BER Analysis of Space-Time Block Codes from Generalized Complex Orthogonal Designs for M-PSK

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***Abstract*— In this paper, we present an analysis for the bit error rate (BER) performance of space-time block codes (STBC) from *generalized* complex orthogonal designs for M-PSK modulation. In STBCs from complex orthogonal designs (COD), the norms of the column vectors are the same (e.g., Alamouti code). However, in generalized COD (GCOD), the norms of the column vectors may not necessarily be the same (e.g., the rate-3/5 and rate-7/11 codes by Su and Xia in [1]). STBCs from GCOD are of interest because of the high rates that they can achieve (in [2], it has been shown that the maximum achievable rate for STBCs from GCOD is bounded by 4/5). While the BER performance of STBCs from COD (e.g., Alamouti code) can be simply obtained from existing analytical expressions for receive diversity with the same diversity order by appropriately scaling the SNR, this can not be done for STBCs from GCOD (because of the unequal norms of the column vectors). Our contribution in this paper is that we derive analytical expressions for the BER performance of any STBC from GCOD. Our BER analysis for the GCOD captures the performance of STBCs from COD as special cases. We validate our results with two STBCs from GCOD reported by Su and Xia in [1], for 5 and 6 transmit antennas (G_5 and G_6 in [1]) with rates $7/11$ and $3/5$, respectively.**

I. Introduction

A generalized complex orthogonal design (GCOD) in variables $x_1, x_2, ..., x_k$ of size n and rate k/p, $p \geq n$ is a $p \times n$ matrix G such that

- the entries of G are complex linear combinations of $x_1, x_2, ..., x_k$ and their complex conjugates $x_1^*, x_2^*, ..., x_k^*$.
- $G^*G = D$ where G^* is the complex conjugate and transpose of G, and D is an $n \times n$ diagonal matrix with the (i,i)th diagonal element of the form

$$l_{i,1}|x_1|^2 + l_{i,2}|x_2|^2 + ... + l_{i,k}|x_k|^2$$

where all the coefficients $l_{i,1}, l_{i,2}, ..., l_{i,k}$ are strictly positive numbers. Complex orthogonal designs (COD) are special cases of GCOD where

$$l_{i,1} = l_{i,2} = ... = l_{i,k}, \ \forall i,$$

Su and Xia, in [1], has given two space-time block codes (STBC) from GCOD (G_5 with 5 transmit antennas and G_6 with 6 transmit antennas).

It is known that the bit error rate (BER) performance of orthogonal STBCs from COD's can be simply obtained from existing analytical results for receive diversity maximal ratio combining (MRC) with the same diversity order by appropriately scaling the SNR [3]. However, this can not be done for STBC's from GCOD's. In this paper, our focus is to derive analytical expressions for the BER performance of *STBCs from GCOD's* for M-PSK modulation. We show that our general BER expressions absorb the results for STBC's from GCOD's as special cases.

The rest of the paper is organized as follows. Section II gives the system model. Section III presents the performance analysis. Section IV gives the numerical results and discussions. Conclusions are given in V.

II. System Model

Consider a wireless communication system with n transmit antennas and m receive antennas. Assume that the $m \times n$ channel matrix $\mathbf{H}$ is static for the code length which is p time slots. The entries of $\mathbf{H}$, h_{ij}'s are independent complex Gaussian random variables (i.e., the fade amplitudes are Rayleigh distributed). The $m \times 1$ receive vector, $\mathbf{y}_t$, at time slot t can be expressed as

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t + \eta_t, \tag{1}$$

where $\mathbf{x}_t$ is the transmitted $n \times 1$ complex symbol vector at time t, and η_t is the $m \times 1$ noise vector with independent zero mean complex Gaussian random variables with variance $N_o/2$ per complex dimension.

For a code of length p, the transmitted code block $\mathbf{X}$ is given by $\mathbf{X} = [\mathbf{x}_t, \mathbf{x}_{t+1}, \cdots, \mathbf{x}_{t+p}]^T$, where $[.]^T$ represents the transpose operation. The corresponding received code block $\mathbf{Y}$ can be expressed as

$$\mathbf{Y} = \tilde{\mathbf{H}}\mathbf{X} + \eta, \tag{2}$$

where $\tilde{\mathbf{H}}$ is the block diagonal channel matrix given by

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H} & 0 & \cdots & 0 \\ 0 & \mathbf{H} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{H} \end{bmatrix}, \tag{3}$$

and the noise block η is given by $\eta = [\eta_t, \eta_{t+1}, \cdots, \eta_{t+p}]^T$.

If $s(l) = Ve^{j\phi_l}$, $l = 1, 2, \cdots, \kappa$ are the complex symbols taken from the M-PSK signal set to be transmitted in p time slots, the transmitted code block $\mathbf{X}$ can be expressed in the form

This work was supported in part by the Swarnajayanti Fellowship, Department of Science and Technology, New Delhi, Government of India, under Project Ref: No.6/3/2002-S.F.

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$$\mathbf{X} = \mathbf{A}\mathbf{v}, \quad (4)$$

where $\mathbf{v}$ a $2\kappa \times 1$ vector, given by

$$\mathbf{v} = [v_{1I}, v_{2I}, \cdots, v_{kI}, v_{1Q}, v_{2Q}, \cdots, v_{kQ}], \quad (5)$$

where v_{lI} and v_{lQ}, respectively, are the real and imaginary parts of the l^{th} complex symbol, $s(l)$. $\mathbf{A}$ is the $np \times 2\kappa$ complex matrix which performs the space-time coding on $\mathbf{v}$. Now, $\mathbf{A} = [\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_p]$, where $\mathbf{A}_i$ performs the linear operation at time slot i. Using this, the received code block $\mathbf{Y}$ in (2) becomes

$$\mathbf{Y} = \mathbf{H}_{eq}\mathbf{v} + \eta, \quad (6)$$

where $\mathbf{H}_{eq} = \tilde{\mathbf{H}}\mathbf{A}$ is a $mp \times 2\kappa$ equivalent channel matrix.

For example, for Alamouti code with one receive antenna (i.e., $n = \kappa = p = 2$ and $m = 1$), the $\mathbf{A}$, $\mathbf{H}_{eq}$, and $\mathbf{v}$ are given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & j & 0 \\ 0 & 1 & 0 & j \\ 0 & -1 & 0 & -j \\ 1 & 0 & -j & 0 \end{bmatrix}, \quad (7)$$

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 & j & 0 \\ 0 & 1 & 0 & j \end{bmatrix}, \quad (8)$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & -1 & 0 & -j \\ 1 & 0 & -j & 0 \end{bmatrix}, \quad (9)$$

$$\mathbf{H}_{eq} = \begin{bmatrix} h_1 & h_2 & jh_1 & jh_2 \\ -h_2 & h_1 & jh_2 & -jh_1 \end{bmatrix}, \quad (10)$$

$$\mathbf{v} = [x_{1I}, x_{2I}, x_{1Q}, x_{2Q}]^T. \quad (11)$$

At the receiver, linear combining is performed. We assume that the channel matrix $\mathbf{H}$ is perfectly known at the receiver. We use the form of the optimum decision metric for the orthogonal STBCs presented in [4], which is given by

$$\tilde{\mathbf{Y}} = \Re(\mathbf{H}_{eq}^{*}\mathbf{Y}) = \mathbf{\Lambda}\mathbf{v} + \tilde{\eta}, \quad (12)$$

where $\Re(z)$ denotes the real part of z, $\mathbf{\Lambda}$ is a $2\kappa \times 2\kappa$ diagonal matrix and $\tilde{\eta} = \Re(\mathbf{H}_{eq}^{*}\eta)$ where $*$ denotes the Hermitian operator. $\tilde{\eta}$ can be shown to be WGN. Hence, the optimal receiver, is taking the real and imaginary values from $\tilde{\mathbf{Y}}$ and performing symbol by symbol detection. From (12), the decision metric for the l^{th} complex symbol is given by

$$Z(l) = \tilde{\mathbf{Y}}(l) + j\tilde{\mathbf{Y}}(l+\kappa), \quad l = 1, 2, \cdots, \kappa. \quad (13)$$

A. Equivalence to Generalized MRC

The system model and the decision metric presented above is valid for STBC's from both GCOD's as well as from COD's In [3], the equivalence of the above decoding to receive diversity MRC has been shown for equal-weight STBCs. In the following, we show that a non-equal weight orthogonal STBC is equivalent to a MRC scheme in which the channels can be classified into independent but not identically distributed sets. From (12), the l^{th} entry of $\tilde{\mathbf{Y}}$ is given by

$$\tilde{\mathbf{Y}}(l) = \mathbf{\Lambda}_{l,l}\mathbf{v}(l) + \tilde{\eta}(l), \quad (14)$$

where $\mathbf{\Lambda}_{l,l}$ is the l^{th} diagonal entry of $\mathbf{\Lambda}$ and is given by [4]

$$\mathbf{\Lambda}_{l,l} = \sum_{f=0}^{n-1} r_{ff} \sum_{i=0}^{p-1} |a_{fl}^{(i)}|^2, \quad (15)$$

where r_{ff} is the $(f,f)^{th}$ component of $\mathbf{H}^{*}\mathbf{H}$, given by $\sum_{i=1}^{n} |h_{fi}|^2$ and $a_{ij}^{(l)}$ is the $(i,j)^{th}$ component of $\mathbf{A}_l$, and $\tilde{\eta}(l) = \Re(\mathbf{H}_{eq}^{(l)*}\eta)$. $\mathbf{H}_{eq}^{(l)}$ is the l^{th} column of $\mathbf{H}_{eq}$. From the definition of $\mathbf{H}_{eq}$, it can be shown that

$$\tilde{\eta}(l) = \Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n} h_{ij}\mathbf{a}_i(l)\eta_j\right), \quad (16)$$

where $\mathbf{a}_i(l) = [a_{i,l}^{(1)}, a_{i,l}^{(2)}, \cdots, a_{i,l}^{(p)}]$ and η_j is $p \times 1$ vector of i.i.d noise samples. Define

$$\tilde{h}_{ij}^{(l)} = h_{ij} \sum_{f=0}^{p-1} |a_{il}^{(f)}|^2 \quad \forall j, \quad (17)$$

$$\tilde{\mathbf{a}}_i(l) = \frac{\mathbf{a}_i(l)}{\sum_{f=0}^{p-1} |a_{il}^{(f)}|^2} \quad \forall i, l. \quad (18)$$

Now (16) can be rewritten as

$$\tilde{\eta}(l) = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{h}_{ij}^{*(l)} \frac{\left(\eta_j^{*}\tilde{\mathbf{a}}_i^{*}(l) + e^{-j2\varphi_{ij}^{(l)}}\tilde{\mathbf{a}}_i(l)\eta_j\right)}{2} \quad (19)$$

where $\varphi_{ij}^{(l)}$ is the angle of $\tilde{h}_{ij}^{(l)}$. Letting

$$\tilde{\tilde{\eta}}_{ij} = \frac{\left(\eta_j^{*}\tilde{\mathbf{a}}_i^{*}(l) + e^{-j2\varphi_{ij}}\tilde{\mathbf{a}}_i(l)\eta_j\right)}{2}, \quad (20)$$

$\{\tilde{\mathbf{a}}_i(l); i = 1, \cdots, n\}$ forms a set of orthonormal vectors for all l (see Appendix A for proof of the orthogonality) Hence, $\tilde{\tilde{\eta}}_{ij}$ forms a set of independent and identically distributed noise variables for all i, j. It is also easy to show that $E(h_{ij}^{(l)}\tilde{\tilde{\eta}}_{ij}) = 0$, and since both are Gaussian random variables, they are independent as well.

The same set of arguments hold for the imaginary part $\tilde{\mathbf{Y}}(l+\kappa)$ as well. Hence the combiner output for the l^{th} symbol, $l = 1, 2, \cdots, \kappa$, can be expressed as

$$Z(l) = \sum_{i=1}^{m}\sum_{j=1}^{n} |\tilde{h}_{ij}^{(l)}|^2 s(l) + \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{h}_{ij}^{(l)*}\tilde{\tilde{\eta}}_{ij}. \quad (21)$$

It is noted that the combined output $Z(l)$ in (21) is the same as that of MRC scheme of nm receive diversity order with *independent but not identically distributed paths*.

III. Performance Analysis

In this section, we derive an exact expression for the BER of the orthogonal STBC schemes with non-equal weights. To do that, we need to obtain the pdf of the angle of the decision variable $Z(l)$ in (21). We observe that our formulation in (21) has dissimilar sets of paths in which the channel gains are independent and identically distributed within the sets and non-identically distributed across the sets.

The pdf of the angle of $\tilde{Z}(l)$, θ_l conditioned on ϕ_l and the instantaneous SNR per bit $\gamma^{(l)}$ is given by [5]

$$\begin{aligned} f_\theta(\theta_l|\phi_l, \gamma^{(l)}) &= \frac{e^{-a\gamma^{(l)}}}{2\pi} \\ &+ \frac{e^{-a\gamma^{(l)}}}{2\pi}\sqrt{4\pi a\gamma^{(l)}}\cos(\phi_l - \theta_l)e^{a\gamma_l \cos^2(\phi_l-\theta_l)} \\ - \frac{e^{-a\gamma^{(l)}}}{4\pi}&\sqrt{4\pi a\gamma^{(l)}}\cos(\phi_l - \theta_l)e^{a\gamma_l \cos^2(\phi_l-\theta_l)}\mathrm{erfc}(\sqrt{a\gamma}\cos(\phi_l - \theta_l)) \end{aligned} \quad (22)$$

where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$, and $a = \log_2 M$. $P(\theta_l \in [\theta_L, \theta_U))$ is the probability of the phase angle θ_l lying in the decision region $[\theta_L, \theta_U)$, which is given by

$$P(\theta_l \in [\theta_L, \theta_U))) = \int_{\theta_L}^{\theta_U}\int_0^\infty f_\theta(\theta_l|\phi_l, \gamma^{(l)}) f_\gamma(\gamma^{(l)}) d\gamma^{(l)} d\theta_l. \quad (23)$$

Let $m_k^{(l)}$ denote the number of values i for which value of $\sum_{f=0}^{p-1}|a_{ip}^{(f)}|^2$ is same, and let $p_k^{(l)}$ denote this value. Let there be $K^{(l)}$ such sets. Note that $\sum_{i=1}^{K^{(l)}} m_k^{(l)} = nm$. The pdf of $\gamma_l^{(l)}$ is obtained as (see Appendix B for the derivation)

$$f_\gamma(\gamma^{(l)}) = \sum_{k=1}^{K^{(l)}}\sum_{i=1}^{m_k^{(l)}} \frac{C_{i,k}^{(l)}}{(i-1)!}\frac{\gamma^{(l)i-1}}{\gamma_k^{(l)(i)}} e^{-\gamma^{(l)}/\gamma_k^{(l)}}, \quad (24)$$

where $\gamma_k^{(l)}$ is the SNR per bit of the k^{th} set and is given by $\gamma_k^{(l)} = p_k^{(l)} E_b/N_o$. Define

$$\begin{aligned} I_N(\theta_U, \theta_L, \gamma_k^{(l)}) &= \int_0^\infty \int_{\theta_L}^{\theta_U} f_\theta(\theta_l|\phi_l, \gamma^{(l)}) d\theta_l \\ &\cdot \frac{1}{(N-1)!}\frac{\gamma^{(l)N-1}}{\gamma^{(l)N}} e^{-\gamma^{(l)}/\gamma_k^{(l)}} d\gamma^{(l)}. \end{aligned} \quad (25)$$

Then

$$P(\theta_l \in [\theta_L, \theta_U))) = \sum_{k=1}^{K^{(l)}}\sum_{i=1}^{m_k^{(l)}} C_{i,k}^{(l)} I_i(\theta_U, \theta_L, \gamma_k^{(l)}). \quad (26)$$

Eqn (25) can be solved in closed-form as in [6] (Eqn. 18).

For equally probable symbols in M-PSK, the symbol error rate on the l^th symbol, $P_s^{(l)}$, is obtained as

$$P_s^{(l)} = 2P(\theta_l \in [\pi, \pi/M)). \quad (27)$$

The bit error on the l^{th} symbol, $P_b^{(l)}$, is obtained as

$$P_b^{(l)} = \frac{\sum_{i=1}^{M} e_i P(\theta_l \in R_i)}{log_2 M}, \quad (28)$$

where $R_i = [\frac{(2i-3)\pi}{M}, \frac{(2i-1)\pi}{M})$ and e_i is the number of bit errors made in the region R_i.

The average symbol error rate and bit error rate is obtained as

$$P_s = \frac{1}{\kappa}\sum_{l=1}^{\kappa} P_s^{(l)}, \qquad P_b = \frac{1}{\kappa}\sum_{l=1}^{\kappa} P_b^{(l)}. \quad (29)$$

For example, for 8-PSK

$$\begin{aligned} P_s &= \frac{2}{\kappa}\sum_{l=1}^{\kappa} P(\theta_l \in [\pi, \pi/8)) \quad (30) \\ P_b &= \frac{1}{3\kappa}\sum_{l=1}^{\kappa}[2P(\theta_l \in [3\pi/8, \pi/8)) \\ &+ 4P(\theta_l \in [5\pi/8, 3\pi/8)) \\ &+ 4P(\theta_l \in [7\pi/8, 5\pi/8)) \\ &+ 4P(\theta_l \in [\pi, 7\pi/8))]. \quad (31) \end{aligned}$$

It is noted that the BER expression in the above absorbs the equal-weight condition the number of sets $K^{(l)}$ equal to the total number of paths nm.

IV. Results and Discussion

In this section, we present some numerical and simulation results that illustrate the BER performance of STBCs from GCOD's G_5 and G_6, which are given by [1]

$$G_5 = \begin{pmatrix} x_1 & x_2 & x_3 & 0 & x_4 \\ -x_2^* & x_1^* & 0 & x_3 & x_5 \\ x_3^* & 0 & -x_1^* & x_2 & x_6 \\ 0 & x_3^* & -x_2^* & -x_1 & x_7 \\ x_4^* & 0 & 0 & -x_7^* & -x_1^* \\ 0 & x_4^* & 0 & x_6^* & -x_2^* \\ 0 & 0 & x_4^* & x_5^* & -x_3^* \\ 0 & -x_5^* & -x_6^* & 0 & x_1 \\ x_5^* & 0 & x_7^* & 0 & x_2 \\ -x_6^* & -x_7^* & 0 & 0 & x_3 \\ x_7 & -x_6 & -x_5 & x_4 & 0 \end{pmatrix} \quad (32)$$

$$
G_6 = \begin{pmatrix}
x_1 & x_2 & x_3 & 0 & x_4 & x_8 \\
-x_2^* & x_1^* & 0 & x_3 & x_5 & x_9 \\
x_3^* & 0 & -x_1^* & x_2 & x_6 & x_{10} \\
0 & x_3^* & -x_2^* & -x_1 & x_7 & x_{11} \\
x_4^* & 0 & 0 & -x_7^* & -x_1^* & x_{12} \\
0 & x_4^* & 0 & -x_6^* & -x_2^* & x_{13} \\
0 & 0 & x_4^* & x_5^* & -x_3^* & x_{14} \\
0 & x_5^* & -x_6^* & 0 & -x_1 & x_{15} \\
x_5^* & 0 & x_7^* & 0 & x_2 & x_{16} \\
x_6^* & x_7^* & 0 & 0 & -x_3 & x_{17} \\
x_7 & -x_6 & -x_5 & x_4 & 0 & x_{18} \\
x_8^* & 0 & 0 & -x_{11}^* & -x_{15}^* & -x_1^* \\
0 & x_8^* & 0 & x_{10}^* & x_{16}^* & -x_2^* \\
0 & 0 & x_8^* & x_9^* & -x_{17}^* & -x_3^* \\
0 & 0 & 0 & x_{18}^* & x_8^* & -x_4^* \\
0 & 0 & -x_{18}^* & 0 & x_9^* & -x_5^* \\
0 & -x_{18}^* & 0 & 0 & x_{10}^* & -x_6^* \\
-x_{18}^* & 0 & 0 & 0 & x_{11}^* & -x_7^* \\
0 & -x_9^* & x_{10}^* & 0 & x_{12}^* & x_1 \\
x_9^* & 0 & x_{11}^* & 0 & x_{13}^* & x_2 \\
-x_{10}^* & -x_{11}^* & 0 & 0 & x_{14}^* & x_3 \\
-x_{12}^* & -x_{13}^* & -x_{14}^* & 0 & 0 & x_4 \\
-x_{16}^* & -x_{15}^* & 0 & -x_{14}^* & 0 & x_5 \\
-x_{17}^* & 0 & x_{15}^* & -x_{13}^* & 0 & x_6 \\
0 & -x_{17}^* & -x_{16}^* & x_{12}^* & 0 & x_7 \\
0 & x_{14} & -x_{13} & -x_{15} & x_{11} & 0 \\
x_{14} & 0 & -x_{12} & -x_{16} & x_{10} & 0 \\
-x_{13} & x_{12} & 0 & x_{17} & x_9 & 0 \\
x_{15} & -x_{16} & x_{17} & 0 & x_8 & 0 \\
-x_{11} & x_{10} & x_9 & -x_8 & x_{18} & 0
\end{pmatrix} \quad (33)
$$

The following Table gives the various parameters for the above codes.

	# symbols κ	Symbol index	# sets $K^{(l)}$	set values $p_k^{(l)}$ $k = 1..K^{(l)}$	cardinality $m_k^{(l)}$ $k = 1..K^{(l)}$
G_5	7	l = 1..4	1	1	nm
		l = 4..7	2	$\{1, 2\}$	$\{(n-1)m, m\}$
G_6	18	l = 1..7	1	1	nm
		l = 8..15	2	$\{1, 2\}$	$\{(n-1)m, m\}$
		l = 16..18	2	$\{1, 2\}$	$\{(n-2)m, 2m\}$

Using the parameters of the codes G_5 and G_6 given in Table I, we computed the analytical BER performance from the expressions obtained in the previous section, for the case of 8-PSK and one receive antenna (i.e., $m = 1$). We also evaluated the same performance through simulations. Fig. 1 shows both the analytical as well as the simulation results of the BER for G_5 and G_6 codes. It can be seen that, because of the larger diversity order (6th order), G_6 code performs better than G_5 code (5th order diversity) as expected, and that there is a close match between the analytical and the simulation results.

V. Conclusion

We presented an analysis for the bit error performance STBCs from GCOD's (G_5 and G_6 codes given by Su and Xia), for M-PSK modulation. We showed that our general BER expressions absorb the results for equal weight STBCs as special cases.

Appendix A

Claim: $\{\mathbf{a}_i(l); i = 1..n\}$ forms a set of orthogonal vectors $\forall\, l$.
Proof: Let $B_l = [\mathbf{a}_1(l), \mathbf{a}_2(l), ..., \mathbf{a}_n(l)]$. Then the GCOD,

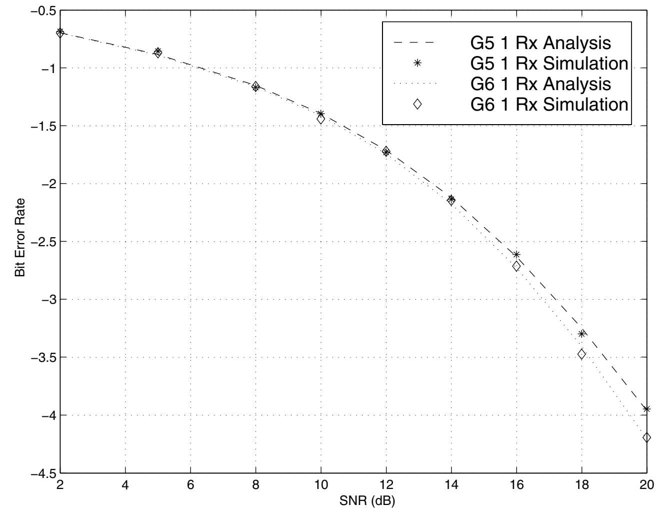

Fig. 1. BER performance of G_5 and G_6 orthogonal STBCs with 8-PSK. One Rx. antenna. Analysis and simulations.

G, can be written as

$$
G = \sum_{i=1}^{\kappa} (v_{iI} B_i + v_{iQ} B_{i+\kappa}). \quad (34)
$$

Now the orthogonality condition, $G^* G = D$, implies that

$$
B_i^* B_i = D_i, i = 1, .., 2\kappa, \quad (35)
$$

$$
B_j^* B_i + B_i^* B_j = 0, 1 \leq i \neq j \leq 2\kappa. \quad (36)
$$

Eqn. (35) implies that $< \mathbf{a}_i(l), \mathbf{a}_j(l) >= 0, \forall\, i \neq j$. Hence the result.

Appendix B

In this appendix, we derive the pdf of $\gamma^{(l)}$. The moment generating function $\gamma^{(l)}$ is given by

$$
M(\nu) = \prod_{k=1}^{K^{(l)}} \left(\frac{1}{1 + j\gamma_k^{(l)} \nu} \right)^{m_k^{(l)}}. \quad (37)
$$

Expanding in partial fractions,

$$
M(\nu) = \sum_{k=1}^{K^{(l)}} \sum_{i=1}^{m_k^{(l)}} \frac{C_{i,k}^{(l)}}{(1 + j\gamma_k^{(l)} \nu)^i}, \quad (38)
$$

where $C_{i,k}^{(l)}$ equals

$$
C_{i,k}^{(l)} = \frac{\left. \left\{ \frac{d^{(m_k^{(l)} - i)}}{d\nu^{(m_k^{(l)} - i)}} (1 + j\gamma_k^{(l)})^{m_k^{(l)}} M(\nu) \right\} \right|_{\nu = -(j\gamma_k^{(l)})^{-1}}}{(m_k^{(l)} - i)! \gamma_k^{m_k^{(l)} - i}}. \quad (39)
$$

The pdf of $\gamma^{(l)}$ in (24) is obtained by taking the Fourier transform of the above.

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