, $g_d = 0.1$, $g_r = 0.1$, $\lambda = 1$, $F = 5, 10$ dB

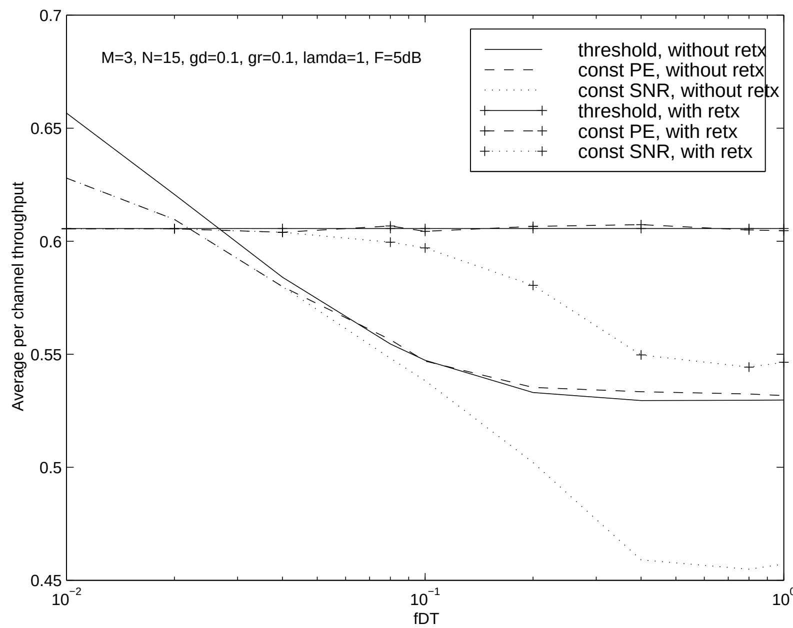

Figure 9: Average per channel throughput, η_c, versus $f_D T$. Protocol *with* and without LL retransmission. $M = 3$, $N = 15$, $g_d = 0.1$, $g_r = 0.1$, $\lambda = 1$, $F = 5$ dB

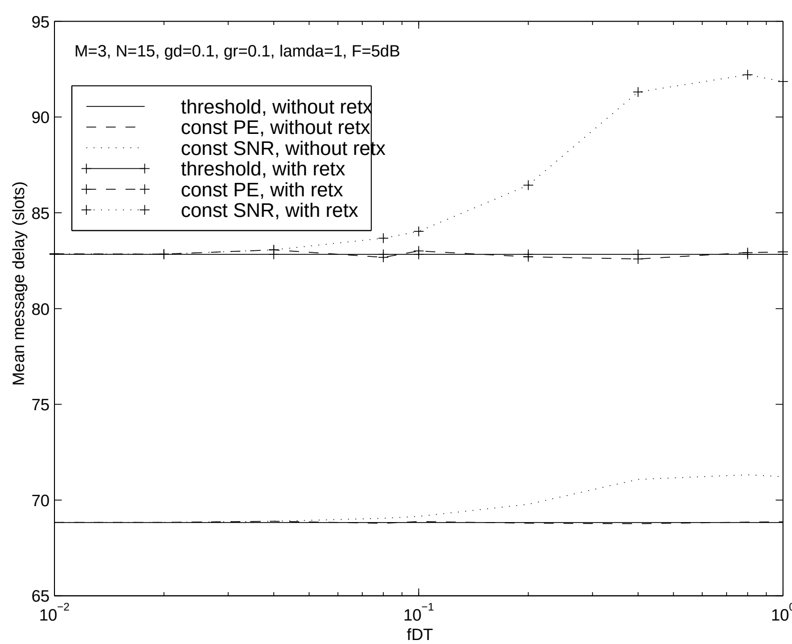

Figure 10: Mean message delay versus $f_D T$. Protocol *with* and without LL retransmission. $M = 3$, $N = 15$, $g_d = 0.1$, $g_r = 0.1$, $\lambda = 1$, $F = 5$ dB