Adaptive ARQ with Energy Efficient Backoff on Markov Fading Links

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Abstract— **In this letter, we are concerned with adaptive ARQ techniques combined with backoff strategies that exploit the bursty nature of wireless links for improved energy savings. Specifically, we propose an adaptive Go-back-N/Stop-and-Wait (GBN/SAW) protocol with backoff and present a renewal reward analysis of the throughput and energy efficiency performance of the proposed scheme. We show that the GBN protocol with a linear backoff (LBO) strategy results in energy savings of about 2 dB compared to GBN with no backoff (NBO) on highly correlated fading channels when the round-trip delays (RTD) are small, and this energy saving decreases as RTD increases. With adaptive GBN/SAW protocol with LBO, however, the energy saving is retained high even for large RTDs. In fact, the proposed adaptive GBN/SAW protocol with LBO performs quite close to that of an ideal (though not practical) backoff scheme which assumes *a priori* knowledge of the channel status in each slot.**

Index Terms— **Adaptive ARQ, backoff, energy efficiency, Go-back-N, Stop-and-Wait, round-trip delay.**

I. Introduction

Automatic repeat request (ARQ) techniques have been widely studied in the literature. Several enhancements to the three basic ARQ types, namely, stop-and-wait (SAW), go-back-N (GBN) and selective repeat (SR), have been proposed to improve throughput and delay performance [1]-[6]. For example, in order to alleviate the throughput degradation of GBN ARQ in high error rate conditions, adaptive techniques which employ multicopy retransmission (adaptation done, for example, based on number of ACKs/NAKs received) have been proposed and analyzed [1]-[4]. Some other interesting enhancements include employing packet combining [7] and adapting packet length [8]. Our focus in this letter is on techniques that can enhance the *energy efficiency performance* of ARQ schemes. Specifically, we propose and analyze a novel ARQ scheme which involves the combined use of two interesting ideas, namely, i) backoff and ii) mode switching between GBN and SAW. The basic idea in backoff is to remain idle (i.e., not transmit) if the channel condition is determined to be poor (e.g., through ACKs/NAKs received) [10]. The idea behind mode switching is to use GBN (aggressive ARQ) in good channel conditions and use SAW (non-aggressive ARQ) in bad channel conditions. The proposed scheme can be easily implemented since both the backoff as well as the mode switching are done based just on the received ACKs/NAKs. We carry out a detailed renewal reward analysis of the throughput and energy efficiency performance of the proposed scheme (defined as the average number of correctly received packets per slot and per unit energy, respectively), and show that the proposed adaptive GBN/SAW protocol with backoff results in about 2 dB of energy savings compared to ARQ without adaptation and backoff on highly correlated fading channels. Also, we note that the improved energy efficiency of the proposed scheme is achieved at the expense of a small loss in the throughput performance.

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II. ARQ Backoff Algorithms

We are interested in link layer (LL) ARQ designs that exploit channel memory with a motivation to improve energy efficiency. The proposed backoff strategy is to remain idle (i.e., not transmit) if the channel condition is determined to be poor. The channel status is determined through the received ACKs/NAKs. The transmitter idles incrementally more if the channel condition remains poor for a long time (e.g., as in the case of long channel memory for a shadowed pedestrian user). Backoff algorithms can be employed in such scenarios to adaptively determine the amount of time to idle (for example, according to the number of retransmission attempts of a LL packet). We consider that the LL packets are of constant size and that the time axis is split into slots of duration equal to one LL packet. Accordingly, the backoff time is expressed in number of slots. An infinite supply of packets at the transmitter is assumed.

In [10], an ARQ backoff protocol was considered for the case of instantaneous feedback. Here we consider the general case in which the feedback about the success/failure of a packet transmitted in slot t is available at the transmitter by the end of slot $t+m-1$ ($m \geq 1$, with $m = 1$ corresponding to instantaneous feedback). As ARQ scheme, we consider here Go-Back-N (GBN) that, although slightly less efficient than Selective Repeat, is easier to implement and to analyze.

The proposed GBN backoff protocol works as follows. As long as packets are correctly received at the receiver, the transmitter continuously sends packets in order. Suppose an error occurs in slot t. As per GBN's rules, the transmitter (that is not yet aware of the erroneous transmission) sends packets in slots $t+1$ through $t+m-1$. By the end of slot $t+m-1$ the transmitter receives the NAK (or it times out) and becomes aware of the need to retransmit the failed packet. The backoff scheme requires that, before retransmitting, the transmitter idles for d_1 slots, after which it retransmits the erroneous packet and successively (as per GBN's rules) the subsequent $m-1$ packets as well. By the end of slot $t+2m+d_1-1$, the transmitter receives feedback about the retransmission of the packet. If it is correctly received (ACK), the error is recovered and transmission continues until the next error. If on the other hand the retransmitted packet is again in error (NAK), it is retransmitted after d_2 idle slots, i.e., in slot $t+2m+d_1+d_2$, and so on until eventually the packet is correctly received.

A specific choice for d_j values corresponds to a particular backoff scheme. In this paper we consider three of these schemes: $i)$ no backoff (NBO), where $d_j = 0, j > 0$, i.e., basic GBN; $ii)$ linear backoff (LBO), where $d_j = j, j > 0$; $iii)$ ideal backoff (IBO), where d_j is the minimum number of slots that should be skipped until a successful transmission can take place. More specifically, in IBO we assume that, after being notified of an erroneous transmission, the sender has *a priori* knowledge of the channel status in each subsequent slot and chooses the first available good channel slot to retransmit the packet (which is then correctly received with certainty). All subsequent packets are then sent in consecutive slots. This IBO scheme, which is obviously impossible to implement in practice, is considered here because it results in the best possible energy performance for a GBN backoff scheme (retransmissions are *never* performed in a bad slot), and can act as a benchmark to compare the energy savings achieved in practically implementable backoff algorithms. As a final point, we remark that LBO is one possibility for deterministic backoff, and other choices for the d_j's, e.g., exponential or even random, could be made [10].

The tradeoff involved in the use of backoff algorithms is that we may lose some LL throughput (because of the possibility of remaining idle in good slots), but will save energy (avoiding transmission during bad slots). The throughput loss, however, is typically small, while energy savings may be significant, particularly when the channel fades are highly correlated and the round-trip delay is small [10]. In the sequel, we present the analysis of LBO for the case of general round-trip delay. The analysis of other backoff strategies can be carried out likewise.

III. Renewal Analysis

We employ a first-order Markov chain representation of the wireless channel [11] with Markov parameters p and $(1-q)$ being the probabilities that the packet transmitted in the kth slot is a success given the packet transmitted in the $(k-1)$st slot is a success or a failure, respectively. In other words, the channel transition probability matrix is

$$M_c = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}, \qquad (1)$$

where $p_{ij},\ i,j \in \{0,1\}$ denotes the transition probability of moving from state i to state j in a single slot, and '0' and '1' are the good and bad channel states, respectively. This simple two-state model is able to effectively capture the error burstiness, that is one of the main features of the channel we consider, while still allowing for an analytical approach and close-form solutions. More sophisticated models could be used as well (e.g., Markov chains with a larger number of states), that would translate in a conceptually similar treatment but would require a significantly more cumbersome notation. Due to space limitations, we do not report this more comprehensive approach in the present paper. The interested reader will find in [12] an example of how Markov protocol analysis can be extended to the N-state case.

In GBN, not all successful slots correspond to useful packet transmissions. More specifically, even though the channel can be good in a slot, the corresponding packet transmission may be discarded by the receiver due to the protocol rules, thereby not contributing to the throughput (e.g., the packets sent in the slots immediately after a channel failure are lost regardless of the channel conditions). Define a *throughput opportunity* as a slot which, if the channel is good, will positively contribute to the throughput count[1].

According to the protocol description and due to the Markov channel model, the packet transmission process is a renewal process [13] where the renewal instants are the times at which a packet is transmitted for the first time in a throughput opportunity. As such, we can count one packet success at each renewal instant, and the throughput (number of successfully delivered packets per slot) is the inverse of the average number of slots between successive renewals. Here, we analytically derive the statistics of the inter-renewal times for the schemes considered in this paper.

Ideal Backoff (IBO): In IBO, consider the case in which a slot contains a successful transmission, i.e., the slot is a successful channel slot and is a throughput opportunity. Assuming that the packet transmission in the current slot ($t = 0$) is successful, we need to determine what is the average time until the next successful packet transmission. After the current slot (which counts as one), if the following slot $t = 1$ is a failure the time before the next retransmission of the failed packet can be computed as follows. There are m transmission slots (the one with the failed packet and the $m-1$ subsequent slots) in all cases. The retransmission occurs in the first good channel slot after then. With probability $p_{10}(m)$, [2] the first available slot ($t = m+1$) is a good one, and the corresponding delay is $m+1$. With probability $p_{11}(m)$, slot $m+1$ is bad, and for the next good slot one needs to wait for a geometric number of slots, with average $1/p_{10}$. This leads to the following expression for the average time between two successful transmissions:

$$1 + p_{01}\left(m + \frac{p_{11}(m)}{p_{10}}\right). \qquad (2)$$

For the average number of packet transmissions, we reason as before, except that we do not count the geometric number of slots (during which no transmissions occur), so we have $1+p_{01}m$.

Deterministic Backoff: In this case, following the j-th failure, the next transmission attempt is performed after d_j idle slots. According to the definition of the backoff schemes, we have $d_j = j$ for LBO and $d_j = 0$ for NBO.

[1]As an example, starting from a good channel state, all slots until (and including) the first bad slot are throughput opportunities, whereas the $m-1$ slots following the first bad slot are not, since even in the presence of good channel conditions the corresponding transmissions are wasted.

[2]$p_{ij}(m)$ is the m-step channel transition probability (i.e., the probability that the channel is in state j at time $t+m$ given that it was in state i at time t), and is the ij-th element of the m-th power of the channel transition matrix, M_c^m.

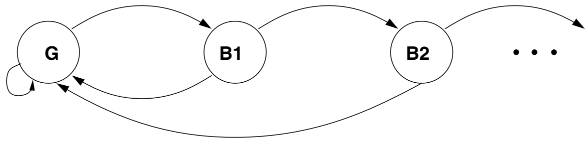

Fig. 1. Flow diagram for GBN with backoff.

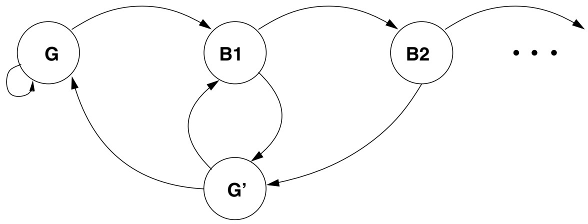

Fig. 2. Flow diagram for adaptive GBN/SAW with backoff.

Consider the following set of states (see Fig. 1): state G corresponds to a successful transmission of a packet (i.e., a packet which is counted as useful throughput according to the protocol rules), whereas states $B_j, j = 1, 2, \ldots$, correspond to the subsequent failed attempts of a packet. That is, as long as packets are correctly sent, the protocol stays in state G. When a packet transmission encounters a bad channel and fails, the protocol goes to state B_1. From state B_1, if the next attempt to transmit the packet ($m + d_1$ slots later according to the BO mechanism) is successful, the protocol goes back to state G, otherwise it goes to state B_2, where it stays for $m + d_2$ slots, and so on. More specifically, the transition structure through this diagram is as follows. From state G we can go back to state G with probability p_{00} or to state B_1 with probability p_{01}. From state B_j we can go to state G with probability $p_{10}(m + d_j)$ or to state B_{j+1} with probability $p_{11}(m + d_j)$. The time associated with a visit to state G is a single slot, whereas for state B_j it is $m + d_j$ slots.

Suppose a packet transmission has failed and the protocol goes to state B_1. We want to compute the average time it takes until state G is again reached. Note in fact that this is the first valid transmission to be counted in throughput, and in fact each visit to state G is counted as a successful packet. From state B_1, the system will visit a number of B states until it eventually goes back to state G. The probability that the last B state visited is B_j (i.e., that the packet is correctly received at its $(j + 1)$st transmission, given that the first transmission was a failure) is found as

$$\left(\prod_{i=1}^{j-1} p_{11}(m + d_i)\right) p_{10}(m + d_j), \quad (3)$$

and the associated average time to go from B_1 to G is then computed as

$$D_{BG} = \sum_{j=1}^{\infty} \left(jm + \sum_{i=1}^{j} d_i\right) \left(\prod_{i=1}^{j-1} p_{11}(m + d_i)\right) p_{10}(m+d_j). \quad (4)$$

The corresponding number of packet transmissions is computed similarly, taking into account that in the backoff slots no transmissions occur:

$$T_{BG} = \sum_{j=1}^{\infty} jm \left(\prod_{i=1}^{j-1} p_{11}(m + d_i)\right) p_{10}(m + d_j). \quad (5)$$

Considering the visits to state G as the renewal instants, we want to compute the average time between successive visits. This time is equal to one slot if the next transmission is successful, i.e., with probability p_{00}, whereas it is equal to $1 + D_{BG}$ with probability p_{01}, i.e.,

$$D_{GG} = 1 + p_{01} D_{BG}. \quad (6)$$

Similarly, for the number of transmissions between successive visits to state G we have

$$T_{GG} = 1 + p_{01} T_{BG}. \quad (7)$$

Finally, the throughput and the energy efficiency of the scheme can be found as $1/D_{GG}$ and $1/T_{GG}$, respectively. The appropriate choice of the variables d_i will correspond to the various backoff schemes, with the scheme without backoff (NBO) corresponding to $d_i = 0$ for all i (notice that in this case we have $T_{GG} = D_{GG}$).

IV. An Adaptive GBN/SAW scheme

This scheme is a modified version of the previous one analyzed above. The idea here is to use GBN (aggressive ARQ) in good channel conditions and SAW (non-aggressive ARQ) in bad channel conditions. GBN is used till a loss is detected. SAW is used till the lost packet is retransmitted successfully. Once successfully retransmitted, the protocol switches back to GBN, and so on. This adaptive GBN/SAW scheme uses a similar concept as in [9], adding backoff to it. As long as correct transmissions occur, the system stays in state G as before (see Fig. 2). As soon as there is an erroneous transmission, the system goes to state B_1. The transition structure from there and the times associated to the various states are the same as before, except for two differences: 1) the number of packet transmissions per transition is now 1 instead of m for all transitions from the B states (SAW instead of GBN); 2) after the eventual correct retransmission of a packet the GBN mode cannot be immediately restored, as the success of that transmission takes m slots to be known at the transmitter. This second point requires that an additional state be introduced. This state, that we call G', corresponds to a correct transmission but is associated to a time of m slots instead of only one. From state G', the system goes to state G with probability $p_{00}(m)$, while it goes to state B_1 with probability $p_{01}(m)$.

The evolution of the system is the following: when leaving state G the system goes to state B_1, from where it eventually reaches state G'. From there, the system either goes once again to B_1, or ends the cycle going to G. The number of times the system loops between B_1 and G' is a geometric random variable with mean $1/p_{00}(m)$. The average time taken in going from B_1 to G' is

$$D_{BG'} = \sum_{j=1}^{\infty} \left(jm + \sum_{i=1}^{j} d_i\right) \left(\prod_{i=1}^{j-1} p_{11}(m + d_i)\right) p_{10}(m+d_j), \quad (8)$$

and the average number of transmissions is

$$T_{BG'} = \sum_{j=1}^{\infty} j \left(\prod_{i=1}^{j-1} p_{11}(m+d_i) \right) p_{10}(m+d_j), \tag{9}$$

where we took into account that the retransmissions are done using SAW, i.e., one per transition. The total delay between two consecutive visits to state G is then found as

$$D_{GG} = 1 + \frac{m + D_{BG'}}{p_{00}(m)} p_{01}, \tag{10}$$

where we accounted for the fact that the number of $B_1 - G'$ loops is a geometric r.v., and that each loop on average lasts $D_{BG'}$ plus the m slots for the transition from G' to B_1 (which in the last case leads to G instead but has the same duration). A similar argument leads to

$$T_{GG} = 1 + \frac{m + T_{BG'}}{p_{00}(m)} p_{01}. \tag{11}$$

Finally, note that in this case the successful transmissions correspond to visits to both states G and G'. Therefore, during the evolution between two consecutive visits to state G we may have more than one success. The average number of successes is found as

$$S_{GG} = 1 + \frac{1}{p_{00}(m)} p_{01}. \tag{12}$$

The throughput and energy efficiency can be calculated as S_{GG}/D_{GG} and S_{GG}/T_{GG}, respectively.

V. Results and Discussions

In this section, we present the analytical results of the throughput and energy efficiency performance (which we have verified by simulations as well). Figure 3 shows the LL throughput performance of GBN without and with backoff, as a function of the number of RTD slots, m. The following three cases are considered: 1) GBN with Ideal Backoff (IBO), 2) GBN with No Backoff (NBO), and 3) GBN with Linear Backoff (LBO). We computed the throughput and energy efficiency using channel parameters $p = 0.998208$ and $q = 0.997313$ corresponding to a fade margin of about 3 dB (we consider this fade margin since ARQ schemes need to be robust even in high error rate conditions) and a normalized Doppler bandwidth $f_dT = 0.001$ (which represents a high channel correlation scenario). For the first-order Markov representation of the multipath Rayleigh fading, the relation between average LL packet error rate (ϵ), fade margin (F), and parameters p and q is given by [11]: $\epsilon = 1 - e^{-1/F} = (1-p)/(2-p-q)$ and $(1-q) = [Q(\theta, \rho\theta) - Q(\rho\theta, \theta)]/(e^{1/F} - 1)$, where $\theta = \sqrt{2/F(1-\rho^2)}$, $\rho = J_0(2\pi f_d \tau)$ is the correlation coefficient between two samples of the complex gain of the fading process taken T seconds apart, f_d is the Doppler bandwidth (related to user speed v and carrier wavelength λ as $f_d = v/\lambda$), $J_0(\cdot)$ is the Bessel function of the first kind and zeroth order, and $Q(\cdot,\cdot)$ is the Marcum Q function. At a carrier frequency of 900 MHz, $f_dT = 0.001$ corresponds to user speed of 1.2 Km/h, link speed of 1 Mbps and LL packet size of 1000 bits. Performance plots for the independent (i.i.d.) error case are also computed and plotted. From Fig. 3 the following observations can be made. As expected, GBN with IBO gives the best LL throughput performance for both $f_d = 0.001$ as well as i.i.d. For $f_dT = 0.001$, the performance of GBN with NBO is quite close to that of GBN with IBO. We pointed out earlier that LBO may lose some throughput (due to the possibility of remaining idle during good slots) compared to NBO. However, as can be seen in Fig. 3, this throughput loss for LBO compared to NBO turns out to be quite small when the channel correlation is high ($f_dT = 0.001$), i.e., there is no major throughput loss due to backoff. On the other hand, the backoff results in significant energy savings as can be observed in Fig. 4. In the i.i.d. error case, there is no significant benefit in using backoff due to the memoryless channel behavior. Channels with short memory will behave similar to the i.i.d. error case, even though some backoff strategy may still be considered. A more detailed analysis of this case is beyond the scope of this paper and is left for future study.

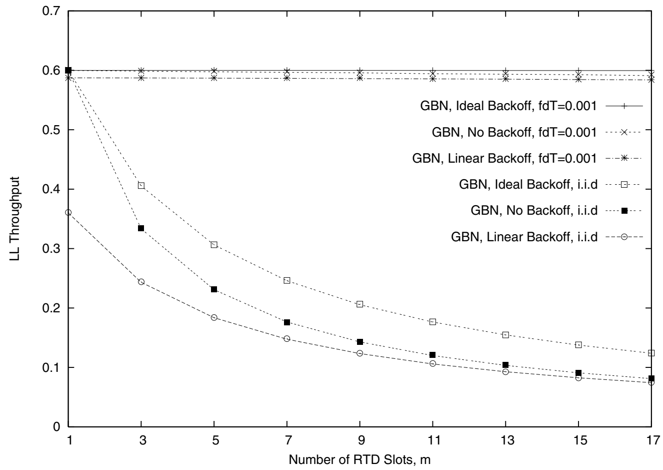

Fig. 3. LL Throughput as a function of number of round-trip delay slots, m, for GBN protocol without and with LL backoff. $F = 3$ dB.

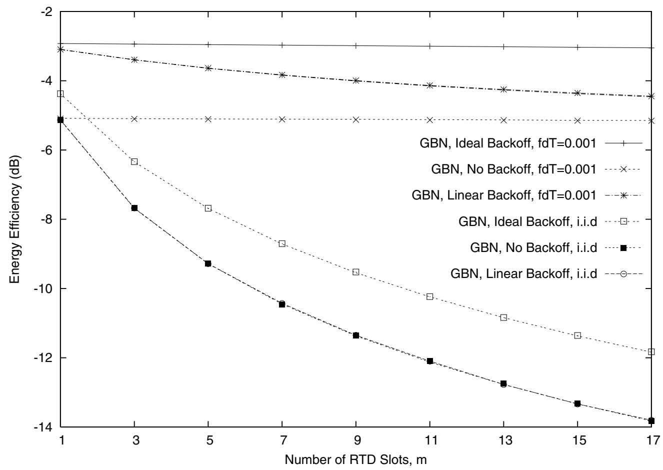

Fig. 4. Energy Efficiency as a function of number of round-trip delay slots, m, for GBN protocol without and with LL backoff. $F = 3$ dB.

For the same schemes and system parameters in Fig. 3, Fig. 4 shows the energy efficiency (normalized by the fading margin, F, so that the unit of energy corresponds to a packet transmission with $F = 1$) as a function of the number of round-trip delay slots, m. It is noted that for $m = 1$, the

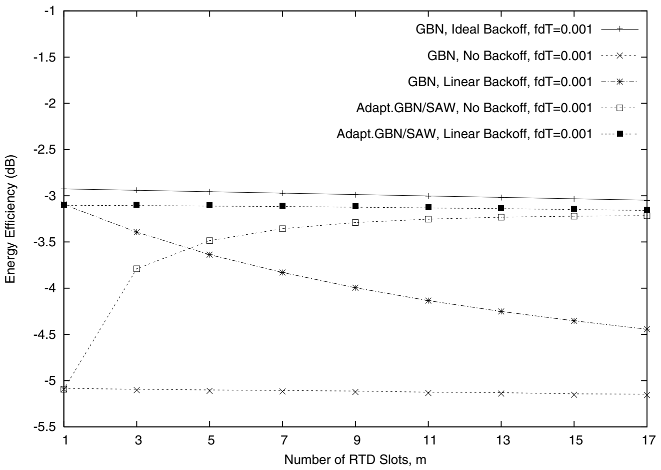

Fig. 5. Energy Efficiency vs number of round-trip delay slots, m, for adaptive GBN/SAW protocol and GBN protocol without and with LL backoff. $F = 3$ dB.

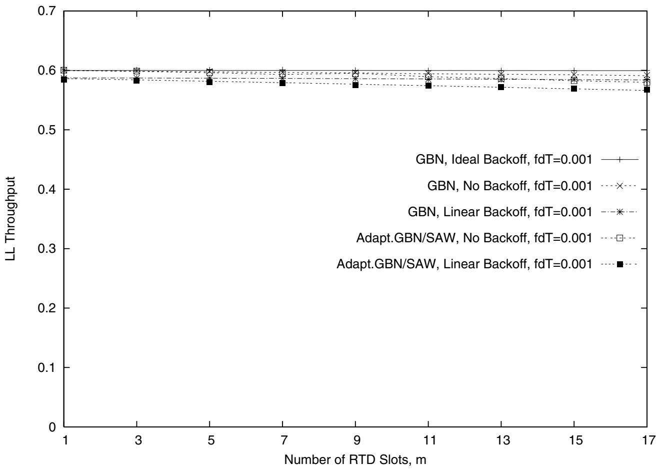

Fig. 6. LL Throughput vs number of round-trip delay slots, m, for adaptive GBN/SAW protocol and GBN protocol without and with LL backoff. $F = 3$ dB.

best possible energy efficiency is $1/F$ (i.e., no transmissions in any of the bad slots) which is approached by IBO. The energy efficiency achieved is much less $\big($equal to $(1-\epsilon)/F\big)$ for $m = 1$ when no backoff is performed (NBO). It can further be observed that when linear backoff is performed (LBO), the energy efficiency improves by about 2 dB compared to NBO. In fact, for $m = 1$, LBO achieves energy efficiency close to that of IBO. But this energy efficiency improvement of LBO over NBO decreases as m increases. This is because GBN is applied even during loss recovery, i.e., a retransmission attempt after a backoff involves transmission in m slots, and when the channel is highly correlated all these m slot transmissions can be lost. This reduces the energy efficiency of LBO with increasing m. This performance behaviour motivates us to use a less aggressive ARQ (e.g., SAW) during loss recovery (bad periods) to get closer to the energy efficiency bound of IBO. Accordingly, adaptive GBN/SAW schemes without and with backoff are considered in Figs. 5 and 6.

Figure 5 shows the energy efficiency of the adaptive GBN/SAW without and with backoff in comparison with GBN without and with backoff for $f_dT = 0.001$. Figure 6 shows the corresponding LL throughput. From Fig. 5 it can be observed that the energy efficiency of adaptive GBN/SAW with no backoff (i.e., no backoff during SAW phase) gets close to that of IBO for large m, but for small m, the energy efficiency is still poor (closer to NBO). This indicates that further improvement may be possible for small m if backoff is introduced during SAW phase (i.e., adaptive GBN/SAW with backoff). As can be seen, the adaptive GBN/SAW with LBO almost achieves the energy efficiency bound of IBO even for large values of m. As can be seen from Fig. 6, the loss in LL throughput due to backoff is very small. Thus, the proposed adaptive GBN/SAW protocol with backoff results in good energy savings without any significant loss in throughput.

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