BER-Optimal Linear Parallel Interference Cancellation for Multicarrier DS-CDMA in Rayleigh Fading

Shamaiah Manohar, Vibhor Tikiya, Ramesh Annavajjala, *Member, IEEE*, and Ananthanarayanan Chockalingam, *Senior Member, IEEE*

***Abstract*—In this paper, we consider the design and bit-error performance analysis of linear parallel interference cancellers (LPIC) for multicarrier (MC) direct-sequence code division multiple access (DS-CDMA) systems. We propose an LPIC scheme where we estimate and cancel the multiple access interference (MAI) based on the soft decision outputs on individual subcarriers, and the interference cancelled outputs on different subcarriers are combined to form the final decision statistic. We scale the MAI estimate on individual subcarriers by a weight before cancellation. In order to choose these weights optimally, we derive exact closed-form expressions for the bit-error rate (BER) at the output of different stages of the LPIC, which we minimize to obtain the optimum weights for the different stages. In addition, using an alternate approach involving the characteristic function of the decision variable, we derive BER expressions for the weighted LPIC scheme, matched filter (MF) detector, decorrelating detector, and minimum mean square error (MMSE) detector for the considered multicarrier DS-CDMA system. We show that the proposed BER-optimized weighted LPIC scheme performs better than the MF detector and the conventional LPIC scheme (where the weights are taken to be unity), and close to the decorrelating and MMSE detectors.**

***Index Terms*—Linear parallel interference cancellation, multicarrier DS-CDMA, optimum weights.**

I. Introduction

RECENTLY, there has been an increased interest in multiuser, multicarrier (MC) systems [e.g., multicarrier code-division multiple access (CDMA), orthogonal frequency division multiple acess (OFDMA)] for broadband wireless communications [1]. The MC approach in CDMA offers several advantages including robustness in fading and interference, operation at lower chip rates (and, hence, lower device clock speeds and device power consumption), and noncontiguous bandwidth operation [2], [3]. Because of their potential to remove multiple access interference (MAI), and hence increase CDMA system capacity, multiuser detection schemes in general [4], and interference cancellation (IC) techniques in particular, applied to multicarrier direct-sequence CDMA (MC DS-CDMA), are getting increased research attention [5]–[7]. Successive and parallel interference cancellers are attractive owing to their implementation simplicity and good performance [4]. Parallel interference cancellation (PIC) lends itself to a multistage implementation where the decision statistics of the users from the previous stage are used to estimate and cancel the MAI in the current stage, and a final decision statistic is obtained at the last stage. In this paper, we are concerned with the design and bit-error performance analysis of linear PICs (LPICs) [10]–[19] for MC DS-CDMA systems. LPICs have the advantages of implementation simplicity, analytical tractability, and good performance.

The conventional way to realize LPIC schemes is to use unscaled values of the soft outputs from different users for MAI estimation. A known problem with this conventional LPIC (CLPIC) approach is that it can perform even worse than the matched filter (MF) detector (where cancellation is not done), particularly at low signal-to-noise ratios (SNRs) [10]. This is because the MAI estimates obtained using unscaled values of soft outputs can become quite inaccurate under poor channel conditions (e.g., low SNRs) to such an extent that it may be better not to do cancellation. This problem can be alleviated by properly weighing (scaling) the MAI estimates before cancellation [10], [11]. A key question in this regard is how to choose these weights (scaling factors) for different stages of the LPIC. For the case of single carrier CDMA, the issue of the choice of the weights in LPIC has been addressed in [11], [17], and [18] for additive white Gaussian noise (AWGN) channels, and in [16] for Rayleigh fading and diversity channels.

In this paper, we propose a weighted LPIC (WLPIC) scheme for a MC DS-CDMA system, where we scale the MAI estimate on individual subcarriers by a weight before cancellation [19]. One way to optimally choose the weights is to derive analytical expressions for the average signal-to-interference ratio (SIR) at the output of the IC stages as a function of the weights, and maximize these SIR expressions to obtain the optimum weights

Paper approved by L. Rasmussen, the Editor for Iterative Detection, Decoding, and ARQ of the IEEE Communications Society. Manuscript received August 11, 2005; revised March 28, 2006 and June 30, 2006. This work was supported in part by the Swarnajayanti Fellowship, Department of Science and Technology, New Delhi, Government of India, under Project 6/3/2002-S.F. This paper was presented in part at the IEEE Global Telecommunications Conference, St. Louis, MO, 2005.

S. Manohar is with the Department of Electrical and Communication Engineering, Indian Institute of Science, Bangalore 560012, India. He is also with the Honeywell Technology Solutions Pvt. Ltd., Bangalore 560076, India (e-mail: manohar_s_126@yahoo.com).

V. Tikiya is with the Department of Electrical and Communication Engineering, Indian Institute of Science, Bangalore 560012, India. He is also with the Indian Institute of Management, Ahmedabad, India (e-mail: vibhor_tikiya@yahoo.com).

R. Annavajjala is with ArrayComm LLC, San Jose, CA 95131 USA (e-mail: ramesh.annavajjala@gmail.com).

A. Chockalingam is with the Department of Electrical and Communication Engineering, Indian Institute of Science, Bangalore 560012, India (e-mail: achockal@ece.iisc.ernet.in).

Digital Object Identifier 10.1109/TCOMM.2007.898860

for different stages, as done in [11] and [16] for single carrier CDMA. However, for the MC DS-CDMA scheme we consider in this paper, obtaining closed-form expressions for the average SIR at the output of different canceller stages is rather difficult. An alternate approach to obtain the optimum weights can be to derive expressions for the average bit-error rate (BER) at the output of each IC stage of the MC DS-CDMA system in terms of the weights and choose those weights that minimize this average BER. A new contribution in this paper, in this context, is that we are able to derive exact closed-form expressions for the average BER for different stages of the LPIC scheme for MC DS-CDMA, which we minimize and obtain the optimum weights for different stages. We point out that the BER analysis does not resort to Gaussian approximation of the interference. Another new contribution is that, using an alternate approach involving the characteristic function of the decision variable, we derive BER expressions for the weighted LPIC scheme, MF detector, decorrelating (DC) detector, and minimum mean square error (MMSE) detector for the considered MC DS-CDMA system. We show that the proposed BER-optimized weighted LPIC scheme for MC DS-CDMA performs better than the MF detector and the conventional LPIC scheme (where the weights are taken to be unity), and close to the DC and MMSE detectors.

The rest of the paper is organized as follows. In Section II, we present the MC DS-CDMA system model. In Section III, we present the proposed weighted LPIC scheme for MC DS-CDMA, and the corresponding BER analysis to obtain optimum weights. Numerical results and discussions are presented in Section IV. An alternate derivation of the BER expressions is presented in Section V. Conclusions are given in Section VI.

II. System Model

We consider a K-user synchronous multicarrier DS-CDMA system (an asynchronous system can be considered likewise). Let M denote the number of subcarriers. Let $b_k \in \{+1,-1\}$ denote the binary data symbol of the kth user, which is sent in parallel on M subcarriers [1]. It is assumed that the channel is frequency nonselective on each subcarrier and the fading is slow (assumed constant over one bit interval) and independent from one subcarrier to the other.

Let $\mathbf{y}^{(i)} = [y_1^{(i)}, y_2^{(i)}, \ldots, y_K^{(i)}]^T$, where $[.]^T$ denotes the transpose operator, denote the K-length received signal vector on the ith subcarrier; i.e., $y_k^{(i)}$ is the output of the kth user's matched filter on the ith subcarrier. Assuming that the intercarrier interference is negligible, the K-length received signal vector on the ith subcarrier $\mathbf{y}^{(i)}$ can be written in the form

$$\mathbf{y}^{(i)} = \mathbf{C}^{(i)}\mathbf{H}^{(i)}\mathbf{b} + \mathbf{n}^{(i)} \tag{1}$$

where $\mathbf{C}^{(i)}$ is the $K \times K$ cross-correlation matrix on the ith subcarrier, given by

$$\mathbf{C}^{(i)} = \begin{bmatrix} 1 & \rho_{12}^{(i)} & \cdots & \rho_{1K}^{(i)} \\ \rho_{21}^{(i)} & 1 & \cdots & \rho_{2K}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K1}^{(i)} & \rho_{K2}^{(i)} & \cdots & 1 \end{bmatrix} \tag{2}$$

where $\rho_{lj}^{(i)}$ is the correlation coefficient between the signature waveforms of the lth and jth users on the ith subcarrier. $\mathbf{H}^{(i)}$ represents the $K \times K$ channel matrix, given by

$$\mathbf{H}^{(i)} = \operatorname{diag}\left\{h_1^{(i)}, h_2^{(i)}, \cdots, h_K^{(i)}\right\} \tag{3}$$

where the channel coefficients $h_k^{(i)}$, $i = 1, 2, \cdots, M$, are assumed to be i.i.d. complex Gaussian r.v's (i.e., fade amplitudes are Rayleigh distributed) with zero mean and $E[(h_{kI}^{(i)})^2] = E[(h_{kQ}^{(i)})^2] = 1$, where $h_{kI}^{(i)}$ and $h_{kQ}^{(i)}$ are the real and imaginary parts of $h_k^{(i)}$. The K-length data vector $\mathbf{b}$ is given by

$$\mathbf{b} = [A_1 b_1 \quad A_2 b_2 \quad \cdots \quad A_K b_K]^T \tag{4}$$

where A_k denotes the transmit amplitude of the kth user, and $b_k \in \{+1,-1\}$ denotes the data bit of the kth user. The K-length noise vector $\mathbf{n}^{(i)}$ is given by

$$\mathbf{n}^{(i)} = \left[n_1^{(i)} \quad n_2^{(i)} \quad \cdots \quad n_K^{(i)}\right]^T \tag{5}$$

where $n_k^{(i)}$ denotes the additive noise component of the kth user on the ith subcarrier, which is assumed to be complex Gaussian with zero mean with $E[n_k^{(i)}(n_j^{(i)})^*] = 2\sigma^2$ when $j = k$ and $E[n_k^{(i)}(n_j^{(i)})^*] = 2\sigma^2\rho_{kj}^{(i)}$ when $j \neq k$.

III. Weighted LPIC Scheme for MC DS-CDMA

The proposed interference cancellation on the ith subcarrier is explained as follows. From (1), the first stage output (i.e., MF output) of the desired user k on the ith subcarrier is given by

$$y_k^{(i)} = A_k b_k h_k^{(i)} + \underbrace{\sum_{j=1\ j\neq k}^{K} \rho_{jk}^{(i)} A_j b_j h_j^{(i)}}_{\text{MAI}} + n_k^{(i)} \tag{6}$$

where the first term is the desired signal term, and the second term is the MAI term which we seek to estimate and cancel. Towards that end, consider the following MAI estimate on the ith subcarrier obtained using the soft values of the MF outputs of the interfering users on the ith subcarrier

$$\begin{aligned} \widehat{\text{MAI}}^{(i)} &= \sum_{j=1,j\neq k}^{K} \rho_{jk}^{(i)} y_j^{(i)} \\ &= \sum_{j=1,j\neq k}^{K} \rho_{jk}^{(i)} \left(A_j b_j h_j^{(i)} + \sum_{r=1\ r\neq j}^{K} \rho_{jr}^{(i)} A_r b_r h_r^{(i)} + n_j^{(i)} \right) \\ &= \underbrace{\sum_{j=1,j\neq k}^{K} \rho_{jk}^{(i)} A_j b_j h_j^{(i)}}_{T_1} + \underbrace{\sum_{j=1,j\neq k}^{K} \left(\rho_{jk}^{(i)}\right)^2 A_k b_k h_k^{(i)}}_{T_2\text{: desired signal leak}} \\ &\quad + \underbrace{\sum_{j=1,j\neq k}^{K} \rho_{jk}^{(i)} \left(\sum_{r=1\ r\neq j,k}^{K} \rho_{jr}^{(i)} A_r b_r h_r^{(i)} + n_j^{(i)} \right)}_{T_3}. \end{aligned} \tag{7}$$

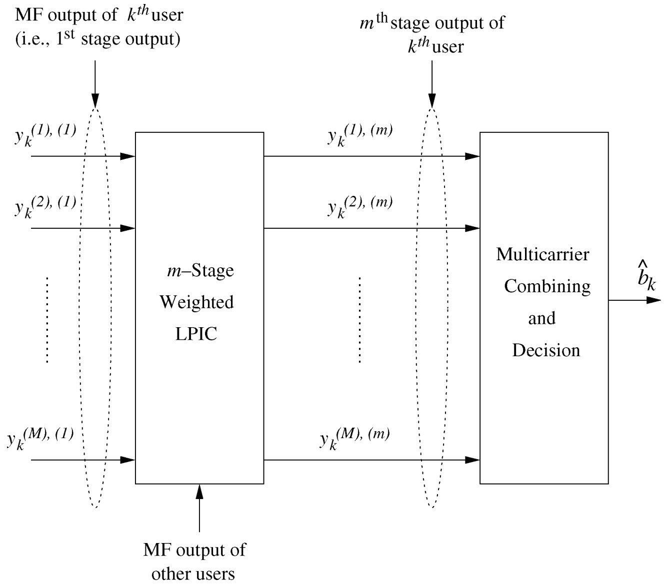


Fig. 1. Multicarrier DS-CDMA receiver with weighted LPIC. $y_k^{(i),(m)}$: mth stage output of the kth user on the ith subcarrier.

Note that the first term T_1 in the above estimate is the same as the MAI term in (6). In addition, the estimate also consists of a desired signal component T_2 ["desired signal" leak term in (7)], and other user and noise components T_3. Subtracting (7) from (6) will perfectly cancel the MAI term in (6); however, in the process some desired signal will also be removed. Since the amount of desired signal removed is rather small (which is proportional to $(\rho_{jk}^{(i)})^2$), the benefit of MAI removal can typically outweigh the effect of desired signal removal. However, when noise is high (i.e., low SNRs), the effect of T_3 term in (7) can dominate to the extent that it may be better not to do cancellation. To alleviate this problem, we scale the MAI estimate by a weight (before cancellation), which we optimize to achieve minimum BER performance. Based on the above, we present the proposed weighted LPIC scheme for the mth stage $m > 1$, in the following subsection.

A. Interference Cancellation on ith Subcarrier in Stage-m

The MC DS-CDMA receiver with weighted LPIC is shown in Fig. 1. For a desired user k, an estimate of the MAI from other interfering users j, $j \neq k$, on a given subcarrier i, for cancellation in a given stage m, $m > 1$, is obtained from the soft outputs of the interfering users' previous stage outputs, $y_j^{(i),(m-1)}$. This MAI estimate is then scaled by a weighting factor, $w_{jk}^{(i),(m)}$, before cancellation. Accordingly, the mth stage interference cancelled output on the ith subcarrier for the desired user k, $y_k^{(i),(m)}$, is given by

$$y_k^{(i),(m)} = y_k^{(i),(1)} - \sum_{j=1,j\neq k}^{K} w_{jk}^{(i),(m)} \rho_{jk}^{(i)} y_j^{(i),(m-1)} \quad (8)$$

where we take the MF output as the first stage output, i.e., $y_k^{(i),(1)} = y_k^{(i)}$, and the second term on the RHS is the weighted MAI estimate. The interference cancelled outputs on all subcarriers of the desired user are then coherently combined to get the combined output, $v_k^{(m)}$, as

$$v_k^{(m)} = \sum_{i=1}^{M} \left(h_k^{(i)}\right)^* y_k^{(i),(m)}. \quad (9)$$

The bit decision at the mth stage output is then obtained as

$$\hat{b}_k^{(m)} = \text{sgn}\left(\text{Re}\left(v_k^{(m)}\right)\right). \quad (10)$$

Note that both the conventional LPIC as well as the MF detector become special cases of the above weighted LPIC for $w_{jk}^{(i),(m)} = 1 \forall i, j, m$ and $w_{jk}^{(i),(m)} = 0 \forall i, j, m$, respectively. In [20], a larger class of polynomial detectors is shown to be described by an equation like (8).

For the weighted LPIC scheme in (8), the choice of the weights $w_{jk}^{(i),(m)}$ can be made based on maximizing the average SIR at the combined output or minimizing the average BER of each stage. For the MC DS-CDMA system considered in the above, obtaining closed-form expressions for the average SIR at the output of different canceller stages is rather difficult. However, we could derive exact closed-form expressions for the average BER of the system which when minimized can give optimum weights. We present the derivation of the BER expressions for the second and third stages of the weighted LPIC in the following subsection.

B. Derivation of BER Expressions

1) Second Stage Output Statistics and BER: From (9) and (8), the weighted interference cancelled and multicarrier combined output of the second stage (i.e., $m = 2$) for the desired user k can be written as

$$\begin{aligned} v_k^{(2)} &= \sum_{i=1}^{M} \left(h_k^{(i)}\right)^* y_k^{(i),(2)} \\ &= \sum_{i=1}^{M} \left(h_k^{(i)}\right)^* \left(y_k^{(i),(1)} - \sum_{j=1,j\neq k}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} y_j^{(i),(1)}\right) \\ &= A_k b_k \sum_{i=1}^{M} \left|h_k^{(i)}\right|^2 \left(1 - \sum_{j=1,j\neq k}^{K} w_{jk}^{(i),(2)} \left(\rho_{jk}^{(i)}\right)^2\right) \\ &\quad + I_2 + N_2 \end{aligned} \quad (11)$$

where

$$I_2 = \sum_{i=1}^{M} \left(h_k^{(i)}\right)^* \left[\sum_{j=1,j\neq k}^{K} \left(1 - w_{jk}^{(i),(2)}\right) A_j b_j h_j^{(i)} \rho_{jk}^{(i)} - \sum_{j=1,j\neq k}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} \sum_{\substack{l=1\\ l\neq j,k}}^{K} \rho_{lj}^{(i)} A_l b_l h_l^{(i)}\right] \quad (12)$$

$$N_2 = \sum_{i=1}^{M} \left(h_k^{(i)}\right)^* \left[n_k^{(i)} - \sum_{j=1,j\neq k}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} n_j^{(i)}\right]. \quad (13)$$

The second step in (11) results by using (6) and (7) in the first step. The terms I_2 and N_2 in (11) represent the interference and noise terms introduced in the second stage output due to imperfect cancellation in using the soft output values from the first (i.e., MF) stage. Conditioned on the desired user's channel coefficients, it can be seen that I_2 and N_2 are independent Gaussian r.v's, each with mean zero. The conditional variances of I_2 and N_2 are denoted by $\sigma^2_{I_2}$ and $\sigma^2_{N_2}$, respectively, which can be derived as follows. Since N_2 in (13) is a Gaussian r.v. with zero mean, $\sigma^2_{N_2}$ is given by

$$\sigma^2_{N_2} = E\left[N_2 N_2^* | \mathbf{h}_k\right] = \sum_{i=1}^{M} \left|h_k^{(i)}\right|^2 \sigma^2_{N_{(i,2)}} \tag{14}$$

where $\mathbf{h}_k = [h_k^{(1)}, h_k^{(2)} \cdots, h_k^{(M)}]$, and

$$\begin{aligned}\sigma^2_{N_{(i,2)}} &= E\left[\left(n_k^{(i)} - \sum_{j=1, j\neq k}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} n_j^{(i)}\right) \times \left(n_k^{(i)} - \sum_{j=1, j\neq k}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} n_j^{(i)}\right)^*\right] \\ &= E\left[\left|n_k^{(i)}\right|^2\right] - 2\sum_{\substack{j=1\\ j\neq k}}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} E\left[n_k^{(i)} \left(n_j^{(i)}\right)^*\right] \\ &\quad + \sum_{\substack{l=1\\ l\neq k}}^{K} w_{lk}^{(i),(2)} \rho_{lk}^{(i)} \sum_{\substack{j=1\\ j\neq k}}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} E\left[n_j^{(i)} \left(n_l^{(i)}\right)^*\right] \\ &= 2\sigma^2 \left(1 - 2\sum_{\substack{j=1\\ j\neq k}}^{K} w_{jk}^{(i),(2)} \left(\rho_{jk}^{(i)}\right)^2 + \sum_{\substack{l=1\\ l\neq k}}^{K} w_{lk}^{(i),(2)} \rho_{lk}^{(i)} \times \sum_{\substack{j=1\\ j\neq k}}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} \rho_{jl}^{(i)}\right)\end{aligned} \tag{15}$$

where we have used $E[n_k^{(i)} (n_j^{(i)})^*] = 2\sigma^2 \rho_{kj}^{(i)}$, for $j \neq k$ and $2\sigma^2$ for $j = k$. To derive $\sigma^2_{I_2}$, note that I_2 in (12) can be rearranged in the form

$$I_2 = \sum_{i=1}^{M} \left(h_k^{(i)}\right)^* \times \underbrace{\sum_{l=1, l\neq k}^{K} A_l b_l h_l^{(i)} \left(\left(1 - w_{lk}^{(i),(2)}\right) \rho_{lk}^{(i)} - \sum_{\substack{j=1\\ j\neq k,l}}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} \rho_{jl}^{(i)}\right)}_{\triangleq W}. \tag{16}$$

Since $h_l^{(i)}$'s are independent complex Gaussian with zero mean, b_l's do not affect the statistics of I_2, $\sigma^2_{I_2}$ is given by

$$\sigma^2_{I_2} = E\left[I_2 I_2^* | \mathbf{h}_k\right] = \sum_{i=1}^{M} \left|h_k^{(i)}\right|^2 \sigma^2_{I_{(i,2)}} \tag{17}$$

where

$$\begin{aligned}\sigma^2_{I_{(i,2)}} &= E[WW^*] \\ &= \sum_{l=1, l\neq k}^{K} 2A_l^2 \left(\left(1 - w_{lk}^{(i),(2)}\right) \rho_{lk}^{(i)} - \sum_{\substack{j=1\\ j\neq k,l}}^{K} w_{jk}^{(i),(2)} \rho_{jk}^{(i)} \rho_{jl}^{(i)}\right)^2.\end{aligned} \tag{18}$$

Now, the bit-error analysis of the decision rule in (10) can be carried out by conditioning with respect to the transmitted bits and the channel coefficients. The transmitted bits from other users can be dropped since they do not affect the distribution of the decision variable. Accordingly, the probability of error conditioned on the channel fade coefficients of the desired user k at the second stage output is given by

$$P_{e,\mathbf{h}_k}^{(2)} = \Pr\left(\operatorname{sgn}\left(\operatorname{Re}\left(v_k^{(2)}\right)\right) < 0 | b_k = 1, \mathbf{h}_k\right). \tag{19}$$

The above equation simplifies to

$$P_{e,\mathbf{h}_k}^{(2)} = Q(Y) \tag{20}$$

where $Q(x) = (1/2\pi) \int_{t=x}^{\infty} e^{-t^2/2} dt$, and Y is of the form

$$Y = \frac{\sum_{i=1}^{M} X_i}{\sqrt{\sum_{i=1}^{M} q_i X_i}} \tag{21}$$

where, from the first term in (11) it can be seen that

$$X_i = A_k \left|h_k^{(i)}\right|^2 \left(1 - \sum_{j=1, j\neq k}^{K} w_{jk}^{(i),(2)} \left(\rho_{jk}^{(i)}\right)^2\right) \tag{22}$$

and that X_i's are exponential r.v's with mean $\overline{X}_i$, given by (since $E[|h_k^{(i)}|^2] = 2$)

$$\overline{X}_i = 2A_k \left(1 - \sum_{j=1, j\neq k}^{K} w_{jk}^{(i),(2)} \left(\rho_{jk}^{(i)}\right)^2\right) \tag{23}$$

and q_i's are given by

$$q_i = \frac{\sigma^2_{I_{(i,2)}} + \sigma^2_{N_{(i,2)}}}{\overline{X}_i}. \tag{24}$$

It is noted that the derivation of (17) and (18) leading to (20) is made possible due to the fading being complex Gaussian, and it is much more computationally complex to determine (17) and (18) leading to (20) for the case of no fading.

Now, unconditioning (20) over X_i's the average bit-error probability for user k at the second stage output is

$$P_e^{(2)} = E\left[Q(Y)\right] \tag{25}$$

which can be derived by using the relation [4]

$$E\left[Q(Y)\right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F_Y(y) e^{-y^2/2} dy \tag{26}$$

where $F_Y(y)$ is the cummulative density function (cdf) of Y defined in (21). Hence, we need the cdf of Y in order to derive an expression for $P_e^{(2)}$. Accordingly, we derived the cdf of Y

and the derivation is given in Appendix A. Substituting the cdf expression (67) into (26) and carrying out the integration, we get the expression for $P_e^{(2)}$, in closed-form, as (the derivation is given in Appendix B)

$$P_e^{(2)} = \sum_{j=1}^{M} \xi_j \sum_{l=1}^{M} \alpha_j(l) \Bigg\{ \frac{\overline{X}_j}{\zeta_j(l)} \left(\frac{1}{2} - \frac{1}{2}\sqrt{\frac{\overline{X}_j}{\overline{X}_j + 2q_j}} \right) - \left(\frac{\overline{X}_j - \zeta_j(l)}{\zeta_j(l)} \right) \left(\frac{1}{2} - \frac{1}{2}\sqrt{\frac{\overline{X}_j}{\overline{X}_j + 2q_j}} - 2V_j(l) \right) \Bigg\} \quad (27)$$

where $V_j(l)$ is given by

$$V_j(l) = \frac{1}{2}\left[-\frac{q_j\zeta_j(l)}{\sqrt{1+\frac{2q_j}{\zeta_j(l)}}} + \zeta_j^2(l)\sqrt{1+\frac{2q_j}{\zeta_j(l)}} - \zeta_j^2(l) \right], \quad \text{if } B_{j,l} = 0 \quad (28)$$

$$V_j(l) = \frac{1}{4B_{j,l}} \left[1 - \sqrt{\frac{\overline{X}_j}{\overline{X}_j + 2q_j}} - \frac{1}{\zeta_j(l)} \times \left(\frac{1}{\sqrt{\left(\frac{2B_{j,l}\zeta_j(l)Z^3}{1+2B_{j,l}\zeta_j(l)q_j}\right)^2 + 1}} - \frac{1}{\sqrt{(2B_{j,l}\zeta_j(l)Z^3)^2 + 1}} \right) \right] \quad \text{if } B_{j,l} \neq 0 \quad (29)$$

where $Z = \sqrt{1 - (1/2B_{j,l}\zeta_j^2(l))}$, and the other parameters ξ_j, $\zeta_j(i)$, $\alpha_j(l)$, and $B_{j,l}$ are defined as

$$\xi_j = \prod_{i=1, i\neq j}^{M} \frac{q_j\overline{X}_j}{q_j\overline{X}_j - q_i\overline{X}_i} \quad (30)$$

$$\zeta_j(i) = \begin{cases} \overline{X}_j & \text{if } i = j \\ \frac{\overline{X}_i\overline{X}_j(q_j - q_i)}{q_j\overline{X}_j - q_i\overline{X}_i} & \text{if } i \neq j \end{cases} \quad (31)$$

$$\alpha_j(l) = \prod_{i=1, i\neq l}^{M} \frac{\zeta_j(l)}{\zeta_j(l) - \zeta_j(i)} \quad (32)$$

$$B_{j,l} = \frac{1}{q_j\overline{X}_j} - \frac{1}{q_j\zeta_j(l)}. \quad (33)$$

2) Third Stage Output Statistics and BER: Following similar steps carried out for deriving the second stage output statistics and BER in the above, the mean $\overline{X}_{i,3}$, and $\sigma^2_{I_{(i,3)}}$ and $\sigma^2_{N_{(i,3)}}$ corresponding to the the third stage outputs are obtained as (34)-(36), as shown at the bottom of the page.

Using (34)-(36) the BER at the third stage output can be derived similar to the second stage BER derivation given before. In a similar way, the expressions for $\overline{X}_{i,m}$, $\sigma^2_{I_{(i,m)}}$, and $\sigma^2_{N_{(i,m)}}$, and hence the BER expressions, for stages beyond the third stage (i.e., $m > 3$) can be obtained. However, we have restricted our derivation only up to the third stage as most cancellation benefit is found to be realized with $m = 3$, and adding more stages typically results in marginal improvement in performance with added complexity.

3) Optimum Weights for second and third Stages: As can be seen, (27) gives the average BER as a function of the weights used in the cancellation. The optimum weights for the second and third stages can be obtained by numerically minimizing

$$\overline{X}_{i,3} = 2A_k \left(1 - \sum_{\substack{j=1\\j\neq k}}^{K} w_{jk}^{(i),(3)} \left(\rho_{jk}^{(i)}\right)^2 \left(1 - w_{kj}^{(i),(2)}\right) + \sum_{\substack{j=1\\j\neq k}}^{K} w_{jk}^{(i),(3)} \rho_{jk}^{(i)} \sum_{\substack{s=1\\s\neq j,k}}^{K} w_{sj}^{(i),(2)} \rho_{sj}^{(i)} \rho_{ks}^{(i)} \right) \quad (34)$$

$$\sigma^2_{I_{(i,3)}} = \sum_{\substack{l=1\\l\neq k}}^{K} 2A_l^2 \Bigg[\sum_{\substack{j=1\\j\neq l,k}}^{K} w_{jk}^{(i),(3)} \rho_{jk}^{(i)} \sum_{\substack{s=1\\s\neq l,j}}^{K} w_{sj}^{(i),(2)} \rho_{sj}^{(i)} \rho_{ls}^{(i)} - \sum_{\substack{j=1\\j\neq k,l}}^{K} w_{jk}^{(i),(3)} \rho_{lj}^{(i)} \rho_{jk}^{(i)} \left(1 - w_{lj}^{(i),(2)}\right) + \rho_{lk} \left(1 - w_{lk}^{(i),(3)} \left(1 - \sum_{\substack{j=1\\j\neq l}}^{K} w_{jl}^{(2)} \left(\rho_{jk}^{(i)}\right)^2 \right) \right) \Bigg]^2 \quad (35)$$

$$\begin{aligned} \sigma^2_{N_{(i,3)}} = {} & 2\sigma^2 \left(1 + \sum_{\substack{l=1\\l\neq k}}^{K} w_{lk}^{(i),(3)} w_{kl}^{(i),(2)} \left(\rho_{lk}^{(i)}\right)^2 \right)^2 + 4\sigma^2 \left(1 + \sum_{\substack{l=1\\l\neq k}}^{K} w_{lk}^{(i),(3)} w_{kl}^{(i),(2)} \left(\rho_{lk}^{(i)}\right)^2 \right) \\ & \times \sum_{\substack{s=1\\s\neq k}}^{K} \rho_{sk}^{(i)} \left(-w_{sk}^{(i),(3)} \rho_{sk}^{(i)} + \sum_{\substack{l=1\\l\neq k,s}}^{K} w_{lk}^{(i),(3)} \rho_{lk}^{(i)} \rho_{sl}^{(i)} w_{sl}^{(i),(2)} \right) + 2\sigma^2 \sum_{\substack{s=1\\s\neq k}}^{K} \sum_{\substack{l=1\\l\neq k}}^{K} \rho_{sl}^{(i)} \\ & \times \left(-\rho_{sk}^{(i)} w_{sk}^{(i),(3)} + \sum_{\substack{j=1\\j\neq k,s}}^{K} w_{jk}^{(i),(3)} \rho_{jk}^{(i)} \rho_{sj}^{(i)} w_{sj}^{(i),(2)} \right) \cdot \left(-\rho_{lk}^{(i)} w_{lk}^{(i),(3)} + \sum_{\substack{j=1\\j\neq k,l}}^{K} w_{jk}^{(i),(3)} \rho_{jk}^{(i)} \rho_{lj}^{(i)} w_{lj}^{(i),(2)} \right) \end{aligned} \quad (36)$$

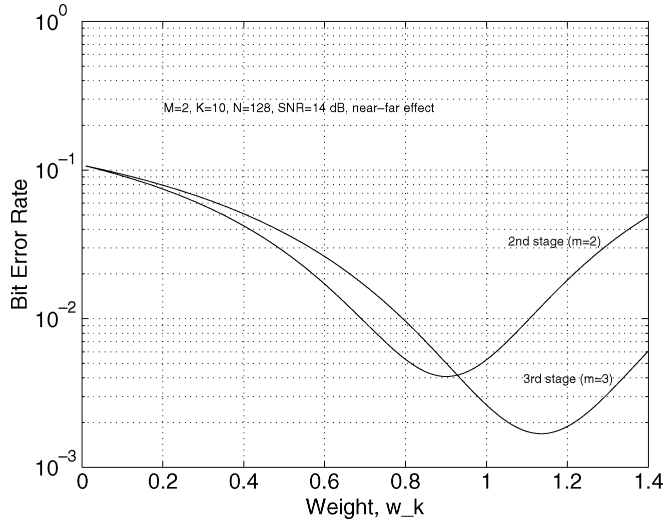


Fig. 2. BER at the second and third stage outputs as a function of the weights in the weighted LPIC scheme. $M = 2, K = 10, N = 128$, average $\mathrm{SNR} =$ 14 dB. Near-far effect: $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$. Random spreading sequences are assumed.

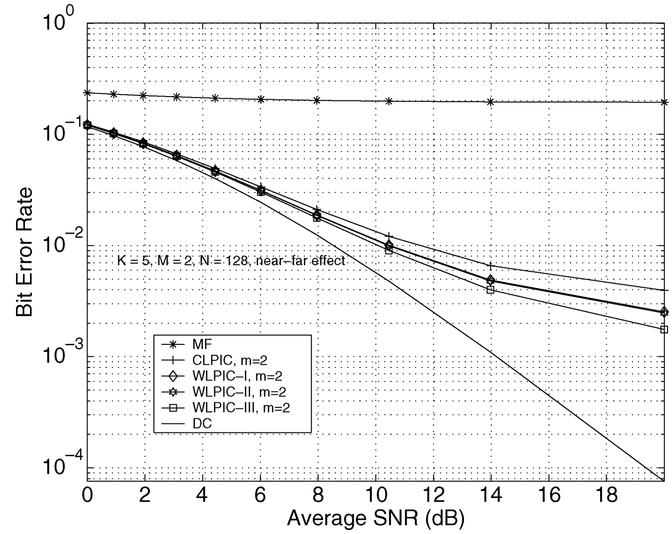


Fig. 3. BER vs average SNR performance at the second stage output of the weighted LPIC schemes-I,II,III. $M = 2, K = 5, N = 128$. Near-far effect. Random spreading sequences are assumed.

their corresponding average BER expressions. The complexity of computing $q_j, X_j, \alpha_j(l), V_j(l), \xi_j(l)$, and $\zeta_j(l)$, respectively, are of order K^3, K, MK, K^5, MK^4, and K^5. With these, the overall complexity in evaluating (27) is of the order of M^4K^{24}. For a given stage, instead of obtaining different weights for different subcarriers and for different users which requires considerable complexity in the numerical optimization, we can consider the simplified case of using the same weight for all other interfering users to a desired user k and for all subcarriers (i.e., $w_{jk}^{(i),(m)} = w_k^{(m)} \forall i, j$), the optimization of which requires less complexity. As we will see in the next section, even this simplified scheme which uses the optimized weights $w_k^{(m)}$ (we will refer to this simplified scheme as WLPIC-I scheme) gives good performance. Also, since the optimum weights are computed using the *average* BER expression, the weights computation can be carried out off-line once (or whenever users exit from or enter into the system which changes the correlation matrix), and this need not add to the per bit detection complexity of the canceller. Like in other PICs, the per bit complexity per stage here also is polynomial in number of users, i.e., MK^2.

In Fig. 2, we illustrate the variation of the average BER performance at the second and third stage outputs as a function of the weights, $w_k^{(m)}$, m =2, 3, for the simplified scheme for $M = 2$, $N = 128$, SNR = 14 dB, and $K = 10$ users. User 1 is taken to be the desired user. A scenario with near-far effect is considered, where $A_1 = A_3 = A_6 = A_7 = A_8 = A_9 = A_{10}$ and $(A_2/A_1) = (A_4/A_1) = (A_5/A_1) = 10$. From Fig. 2, it can be observed that $w_{k,\text{opt}}^{(m)}$ is about 0.9 for $m = 2$ and about 1.15 for $m = 3$. It is pointed out that since the MAI estimate is imperfect, the optimum weights that minimize the average BER can be greater than one, which is explained in detail in [16] for a single carrier CDMA system. Also, from Fig. 2, it can be observed that the minimum achievable BER (corresponding to the optimum weights) is better than that of the conventional LPIC (for which $w_k^{(m)} = 1$) and the MF detector (for which $w_k^{(m)} = 0$).

IV. Results and Discussions

In this section, we present numerical results of the BER performance of the proposed weighted LPIC for MC DS-CDMA. We computed the analytical BER performance for the second and third stages of the weighted LPIC for different number of subcarriers, M, and number of users K. We used random binary sequences of length N as the spreading sequences on each subcarrier. In all the performance plots, NM is taken to be 256 (i.e., the number of chips per bit on each subcarrier is chosen such that the total system bandwidth is fixed regardless of the number of subcarriers used). We take the number of subcarriers M to be 1, 2, and 4. We also keep the total transmit power the same irrespective of the number of subcarriers used. BER performance is computed in near-far scenarios where some users transmit with higher powers than the desired user. We take user 1 as the desired user.

In Fig. 3, we present the BER performance of the desired user at the second stage output of the weighted LPIC schemes as a function of average SNR (given by A_k^2/σ^2), for $M = 2$, $N = 128$, and $K = 5$ users. A near-far scenario with $A_2/A_1 = 15$, $A_3/A_1 = 10$, $A_4/A_1 = 20$, and $A_5/A_1 = 25$. We show three plots for the weighted LPIC schemes, viz: 1) WLPIC-I, which corresponds to optimizing the same weight for all interfering users to the desired user k and for all subcarriers (i.e., $w_{k,\text{opt}}^{(m)}$); 2) WLPIC-II, which corresponds to optimizing the same weight for all interfering users to the desired user k but different weights for different subcarriers, (i.e., $w_{k,\text{opt}}^{(i),(m)}$); and 3) WLPIC-III, which corresponds to optimizing different weights for different interfering users to the desired user k but same weights for all subcarriers (i.e., $w_{jk,\text{opt}}^{(m)}$). For the purpose of comparison, we also plot the BER performance of other detectors including the MF detector, conventional LPIC and decorrelating detector.[1] It can be noted that in terms of optimization complexity WLPIC-III is most complex and

[1]Exact analytical BER expressions for the MF, decorrelating and MMSE detectors for the considered MC DS-CDMA system are derived in Section V.

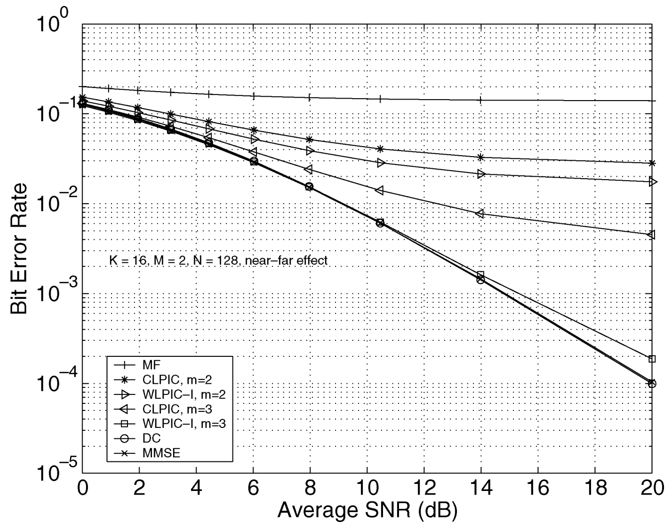


Fig. 4. BER vs average SNR performance at the second and third stage outputs of the weighted LPIC-I scheme. $K = 16$, $M = 2$, $N = 128$. Near-far effect: $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$. Random spreading sequences are assumed.

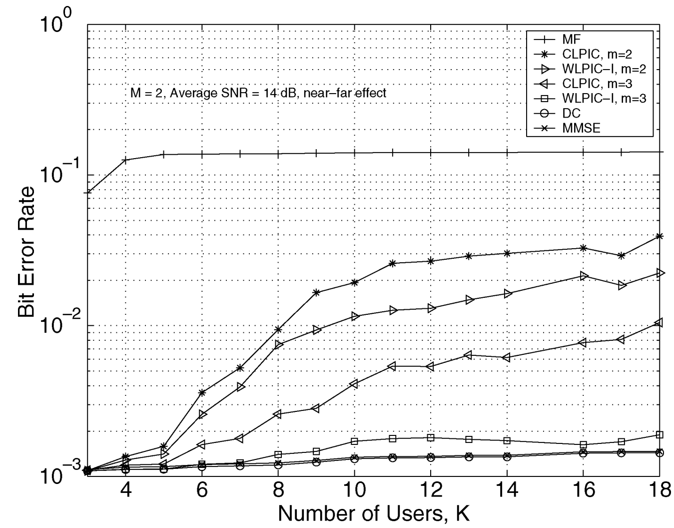


Fig. 5. BER vs number of users, K, performance of the second and third stage outputs of the weighted LPIC-I scheme. $M = 2$, $N = 128$, average $\mathrm{SNR} =$ 14 dB. Near-far effect: $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$. Random spreading sequences are assumed.

WLPIC-I is least complex, and in terms of BER performance WLPIC-III is expected to perform best. As expected, from Fig. 3, it can be observed that the WLPIC-III scheme performs better than the WLPIC-II and WLPIC-I schemes, and the performance of the WLPIC-I and WLPIC-II schemes are very close, which can be expected since the subcarriers correspond to independent channels. Even the WLPIC-I scheme, which has the least optimization complexity among all, clearly performs better than the MF detector as well as the conventional LPIC. This is expected since in MF detector there is no cancellation, whereas, in conventional LPIC there is cancellation but the weights are not optimum.

In Fig. 4, we plot the BER performance of the WLPIC-I scheme at the second and third stage outputs for $M = 2$, $N = 128$, and $K = 16$ users. A near-far scenario is considered where $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$, and users other than users 2, 4, and 5 transmit with same amplitude as the desired user 1. It can be seen that the performance at the third stage output of the WLPIC-I scheme is quite close to those of the decorrelating and MMSE detectors. We have also evaluated the BER performance through simulations and compared with the analytical results. The analytical and simulation results matched as there are no approximations involved in the analysis.

Fig. 5 shows the performance comparison as a function of number of users, K, for $M = 2$, $N = 128$, average $\mathrm{SNR} = 14$ dB with near-far effect such that $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$, and users other than users 2, 4, and 5 transmit with the same amplitude as the desired user 1. Again, the WLPIC-I scheme clearly performs better than the MF detector as well as the conventional LPIC and quite close to the decorrelating and MMSE detectors. Fig. 6 shows the performance of the second stage of the WLPIC-I for different number of subcarriers, M =1, 2, 4 for $NM = 256$, average $\mathrm{SNR} = 10$ dB with near-far effect such that $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$. The performance of $M = 4$ is better than $M = 2$ and $M = 1$ because of frequency diversity effect. For a given NM, increasing K to large values results in a cross-over in BER performance in favour of smaller M (e.g., $M = 2$ performing worse than $M = 1$ for $K > 16$ in Fig. 6), which occurs due to the frequency diversity benefit being overridden by effects of large uncancelled MAI.

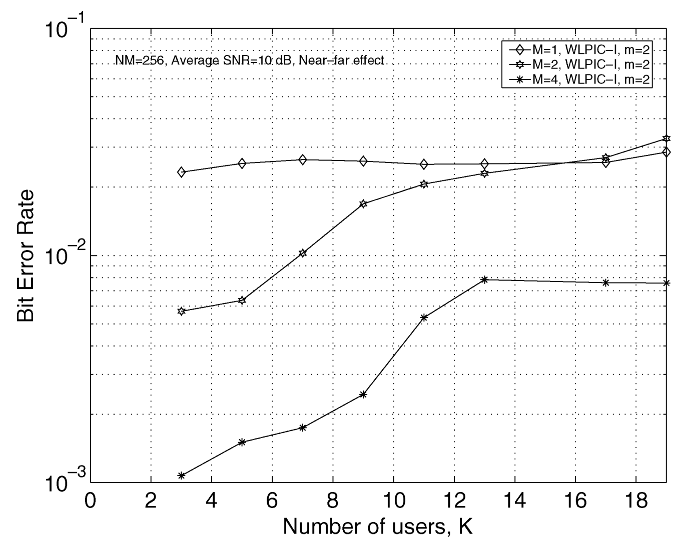


Fig. 6. BER vs number of users, K, performance at the second stage output of the weighted LPIC-I scheme for different number of subcarriers, $M = 1, 2, 4$, $NM = 256$, average $\mathrm{SNR} = 10$ dB. Near-far effect: $A_2/A_1 = A_4/A_1 = A_5/A_1 = 10$. Random spreading sequences are assumed.

A. Cancellation at the Multicarrier Combined Output

In the cancellers proposed and analyzed in the above, we have performed cancellation on each subcarrier first, followed by multicarrier combining. An alternate approach can be to perform multicarrier combining first and cancellation next. The second stage of such a canceller, which we refer to as *LPIC after*

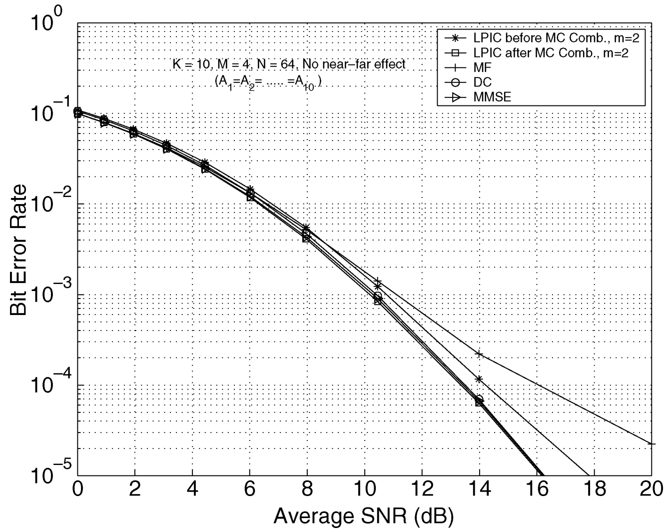


Fig. 7. Comparison of the performance of the LPICs before and after multicarrier combining. $K = 10$, $M = 4$, $N = 64$, no near-far effect: $A_1 = A_2 = \cdots = A_{10}$. Random spreading sequences are assumed.

MC Combining, can be realized as follows. Multicarrier combining of the MF outputs $y_k^{(i)}$'s results in the combined output y_k^{comb} as

$$\begin{aligned} y_k^{\text{comb}} &= \sum_{i=1}^{M} \left(h_k^{(i)}\right)^* y_k^{(i)} \\ &= A_k b_k \sum_{i=1}^{M} \left|h_k^{(i)}\right|^2 + \sum_{i=1}^{M} \left(h_k^{(i)}\right)^* \sum_{j=1, j\neq k}^{K} A_j b_j h_j^{(i)} \rho_{jk}^{(i)} \\ &\quad + \sum_{i=1}^{M} \left(h_k^{(i)}\right)^* n_k^{(i)}. \end{aligned} \tag{37}$$

LPIC on the above multicarrier combined output (to cancel the second term in the above equation) can be performed as

$$\begin{aligned} y_k^{\text{cancelled } o/p} =& y_k^{\text{comb}} \\ & - \sum_{i=1}^{M} \sum_{j=1, j\neq k}^{K} \left(\frac{h_j^{(i)} \left(h_k^{(i)}\right)^*}{\sum_{i_1=1}^{M} \left|h_j^{(i_1)}\right|^2} \right) \rho_{jk}^{(i)} y_j^{\text{comb}} \end{aligned} \tag{38}$$

where $y_k^{\text{cancelled } o/p}$ is the interference cancelled output. Since the analytical derivation of the BER expression for this LPIC after MC combining is rather difficult, we evaluated its performance through simulations and compared with the performance of the LPIC before MC combining. Fig. 7 shows the BER performance of the LPICs before and after MC combining for $K = 10$, $M = 4$, $N = 64$, and no near-far effect. The performance of the decorrelating and MMSE detectors before combining are also plotted for comparison. From Fig. 7, it can be seen that the LPIC after MC combining performs better than the LPIC before combining and quite close to the decorrelating and MMSE detectors before combining. This can be expected because MC combining before cancellation can enhance the reliability of the MAI estimate. However, the LPIC before MC combining has the benefit of tractable analysis (that results in closed-form expression for the BER) and weights optimization.

TABLE I
$\mathbf{D}^{(i)}$ MATRIX OF SIZE $2K \times 2K$ FOR THE SECOND STAGE OF THE WLPIC

$\mathbf{D}^{(i)}(q,j) =$	$\left(1 - w_1^{(i),(m)} \sum_{l=2}^{K} (\rho_{l1}^{(i)})^2\right) A_1 b_1$	for $q = j = 1$
	$\frac{1}{2}\Big[(1 - w_1^{(i),(m)})\rho_{1j}^{(i)} A_j$ $- w_1^{(i),(m)} A_j \sum_{l=1,l\neq j,1}^{K} \rho_{l1}^{(i)} \rho_{lj}^{(i)}\Big] b_j$	for $q = 1, j = 2, .., K$
	$\frac{1}{2}\Big[(1 - w_1^{(i),(m)})\rho_{1q}^{(i)} A_q$ $- w_1^{(i),(m)} A_q \sum_{l=1,l\neq q,1}^{K} \rho_{l1}^{(i)} \rho_{lq}^{(i)}\Big] b_q$	for $q = 2, .., K, j = 1$
	$\frac{1}{2}$	for $q = K+1, j = 1$, and $j = K+1, q = 1$
	$-\frac{1}{2} w_1^{(i),(m)} \rho_{(q-K)1}^{(i)}$	for $q = K+2, ..2K, j = 1$
	$-\frac{1}{2} w_1^{(i),(m)} \rho_{(j-K)1}^{(i)}$	for $j = K+2, ..2K, q = 1$
	0	otherwise

V. ALTERNATE DERIVATION OF BER EXPRESSIONS

In this section, we derive exact BER expressions at the outputs of the different stages of the proposed weighted LPIC (before multicarrier combining) for MC DS-CDMA, using an alternate approach involving the characteristic function of the decision variable. This approach is different from the one presented in Section III-B. But the expressions derived using both the approaches produce the same results. Using the same approach, we also derive exact BER expressions for MF and decorrelating detectors for the considered MC DS-CDMA system.

Considering user 1 as the desired user, the bit decision for the desired user at the mth stage output is given by

$$\hat{b}_1^{(m)} = \text{sgn}\left(\text{Re}\left(v_1^{(m)}\right)\right) \tag{39}$$

where $v_k^{(m)}$ is given by (9). We note that the real part of $v_1^{(m)}$ can be written in the form [21]

$$\text{Re}\left(v_1^{(m)}\right) = \mathbf{V}^H \mathbf{Q} \mathbf{V} \tag{40}$$

where $\mathbf{V}$ is a $2KM \times 1$ vector consisting of the complex Gaussian r.v's $h_k^{(i)}$'s and $n_k^{(i)}$'s, given by

$$\begin{aligned} \mathbf{V}_{2MK\times 1} = \Big[& h_1^{(1)} \ \cdots \ h_K^{(1)} \ n_1^{(1)} \ \cdots \ n_K^{(1)} \ \cdots \\ & h_1^{(M)} \ \cdots \ h_K^{(M)} \ n_1^{(M)} \ \cdots \ n_K^{(M)}\Big]^T \end{aligned} \tag{41}$$

and $\mathbf{Q}$ is a $2KM \times 2KM$ block-diagonal matrix, given by

$$\mathbf{Q}_{2MK\times 2MK} = \begin{bmatrix} \mathbf{D}^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{D}^{(2)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{D}^{(M)} \end{bmatrix} \tag{42}$$

where $\mathbf{D}^{(i)}$ is a $2K \times 2K$ 'coefficient matrix' corresponding to the ith subcarrier, which consists of the coefficients of the quadratic equation in complex Gaussian variables in (40) (i.e., coefficients involving A_j's, b_j's, ρ_{jk}'s, and $w_j^{(m)}$'s). It is noted that $\mathbf{D}^{(i)}$'s will be different for different stages of the weighted LPIC. Accordingly, the $\mathbf{D}^{(i)}$ matrix for the second (i.e., $m = 2$)

TABLE II
$\mathbf{D}^{(i)}$ MATRIX OF SIZE $2K \times 2K$ FOR THE THIRD STAGE OF THE WLPIC

$\mathbf{D}^{(i)}(q,j) =$	$\Big[1 - w_1^{(i),(m)}\Big(\sum_{l=2}^{K}(\rho_{l1}^{(i)})^2(1 - w_l^{(i),(m-1)}) - \sum_{l=2}^{K}\sum_{\substack{r=1,\\ r\neq l,1}} \rho_{l1}^{(i)} w_l^{(i),(m)} \rho_{rl}^{(i)} \rho_{r1}^{(i)}\Big)\Big] A_1 b_1$	for $q = j = 1$
	$\frac{1}{2}A_q b_q\Big[w_1^{(i),(m-1)}\sum_{\substack{l=1,\\ l\neq q,1}}^{K}\sum_{\substack{r=1,\\ r\neq q,l}}^{K} w_l^{(i),(m-1)}\rho_{l1}^{(i)}\rho_{lr}^{(i)}\rho_{rq}^{(i)} - w_1^{(i),(m)}\sum_{\substack{l=2,\\ l\neq q}}^{K}\rho_{l1}^{(i)}\rho_{lq}^{(i)}(1 - w_l^{(i),(m-1)}) + \rho_{q1}^{(i)}(1 - w_1^{(i),(m)}(1 - w_q^{(i),(m-1)}\sum_{\substack{l=1,\\ l\neq q}}(\rho_{lq}^{(i)})^2))\Big]$	for $q = 2,..,K, j = 1$
	$\frac{1}{2}A_j b_j\Big[w_1^{(i),(m-1)}\sum_{\substack{l=1,\\ l\neq j,1}}^{K}\sum_{\substack{r=1,\\ r\neq j,l}}^{K} w_l^{(i),(m-1)}\rho_{l1}^{(i)}\rho_{lr}^{(i)}\rho_{rj}^{(i)} - w_1^{(i),(m)}\sum_{\substack{l=2,\\ l\neq j}}^{K}\rho_{l1}^{(i)}\rho_{lj}^{(i)}(1 - w_l^{(i),(m-1)}) + \rho_{j1}^{(i)}(1 - w_1^{(i),(m)}(1 - w_q^{(i),(m-1)}\sum_{\substack{l=1,\\ l\neq j}}(\rho_{lj}^{(i)})^2))\Big]$	for $q = 2,..,K, j = 1$
	$\frac{1}{2}\Big[1 + w_1^{(i),(m)}\sum_{l=2}^{K}\Big(\rho_{l1}^{(i)}\Big)^2 w_l^{(i),(m-1)}\Big]$	for $q = K+1, j = 1$, and $j = K+1, q = 1$
	$\frac{1}{2}w_1^{(i),(m)}\Big[-\rho_{(q-K)1}^{(i)} + \sum_{\substack{l=1,\\ l\neq(q-K),1}}^{K}\rho_{l1}^{(i)}\rho_{l(q-K)}^{(i)} w_l^{(i),(m-1)}\Big]$	for $q = K+2,..,2K$, $j = 1$
	$\frac{1}{2}w_1^{(i),(m)}\Big[-\rho_{(j-K)1}^{(i)} + \sum_{\substack{l=1,\\ l\neq(j-K),1}}^{K}\rho_{l1}^{(i)}\rho_{l(m-K)}^{(i)} w_l^{(i),(m-1)}\Big]$	for $j = K+2,..,2K$, $q = 1$
	0	otherwise

and third (i.e., $m = 3$) stages of the weighted LPIC can be written as shown in Tables I and II, respectively. The correlation matrix of $\mathbf{V}$ is given by

$$\mathbf{L}_{2MK\times 2MK} = \begin{bmatrix} \mathbf{L}^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{L}^{(2)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{L}^{(M)} \end{bmatrix} \tag{43}$$

where the $\mathbf{L}^{(i)}$ matrix of size $2K \times 2K$ is given by

$$\mathbf{L}_{2K\times 2K}^{(i)} = \begin{bmatrix} 2\mathbf{I}_{K\times K} & \mathbf{0} \\ \mathbf{0} & 2\sigma^2\mathbf{C}^{(i)} \end{bmatrix}. \tag{44}$$

The characteristic function of (40) can be obtained as [21]

$$\psi(i\omega) = \prod_{j=1}^{P}\frac{1}{1 - i\omega\lambda_j} \tag{45}$$

where λ_j's are the eigenvalues of the matrix $\mathbf{LQ}$ and P is the number of eigenvalues of $\mathbf{LQ}$. As before, the bit-error analysis of the decision rule in (39) can be carried out by conditioning with respect to the transmitted bits (in matrix $\mathbf{Q}$) and the channel coefficients (in vector $\mathbf{V}$). Also, the binary coefficients corresponding to the transmitted bits in the above can be dropped since they do not affect the distribution of the decision variable. Hence, from (45), we get the average bit-error probability as

$$P_e^{(m)} = \frac{1}{2\pi}\int_{-\infty}^{0}\int_{-\infty}^{\infty}\left(\prod_{j=1}^{P}\frac{1}{1 - i\omega\lambda_j}\right)e^{-i\omega x}d\omega dx. \tag{46}$$

Ignoring the positions where $\lambda_j = 0$ since the product term is unaltered, the above integral can be evaluated by splitting the product term in (46) into partial fractions. Let the number of distinct eigenvalues be Z. Let the multiplicity of eigenvalue λ_l be $\mathcal{K}_l$. Splitting the product term into partial fractions, we get

$$P_e = \frac{1}{2\pi}\int_{-\infty}^{0}\int_{-\infty}^{\infty}\sum_{\substack{l=1\\ \lambda_l\neq 0}}^{Z}\sum_{j=1}^{\mathcal{K}_l}\frac{A_l^{(j)}}{(1 - i\omega\lambda_l)^j}e^{-i\omega x}d\omega dx. \tag{47}$$

Using [22, Eqn. 3.382 ET 1 118(3) and 118(4)], it can be shown that

$$P_e = \sum_{\substack{l=1\\ \lambda_l<0}}^{Z}\sum_{j=1}^{\mathcal{K}_l}A_l^{(j)}. \tag{48}$$

For the case of distinct eigenvalues, $A_i^{(1)}$'s in the above equation can be calculated as

$$A_i^{(1)} = \prod_{\substack{j=1\\ j\neq i\\ \lambda_j\neq 0}}^{P}\frac{\frac{1}{\lambda_j}}{\frac{1}{\lambda_j} - \frac{1}{\lambda_i}}. \tag{49}$$

We point out that computing the BER in (48) requires the computation of the eigenvalues of the matrix $\mathbf{LQ}$. Also, though derived using different analytical approaches, the expressions in both (48) as well as (27) give the same BER values. We further note that (48) can also be used for optimizing the weights.

A. BER Expressions for MF, DC, and MMSE Detectors

Next, in addition to deriving the exact BER expression for the weighted LPIC in the above, we are able to obtain exact BER expressions for the MF detector, decorrelating (DC) detector, and MMSE detector for MC DS-CDMA, again using the $\mathrm{Re}(v_1^{(1)}) = \mathbf{V}^H\mathbf{QV}$ formulation, which we present in the following.

In the case of MF detector (i.e., $m = 1$), the bit decision is given by

$$\hat{b}_1^{(1)} = \mathrm{sgn}\left(\mathrm{Re}\left(v_1^{(1)}\right)\right). \tag{50}$$

Accordingly, for the MF detector, the coefficient matrix corresponding to the ith subcarrier, $\mathbf{D}^{(i)}$, in (42) is given by

$$\mathbf{D}^{(i)} = \begin{bmatrix} A_1 b_1 & \frac{A_2 b_2 \rho_{12}^{(i)}}{2} & \cdots & \frac{A_K b_K \rho_{1K}^{(i)}}{2} & \frac{1}{2} & 0 & \cdots & 0 \\ \frac{A_2 b_2 \rho_{12}^{(i)}}{2} & 0 & \cdots & & & & & \\ 0 & \ddots & & 0 & & & & \\ \frac{A_K b_K \rho_{1K}^{(i)}}{2} & 0 & \cdots & \cdots & 0 & & 0 & \\ \frac{1}{2} & 0 & \cdots & \cdots & 0 & & & \\ \vdots & \ddots & & & & & & \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & & 0 \end{bmatrix}. \tag{51}$$

In the case of decorrelating detector, the decorrelation operation is given by

$$\widehat{\mathbf{y}}_{dc} = \sum_{i=1}^{M} \left(\mathbf{H}^{(i)}\right)^H \left(\mathbf{C}^{(i)}\right)^{-1} \mathbf{y}^{(i)} \tag{52}$$

and the bit estimate for the desired user 1 is given by

$$\widehat{b}_1 = \mathrm{sgn}\left(\mathbf{e}_1^T \mathrm{Re}\left(\widehat{\mathbf{y}}_{dc}\right)\right). \tag{53}$$

The $\mathbf{D}^{(i)}$ matrix for the decorrelating detector can then be written as

$$\mathbf{D}^{(i)}(q,j) = \begin{cases} A_1 b_1, & \text{for } q = j = 1 \\ \frac{\left(\mathbf{C}^{(i)}\right)^{-1}_{(q-K,1)}}{2}, & \text{for } K+1 \le q \le 2K, j = 1 \\ \frac{\left(\mathbf{C}^{(i)}\right)^{-1}_{(j-K,1)}}{2}, & \text{for } q = 1, K+1 \le j \le 2K \\ 0, & \text{otherwise.} \end{cases} \tag{54}$$

In the case of MMSE detector, the output vector is given by

$$\widehat{\mathbf{y}}_{\mathrm{mmse}} = \sum_{i=1}^{M} \left(\mathbf{H}^{(i)}\right)^H \mathbf{C}_{\mathbf{1}}^{(i)} \mathbf{y}^{(i)} \tag{55}$$

where

$$\mathbf{C}_{\mathbf{1}}^{(i)} = \mathbf{A}^{-1}\left(\mathbf{C}^{(i)} + \sigma^2 \mathbf{A}^{-2}\right)^{-1}. \tag{56}$$

$\mathbf{A} = \mathrm{diag}\{A_1, A_2, \cdots, A_K\}$, and the bit estimate for the desired user 1 is given by

$$\widehat{b}_1 = \mathrm{sgn}\left(\mathbf{e}_1^T \mathrm{Re}\left(\widehat{\mathbf{y}}_{\mathrm{mmse}}\right)\right). \tag{57}$$

Accordingly, the $\mathbf{D}^{(i)}$ matrix for the MMSE detector can be written as

$$\mathbf{D}^{(i)}(q,j) = \begin{cases} A_1 b_1 \left(\mathbf{C}_{\mathbf{2}}^{(i)}\right)_{(1,1)}, & \text{for } q = j = 1 \\ \frac{\left(\mathbf{C}_{\mathbf{2}}^{(i)}\right)_{(j,q)}}{2} A_q b_q, & \text{for } 2 \le q \le K, j = 1 \\ \frac{\left(\mathbf{C}_{\mathbf{2}}^{(i)}\right)_{(q,j)}}{2} A_j b_j, & \text{for } 2 \le j \le K, q = 1 \\ \frac{\left(\mathbf{C}_{\mathbf{1}}^{(i)}\right)_{(q-K,1)}}{2}, & \text{for } K+1 \le q \le 2K, j = 1 \\ \frac{\left(\mathbf{C}_{\mathbf{1}}^{(i)}\right)_{(j-K,1)}}{2}, & \text{for } q = 1, K+1 \le j \le 2K \\ 0, & \text{otherwise} \end{cases} \tag{58}$$

where $\mathbf{C}_{\mathbf{2}}^{(i)} = \mathbf{C}_{\mathbf{1}}^{(i)} \mathbf{C}^{(i)}$.

VI. Conclusion

We presented the design and BER analysis of a weighted LPIC scheme for MC DS-CDMA systems. In the proposed scheme, partial multiuser interference is cancelled at each stage, which is controlled by a weight that is optimized based on minimizing the BER per stage. The BER at each stage is computed based on an exact closed-form formula, which has relatively low complexity. In addition, using an alternate approach involving the characteristic function of the decision variable, we derived another exact expression for the BER at the second and third stages of the weighted LPIC for the MC DS-CDMA system. Using the same approach, we derived exact BER expressions for the MF, decorrelating and MMSE detectors for the considered MC DS-CDMA system. We showed that the proposed BER-optimized wighted LPIC scheme performs significantly better than the MF detector and close to the decorrelating and MMSE detectors.

Appendix A

In this Appendix, we derive the cdf of the random variable $Y = (\sum_{i=1}^{M} X_i / \sqrt{\sum_{i=1}^{M} q_i X_i})$ given in (21).

Let us define $Z_1 = \sum_{i=1}^{M} X_i$, and $Z_2 = \sum_{i=1}^{M} q_i X_i$, where $0 < q_1 < q_2 < \cdots < q_M$, and $X_1, X_2, \ldots, X_M$ are independent exponential random variables. We assume that X_i has mean $\overline{X}_i$. To obtain the cdf, $F_Y(y)$, of $Y = Z_1/\sqrt{Z_2}$, we first derive the joint pdf, $f_{Z_1,Z_2}(z_1, z_2)$, of Z_1 and Z_2. Toward this end, we derive the joint Laplace transform $\mathcal{L}_{Z_1,Z_2}(s_1, s_2) = E[\exp(-s_1 Z_1 - s_2 Z_2)]$ whose inversion gives us the joint pdf, $f_{Z_1,Z_2}(z_1, z_2)$. The derivation is as follows.

Since X_i's are independent and exponential with mean $\overline{X}_i$'s

$$\mathcal{L}_{Z_1,Z_2}(s_1, s_2) = \left\{\prod_{i=1}^{M} \frac{1}{1 + s_1 \overline{X}_i}\right\} \left\{\prod_{i=1}^{M} \frac{1}{1 + s_2 \beta_i(s_1)}\right\} \tag{59}$$

where $\beta_i(s_1) \triangleq q_i \overline{X}_i / (1 + s_1 \overline{X}_i)$. Consider the case when all $\overline{X}_i$'s are distinct. Referring to (59), let us define

$$R_1(s_1) \triangleq \prod_{i=1}^{M} \frac{1}{1 + s_1 \overline{X}_i} \tag{60}$$

and expand the expression in the second $\{\cdot\}$ of (59) using partial fractions, we obtain

$$\mathcal{L}_{Z_1,Z_2}(s_1,s_2)=\sum_{j=1}^{M}\mu_j(s_1)R_1(s_1)\frac{1}{1+s_2\beta_j(s_1)} \quad (61)$$

where

$$\begin{aligned}\mu_j(s_1)&=\prod_{i=1,i\neq j}^{M}\frac{\beta_j(s_1)}{\beta_j(s_1)-\beta_i(s_1)}\\&=\frac{1}{R_1(s_1)}\frac{1}{1+s_1\overline{X}_j}\\&\quad\times\prod_{i=1,i\neq j}\frac{q_j\overline{X}_j}{(q_j\overline{X}_j-q_i\overline{X}_i)+s_1\overline{X}_i\overline{X}_j(q_j-q_i)}. \quad (62)\end{aligned}$$

Using ξ_j, $\zeta_j(i)$ and $\alpha_j(l)$ defined in (30)–(32), respectively, we can simplify (61) as

$$\begin{aligned}&\mathcal{L}_{Z_1,Z_2}(s_1,s_2)\\&=\sum_{j=1}^{M}\xi_j\sum_{l=1}^{M}\alpha_j(l)\left(\frac{1}{1+s_1\zeta_j(l)}\right)\left(\frac{1}{1+s_2\beta_j(s_1)}\right). \quad (63)\end{aligned}$$

Upon inverting (63) w.r.t s_2, we obtain

$$\begin{aligned}&\mathcal{L}_{Z_1|Z_2}(s_1|z_2)\\&=\sum_{j=1}^{M}\xi_j\sum_{l=1}^{M}\alpha_j(l)\frac{1}{1+s_1\zeta_j(l)}\frac{1}{\beta_j(s_1)}\exp\left(-\frac{z_2}{\beta_j(s_1)}\right). \quad (64)\end{aligned}$$

Using the fact that $\beta_j(s_1)=q_j\overline{X}_j/(1+s_1\overline{X}_j)$, and the following result on an inverse Laplace transform [22]

$$\begin{aligned}\mathcal{L}^{-1}\left[\left(\frac{1+as_1}{1+bs_1}\right)e^{-cs_1}\right]&=\frac{a}{b}\delta(z_1-c)\\&-\left(\frac{a-b}{b}\right)\frac{U[z_1-c]}{b}\exp\left(-\left[\frac{t-c}{b}\right]\right) \quad (65)\end{aligned}$$

for $a>0$, $b>0$, $c\geq 0$, where $\delta(x-a)$ is the Dirac-delta function, we can obtain the joint pdf of Z_1, Z_2, after an inversion of (64) w.r.t s_1, as

$$\begin{aligned}f_{Z_1,Z_2}(z_1,z_2)&=\sum_{j=1}^{M}\xi_j\sum_{l=1}^{M}\alpha_j(l)\exp\left(-\frac{z_2}{q_j\overline{X}_j}\right)\frac{1}{q_j\overline{X}_j}\\&\cdot\left\{\frac{\overline{X}_j}{\zeta_j(l)}\delta\left(z_1-\frac{z_2}{q_j}\right)-\left(\frac{\overline{X}_j-\zeta_j(l)}{\zeta_j^2(l)}\right)U\right.\\&\left.\times\left[z_1-\frac{z_2}{q_j}\right]\exp\left(-\frac{\left[z_1-\frac{z_2}{q_j}\right]}{\zeta_j(l)}.\right)\right\}. \quad (66)\end{aligned}$$

Having (66) at hand, we can derive the cdf of Y as

$$\begin{aligned}F_Y(y)&=\int_{z_2=0}^{\infty}dz_2\int_{z_1=0}^{y\sqrt{z_2}}dz_1 f_{Z_1,Z_2}(z_1,z_2)\\&=\sum_{j=1}^{M}\xi_j\sum_{l=1}^{M}\frac{\alpha_j(l)}{q_j\overline{X}_j}\\&\quad\times\left\{\frac{q_j\overline{X}_j^2}{\zeta_j(l)}\left[1-\exp\left(-\frac{y^2q_j}{\overline{X}_j}\right)\right]\right.\\&\quad\left.-\left(\frac{\overline{X}_j-\zeta_j(l)}{\zeta_j^2(l)}\right)\mathcal{J}_1(y,q_j,G_{j,l},B_{j,l})\right\} \quad (67)\end{aligned}$$

where $B_{j,l}$ is given by (33) and

$$G_{j,l}=\frac{1}{\zeta_j(l)} \quad (68)$$

and

$$\begin{aligned}\mathcal{J}_1(y,q_j,G_{j,l},B_{j,l})&=\frac{q_j}{G_{j,l}}\times\frac{1}{G_{j,l}+q_jB_{j,l}}\\&\quad\times\left[1-\exp\left(-q_jG_{j,l}y^2-q_j^2B_{j,l}y^2\right)\right]\\&\quad-\frac{2}{G_{j,l}}\mathcal{J}_2(-G_{j,l}y,-B_{j,l},q_jy) \quad (69)\end{aligned}$$

and

$$\begin{aligned}&\mathcal{J}_1(y,q_j,G_{j,l},B_{j,l})\\&\quad=\frac{q_j}{G_{j,l}}\times\frac{1}{G_{j,l}+q_jB_{j,l}}\left[1-\exp\left(-q_jG_{j,l}y^2-q_j^2B_{j,l}y^2\right)\right]\\&\quad-\frac{2}{G_{j,l}}\mathcal{J}_2(-G_{j,l}y,-B_{j,l},q_jy), \quad (69)\\&\mathcal{J}_2(P_1,P_2,P_3)\\&\quad\triangleq\int_{x=0}^{P_3}x\exp(P_1x+P_2x^2)dx\\&\quad=\frac{e^{-\frac{P_1^2}{4P_2}}}{4P_2^{\frac{3}{2}}}\left\{\left(2\sqrt{P_2}e^{\frac{P_1^2}{4P_2}}\left[-1+e^{P_3(P_1+P_2P_3)}\right]\right)+P_1\sqrt{\pi}\right.\\&\quad\left.\times\left(\text{Erfi}\left[\frac{P_1}{2\sqrt{P_2}}\right]-\text{Erfi}\left[\frac{P_1+2P_2P_3}{2\sqrt{P_2}}\right]\right)\right\} \quad (70)\end{aligned}$$

where $\text{Erfi}(x)=\text{erf}(jx)/j$, $j=\sqrt{-1}$ [22] and $\text{erf}(x)$ is the standard error function defined for the real-valued x.

APPENDIX B

By substituting the cdf of Y in (67) into (26), we can derive the expression for $P_e^{(2)}$ as follows. Substituting (67) into (26), we see that we need to evaluate the following integrals: $(1/\sqrt{2\pi})\int_0^{\infty}[1-\exp(-(y^2q_j/\overline{X}_j))]e^{-y^2/2}dy$ and $(1/\sqrt{2\pi})\int_0^{\infty}\mathcal{J}_1(y,q_j,G_{j,l},B_{j,l})e^{-y^2/2}dy$. First, using the relation [22]

$$\frac{1}{\sqrt{2\pi}}\int_0^{\infty}e^{-ay^2}dy=\frac{1}{2\sqrt{2a}}, \quad \text{for } \operatorname{Re}(a)>0 \quad (71)$$

we get

$$\begin{aligned}&\frac{1}{\sqrt{2\pi}}\int_0^{\infty}\left[1-\exp\left(-\frac{y^2q_j}{\overline{X}_j}\right)\right]e^{-y^2/2}dy\\&\qquad=\frac{1}{2}\left(1-\sqrt{\frac{\overline{X}_j}{\overline{X}_j+2q_j}}\right) \quad (72)\end{aligned}$$

where we have used the substitution $a=1/2$ for evaluating the first term in the integral and $a=(q_j/\overline{X}_j)+(1/2)$ for evaluating the second term. Next, consider the first term in (69). Noting that

$$q_jG_{j,l}+q_j^2B_{j,l}+\frac{1}{2}=\frac{q_j}{\overline{X}_j}+\frac{1}{2} \quad (73)$$

and again using (71), we have

$$\frac{1}{\sqrt{2\pi}}\int_0^\infty \left[1-\exp\left(-q_j G_{j,l} y^2 - q_j^2 B_{j,l} y^2\right)\right] e^{-y^2/2} dy = \frac{1}{2}\left(1-\sqrt{\frac{\overline{X_j}}{\overline{X_j}+2q_j}}\right). \quad (74)$$

Finally, consider the second term in (69). Consider the two cases when $B_{j,l} = 0$ and $B_{j,l} \neq 0$. If $B_{j,l} = 0$, then from (70), $\mathcal{J}_2(-G_{j,l}y, -B_{j,l}, q_j y)$ is given by

$$\mathcal{J}_2(-G_{j,l}y, 0, q_j y) = \int_0^{q_j y} t \exp(-G_{j,l} y t) dt = -\frac{q_j}{G_{j,l}}\exp(-q_j G_{j,l} y^2) + \frac{1-\exp(-q_j G_{j,l} y^2)}{G_{j,l}^2 y^2}. \quad (75)$$

Using the relation [22]

$$\frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{\exp(-gy^2)}{y^2} dy = -\sqrt{\frac{g}{2}} \quad (76)$$

we get

$$\frac{1}{\sqrt{2\pi}}\int_0^\infty \mathcal{J}_2(-G_{j,l}y, 0, q_j y) e^{-y^2/2} dy = \frac{1}{2}\left(-\frac{q_j}{G_{j,l}\sqrt{1+2q_j G_{j,l}}} - \frac{1}{G_{j,l}^2} + \frac{\sqrt{1+2q_j G_{j,l}}}{G_{j,l}^2}\right), \quad \text{for } B_{j,l} = 0 \quad (77)$$

where, to get (77), we have used the substitutions $g = 1/2$ and $g = (1/2) + q_j G_{j,l}$ in (76), $a = (1/2) + q_j G_{j,l}$ in (71). For the case of $B_{j,l} \neq 0$, we make use of the substitutions $a = 1/2$ and $a = (q_j/\overline{X_j}) + (1/2)$ in (71), and use the relations $\mathrm{erf}(-x) = -\mathrm{erf}(x)$, $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$ and $Q(x) = (1/\sqrt{2})\mathrm{erfc}(x/\sqrt{2})$, to obtain

$$\frac{1}{\sqrt{2\pi}}\int_0^\infty \mathcal{J}_2(-G_{j,l}y, -B_{j,l}, q_j y) e^{-y^2/2} dy = \frac{1}{4B_{j,l}}\left[1-\sqrt{\frac{\overline{X}_j}{\overline{X}_j+2q_j}} - \frac{1}{\zeta_j(l)} \times \left(\frac{1}{\sqrt{\left(\frac{2B_{j,l}\zeta_j(l)Z^3}{1+2B_{j,l}\zeta_j(l)q_j}\right)^2+1}} - \frac{1}{\sqrt{(2B_{j,l}\zeta_j(l)Z^3)^2+1}}\right)\right], \quad \text{for } B_{j,l} \neq 0 \quad (78)$$

where $Z = \sqrt{1-(1/2B_{j,l}\zeta_j^2(l))}$. Combining (78), (77), (74), and (72), we get the closed-form expression for $P_e^{(2)}$ given in (27).

REFERENCES

[1] L. Hanzo, L.-L. Yang, E.-L. Kuan, and K. Yen, *Single- and Multi-Carrier DS-CDMA: Multiuser Detection, Space-Time Spreading, Synchronization and Standards*. Piscataway, NJ: IEEE Press, 2003.

[2] L.-L. Yang and L. Hanzo, "Performance of generalized multicarrier DS-CDMA over Nakagami- m fading channels," *IEEE Trans. Commun.*, vol. 50, no. 6, pp. 956–966, Jun. 2002.

[3] S. Kondo and L. B. Milstein, "Performance of multicarrier DS-CDMA systems," *IEEE Trans. Commun.*, vol. 44, no. 2, pp. 238–246, Feb. 1996.

[4] S. Verdu, *Multiuser Detection*. Cambridge, U.K.: Cambridge Univ. Press, 1998.

[5] S. L. Miller and B. J. Rainbolt, "MMSE detection of multicarrier CDMA," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 11, pp. 2356–2362, Nov. 2000.

[6] P. Zong, K. Wang, and Y. Bar-Ness, "Partial sampling MMSE interference suppression in asynchronous multicarrier CDMA system," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 8, pp. 1605–1613, Aug. 2001.

[7] L. Fang and L. B. Milstein, "Successive interference cancellation in multicarrier DS-CDMA," *IEEE Trans. Commun.*, vol. 48, no. 9, pp. 1530–1540, Sep. 2000.

[8] M. Varanasi and B. Aazhang, "Multistage detection in asynchronous code-division multiple-access," *IEEE Trans. Commun.*, vol. 38, no. 4, pp. 509–519, Apr. 1990.

[9] R. Chandrasekaran and J. J. Shynk, "Analysis of parallel interference canceller for DS-CDMA signals," in *Proc. 37th Allerton Conf. Communication, Control and Computing*, Urbana, IL, Sep. 1999, pp. 967–976.

[10] D. Divsalar, M. K. Simon, and D. Raphaeli, "Improved parallel interference cancellation for CDMA," *IEEE Trans. Commun.*, vol. 46, no. 2, pp. 258–268, Feb. 1998.

[11] D. Guo, L. K. Rasmussen, and T. J. Lim, "Linear parallel interference cancellation in long-code CDMA multiuser detection," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 12, pp. 2074–2081, Dec. 1999.

[12] D. Guo, L. K. Rasmussen, S. Sun, and T. J. Lim, "A matrix-algebraic approach to linear parallel interference cancellation in CDMA," *IEEE Trans. Commun.*, vol. 48, no. 1, pp. 152–161, Jan. 2000.

[13] D. R. Brown, M. Motani, V. Veeravalli, H. V. Poor, and C. R. Johnson, Jr., "On the performance of linear parallel interference cancellation," *IEEE Trans. Inf. Theory*, vol. 47, no. 5, pp. 1957–1970, Jul. 2001.

[14] C.-H. Hwang, C.-S. Kim, and C.-C. J. Kuo, "Analysis of multistage linear parallel interference cancellation in CDMA systems using graphical representation," in *Proc. IEEE ICC'2002*, Apr. 2002, pp. 288–292.

[15] G. F. Trichard, J. S. Evans, and I. B. Collings, "Large system analysis of linear multistage parallel interference cancellation," *IEEE Trans. Commun.*, vol. 50, no. 11, pp. 1778–1786, Nov. 2002.

[16] V. Tikiya, S. Manohar, and A. Chockalingam, "SIR-optimized weighted linear parallel interference canceller on fading channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 8, pp. 1998–2003, Aug. 2006.

[17] Y.-H. Li, M. Chen, and S.-X. Chen, "Determination of cancellation factors for soft decision partial PIC detector in DS-CDMA systems," *Electron. Lett.*, pp. 239–241, Feb. 3, 2000.

[18] M. Ghotbi and M. R. Soleymani, "A simple method for computing partial cancellation factors in CDMA using PPIC receiver," in *Proc. IEEE ICASSP*, 2004, pp. 973–976.

[19] S. Manohar, V. Tikiya, R. Annavajjala, and A. Chockalingam, "BER-optimized linear parallel interference cancellers for multicarrier DS-CDMA," in *Proc. IEEE GLOBECOM'2005*, St. Louis, MO, 2005, pp. 3592–3596.

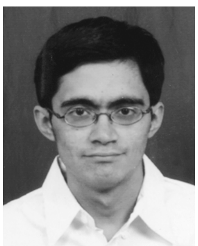
[20] D. Guo, S. Verdu, and L. K. Rasmussen, "Asymptotic normality of linear multiuser receiver outputs," *IEEE Trans. Inf. Theory*, vol. 48, no. 12, pp. 3080–3094, Dec. 2002.

[21] G. L. Turin, "The characteristic function of Hermitian quadratic forms in complex normal variables," *Biometrika*, vol. 1997, pp. 199–201, 1960.

[22] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, And Products*, 6th ed. New York: Academic, 2000.

Shamaiah Manohar received the B.E. degree in electronics and communication engineering from Visveswaraiah Technological University, Belgaum, Karnataka, India, in 2003 and the M.E. degree in electrical communication engineering from Indian Institute of Science, Bangalore, India, in 2005.

He is currently working with Honeywell Technology Solutions Labortory Private Limited, Bangalore, India, as a Senior Engineer. His research interests include UWB, CDMA, and OFDM systems.



Vibhor Tikiya received the B.E. degree from Mumbai University, Bombay, India, in 2002 and the M.E. degree in electrical communication engineering from Indian Institute of Science, Bangalore, India, in 2004. He is currently a student at the Indian Institute of Management, Ahmedabad, India.

His research interests are in the area of wireless communications and pricing in telecommunication networks

Ramesh Annavajjala (M'01) received the Bachelors degree in electronics and communication engineering from the National Institute of Technology (NIT), Warangal, India, in May 1998, the Masters degree in telecommunications from the Indian Institute of Science (IISc), Bangalore, India, in January 2001, and the Ph.D. degree in electrical engineering, majoring in communications theory and systems, from the University of California at San Diego (UCSD) in June 2006.

From July 1998 to July 1999, he was with the CMC R&D Center in Hyderabad, India, working in real time systems. From February 2001 to August 2002, he was with the Systems Group at Synopsys Inc., Bangalore Center, working for the development of the EDGE modem for 2.5G Wireless Standard. From June 2006 to September 2006, he was a postgraduate researcher at UCSD. In September 2006, he joined the research team at ArrayComm LLC., in San Jose, CA, where his current focus is on system level aspects of the IEEE 802.16e WiMax wireless standard. His current research interests lie in the areas of wireless communications, distributed user cooperation diversity, multihop wireless mesh networks, detection and estimation problems in the context of cognitive radio systems, design and analysis of multipleantenna systems, and bandwidth constrained crosslayer optimization for wideband multimedia code division multiple access (CDMA) systems.

Dr. Annavajjala was the recipient of the Purkayastha/TimeLine Venturs Graduate Fellowship Award for the year 2002–2003 and is a guest editor for the special issue Cooperative Communication Networks for the EURASIP Journal on Advanced Signal Processing.

Ananthanarayanan Chockalingam (S'92–M'93–SM'98) received the B.E. (hons.) degree in electronics and communication engineering from the P. S. G. College of Technology, Coimbatore, India, in 1984, the M.Tech. degree with specialization in satellite communications from the Indian Institute of Technology, Kharagpur, India, in 1985, and the Ph.D. degree in electrical communication engineering from the Indian Institute of Science (IISc), Bangalore, India, in 1993.

During 1986–1993, he worked with the Transmission Research and Development Division of the Indian Telephone Industries Limited, Bangalore. From December 1993 to May 1996, he was a Postdoctoral Fellow and an Assistant Project Scientist at the Department of Electrical and Computer Engineering, University of California, San Diego (UCSD). From May 1996 to December 1998, he was with Qualcomm, Inc., San Diego, CA, as a Staff Engineer/Manager in the systems engineering group. In December 1998, he joined the faculty of the Department of Electrical and Communication Engineering, IISc, Bangalore, India, where he is an Associate Professor, working in the area of wireless communications and networking. He was a visiting faculty to UCSD during summers of 1999–2002.

Dr. Chockalingam is a recipient of the Swarnajayanti Fellowship from the Department of Science and Technology, Government of India. From May 2003 to May 2007, he served as an Associate Editor of the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY. He currently serves as an Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He is a guest editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS (JSAC) Special Issue on Multiuser Detection for Advanced Communication Systems and Networks. He is a Fellow of the Indian National Academy of Engineering.