Spatial Modulation and Space Shift Keying in Single Carrier Communication

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***Abstract*—Spatial modulation (SM) and space shift keying (SSK) are relatively new modulation techniques which are attractive in multi-antenna communications. Single carrier (SC) systems can avoid the peak-to-average power ratio (PAPR) problem encountered in multicarrier systems. In this paper, we study SM and SSK signaling in cyclic-prefixed SC (CPSC) systems on MIMO-ISI channels. We present a diversity analysis of MIMO-CPSC systems under SSK and SM signaling. Our analysis shows that the diversity order achieved by (n_t, n_r) SSK scheme and (n_t, n_r, Θ_M) SM scheme in MIMO-CPSC systems under maximum-likelihood (ML) detection is n_r, where n_t, n_r denote the number of transmit and receive antennas and Θ_M denotes the modulation alphabet of size M. Bit error rate (BER) simulation results validate this predicted diversity order. Simulation results also show that MIMO-CPSC with SM and SSK achieves much better performance than MIMO-OFDM with SM and SSK.**

***Keywords:** Spatial modulation, space shift keying, cyclic-prefixed single carrier systems, MIMO-CPSC, diversity order.*

I. INTRODUCTION

The use of multiple antennas at the transmitter and receiver sides can significantly enhance the capacity and reliability of wireless links [1]. However, multi-antenna operation faces significant challenges due to complexity and cost of the hardware owing to the requirement of inter-antenna synchronization and maintenance of multiple radio-frequency (RF) chains [2]. Spatial modulation (SM) is a relatively new modulation technique for multiple antenna systems which addresses these issues [3]. This modulation technique was first proposed in [4], and later improved further in [5]-[7]. Space shift keying (SSK) is another signaling technique which can be thought of as the special case of SM [8]-[11].

High data rate communication system designs necessitate larger bandwidths, which, in turn, result in frequency selectivity of the wireless channel. In frequency selective channels, the presence of multipath components cause inter-symbol interference (ISI). To mitigate ISI, use of multi-carrier techniques such as orthogonal frequency division multiplexing (OFDM) is popular due to simple equalization/receiver complexity. However, high peak to average power ratio (PAPR) at the OFDM transmitter caused by the IFFT operation makes the design of amplifier a challenging task. Single carrier communication [12], [13] in frequency selective channels is of interest because it does not suffer from PAPR limitations.

In single carrier (SC) systems, each data block is transmitted along with either zero padding (ZP) or cyclic prefix (CP), which avoids inter-datablock interference. CP converts the linear convolution of channel with data to circular convolution, which is equivalent to multiplication in frequency domain. This enables low complexity frequency domain processing at the receiver. Comparison between OFDM and single carrier systems has been done extensively in the literature both for single antenna scenario [14]-[19], as well as MIMO scenario [20], [21]. Wireless standards like 3GPP LTE has adopted single carrier system in the uplink [22].

In the context of SM and SSK in frequency selective channels, the focus so far has been on the MIMO-OFDM [5],[23]-[24]. [5] presented a matched filter (MF) based detection algorithm for SM on each subchannel. [6] presented the optimal detection scheme for SM in flat fading MIMO channels. This scheme can be easily extended to MIMO-OFDM scenario on individual subcarriers. However, to the best of our knowledge, neither SM nor SSK has been studied for single carrier systems.

In this paper, we study SM and SSK signaling in cyclic-prefix single carrier (CPSC) systems on MIMO-ISI channels. We present a diversity analysis of the MIMO-CPSC system under SSK and SM signaling. Our analysis shows that the diversity order achieved by (n_t, n_r) SSK scheme and (n_t, n_r, Θ_M) SM scheme in MIMO-CPSC systems under maximum-likelihood (ML) detection is n_r, where n_t, n_r denote the number of transmit and receive antennas and Θ_M denotes the modulation alphabet of size M. Bit error rate (BER) simulation results for SSK and SM are presented to validate this predicted diversity order. In addition, we present a comparison between MIMO-CPSC and MIMO-OFDM performance under SSK and SM signaling. Simulation results show that the optimal BER performance of SM and SSK in MIMO CPSC are significantly better compared to those in MIMO-OFDM.

II. SYSTEM MODEL

Consider a MIMO channel with n_t transmit and n_r receive antennas. The channel between each pair of transmit and receive antennas is assumed to be frequency selective with L multipath components. Let $h_{j,i}^{(l)}$ denote the channel gain from ith transmit antenna to the jth receive antenna on the lth multipath component (MPC), which is modeled as $\mathcal{CN}(0, \Omega_l)$. The following power-delay profile of the channel is considered:

$$\Omega_l = \mathbb{E}[|h_{j,i}^{(l)}|^2] = \Omega_0 10^{-\beta l}, \quad l = 0, \cdots, L-1, \qquad (1)$$

where β denotes the rate of decay of the average power in the MPCs in dB. The power in the channel is assumed to be normalized to unity, i.e., $\sum_{l=0}^{L-1} \Omega_l = 1$.

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Consider a CPSC scheme, where transmission is carried out in frames. Each frame consists of a cyclic prefix (CP) part followed by a data part. The length of the CP part must be at least $L-1$ channel uses in order to avoid inter-frame interference. Let K denote the length of the data part in number of channel uses. The total frame length is $K+L-1$ channel uses. In each channel use in the data part, a data vector is transmitted using n_t transmit antennas. Let $\mathbf{x}_q$ denote the transmitted symbol vector at time q, $0 \leq q \leq K-1$. *The entries of the* $\mathbf{x}_q$ *vector depends on the type of modulation used. Here, we consider spatial modulation (SM) and space shift keying (SSK)*, which are described in the following subsections. We assume that the channel gains remain constant over one frame duration. The received signal vector at time q can be written as

$$\mathbf{y}_q = \sum_{l=0}^{L-1} \mathbf{H}_l \, \mathbf{x}_{q-l} + \mathbf{n}_q, \quad q = 0, \cdots, K-1, \quad (2)$$

where $\mathbf{y}_q \in \mathbb{C}^{n_r \times 1}$, $\mathbf{H}_l \in \mathbb{C}^{n_r \times n_t}$ is the channel gain matrix for the lth MPC such that its on jth row and ith column of is $h_{j,i}^{(l)}$. After removing the CP at the receiver, the vector channel equation can be written in an equivalent form as

$$\mathbf{y} = \mathbf{H}\,\mathbf{x} + \mathbf{n}, \quad (3)$$

where $\mathbf{H}$ is the $Kn_r \times Kn_t$ equivalent channel matrix given at the bottom of this page, $\mathbf{x}$ is the combined vector of data symbols transmitted in one data frame, given by

$$\mathbf{x} = \left[\mathbf{x}_0^T \, \mathbf{x}_1^T \, \cdots \, \mathbf{x}_{K-1}^T\right]^T.$$

Likewise, the vectors $\mathbf{y}$ and $\mathbf{n}$ of size $Kn_r \times 1$ are constructed as

$$\mathbf{y} = \left[\mathbf{y}_0^T \, \mathbf{y}_1^T \, \cdots \, \mathbf{y}_{K-1}^T\right]^T, \quad \mathbf{n} = \left[\mathbf{n}_0^T \, \mathbf{n}_1^T \, \cdots \, \mathbf{n}_{K-1}^T\right]^T,$$

where each entry in the additive noise vector $\mathbf{n}$ is $\mathcal{CN}(0, \sigma^2)$, so that the received SNR is given by $\gamma = \frac{E_s}{\sigma^2}$, where E_s is the average symbol energy.

A. Space Shift Keying (SSK)

In SSK, a group of information bits are used to choose one transmit antenna, i.e., an m-bit sequence chooses one antenna from a total of $n_t = 2^m$ antennas. A known signal (which is known to the receiver) is transmitted on this chosen antenna. The remaining $n_t - 1$ antennas remain silent. By doing so, the problem of detection at the receiver becomes one of merely finding out which antenna is transmitting. This leads to a significantly reduced complexity at the receiver. The index of each transmit antenna represents a certain combination of information bits. For example, with $n_t = 4$, transmit antennas with indices $1, 2, 3, 4$ can be mapped to bit sequences $00, 01, 10, 11$, respectively. Therefore, at the receiver, detection of a certain transmit antenna index leads to conveying the group of information bits associated with that index. The spectral efficiency in SSK grows logarithmically with number of transmit antennas, i.e., $\log_2 n_t$ bits per channel use (bpcu). A consequence of this type of spatial mapping is that number of transmit antennas has to be a power of 2.

Let us take the known signal transmitted by the active antenna to be 1. Then, for a SSK system with two transmit antennas, the possible signal set is given by $\mathbb{S}_2 \equiv \{[1, 0]^T, [0, 1]^T\}$. A zero in the signal vectors above is silence in the corresponding transmit antenna. In general, a SSK signal set for n_t transmit antenna system is given by

$$\mathbb{S}_{n_t} \equiv \{\mathbf{s}_j : j = 1, \cdots, n_t\}, \quad (5)$$
$$\text{s.t. } \mathbf{s}_j = [0, \cdots, 0, \underbrace{1}_{j\text{th coordinate}}, 0, \cdots, 0]^T.$$

In the CPSC system under consideration, $\mathbf{x}_q \in \mathbb{S}_{n_t}$, $\forall q, q = 0, \cdots, K-1$, i.e., $\mathbf{x}_q$ vector in (2) is drawn from $\mathbb{S}_{n_t}$ in (5).

B. Spatial Modulation (SM)

Spatial modulation is a generalization of SSK signal design. In SM, the stream of bits to be transmitted in one channel use are divided into two groups. One group determines the transmit antenna index on which transmission will take place, and the second group determines the symbol to be transmitted from the chosen antenna. The symbol transmitted on the chosen antenna can be from a regular modulation alphabet like M-QAM, denoted by $\Theta_M \equiv \{\theta_m : m = 1, \cdots, M\}$. Therefore, the number of bits that belong to the first and second group, respectively, are $\log_2 n_t$ and $\log_2 M$. This gives a spectral efficiency of $(\log_2 n_t + \log_2 M)$ bpcu. The SM signal set for n_t antenna system, $\mathbb{T}_{n_t,M}$, is given by

$$\mathbb{T}_{n_t,M} \equiv \{\mathbf{t}_{j,m} : j = 1, \cdots, n_t, \ m = 1, \cdots, M\}, \quad (6)$$
$$\text{s.t. } \mathbf{t}_{j,m} = [0, \cdots, 0, \underbrace{\theta_m}_{j\text{th coordinate}}, 0, \cdots, 0]^T, \ \theta_m \in \Theta_M.$$

In the considered CPSC scheme, $\mathbf{x}_q$ vector is drawn from the SM signal set $\mathbb{T}_{n_t,M}$ in (6).

$$\mathbf{H} = \begin{bmatrix}
\mathbf{H}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{H}_{L-1} & \mathbf{H}_{L-2} & \mathbf{H}_{L-3} & . & \mathbf{H}_2 & \mathbf{H}_1 \\
\mathbf{H}_1 & \mathbf{H}_0 & \mathbf{0} & \mathbf{0} & . & \mathbf{0} & \mathbf{0} & \mathbf{H}_{L-1} & \mathbf{H}_{L-2} & . & \mathbf{H}_3 & \mathbf{H}_2 \\
\mathbf{H}_2 & \mathbf{H}_1 & \mathbf{H}_0 & \mathbf{0} & . & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_{L-1} & . & \mathbf{H}_4 & \mathbf{H}_3 \\
\vdots & \vdots & \vdots & . & . & . & . & . & . & . & . & . \\
\mathbf{H}_{L-2} & \mathbf{H}_{L-3} & \mathbf{H}_{L-4} & . & \mathbf{H}_0 & \mathbf{0} & . & . & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_{L-1} \\
\mathbf{H}_{L-1} & \mathbf{H}_{L-2} & \mathbf{H}_{L-3} & . & \mathbf{H}_1 & \mathbf{H}_0 & \mathbf{0} & . & . & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{H}_{L-1} & \mathbf{H}_{L-2} & . & \mathbf{H}_2 & \mathbf{H}_1 & \mathbf{H}_0 & \mathbf{0} & . & . & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{H}_{L-1} & . & . & \mathbf{H}_2 & \mathbf{H}_1 & \mathbf{H}_0 & \mathbf{0} & . & . & \mathbf{0} \\
\vdots & \vdots & \vdots & . & . & . & . & . & . & . & . & . \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & . & . & . & . & . & . & \mathbf{H}_2 & \mathbf{H}_1 & \mathbf{H}_0
\end{bmatrix}. \quad (4)$$

C. Detection

Frequency selectivity in the channel causes inter-frame interference (IFI) and inter-symbol interference (ISI). Though IFI is avoided using CP in this system, the receiver processing still has to deal with ISI within the frame. Joint detection of the entire frame of data optimally is of interest. In particular, the diversity order achieved by optimal detection in SSK and SM with CPSC signaling needs to be established. These are addressed in the next section. This, to our knowledge, has not been reported before for SSK and SM with CPSC signaling in MIMO-ISI channels. Also, the performance of optimal detection is compared with the performance of low complexity linear receivers like ZF and MMSE receivers. A performance comparison between MIMO-CPSC and MIMO-OFDM is also carried out that illustrates the performance advantage in MIMO-CPSC.

III. Optimal Detection and Diversity Analysis

In this section, we present the optimum signal detection scheme for SSK and SM with CPSC signaling. We also present an analysis of the diversity achieved by ML detection.

A. Optimum Detection

Assuming that all the signal vectors in the signal set are equally likely, the maximum likelihood (ML) decision rule for SSK in CPSC system is given by

$$\begin{aligned}&[\mathbf{x}_{q,ML} : q = 1, \cdots, K]\\ &= \underset{\mathbf{x}_q \in \mathbb{S}_{n_t} : q=0,\cdots,K-1}{\arg\max} P(\mathbf{y}|\mathbf{H}, \mathbf{x}_q : q = 0, \cdots, K-1)\\ &= \underset{\mathbf{x}_q \in \mathbb{S}_{n_t} : q=0,\cdots,K-1}{\arg\min} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2. \qquad (7)\end{aligned}$$

Likewise, the optimum decision rule for SM is given by

$$\begin{aligned}&[\mathbf{x}_{q,ML} : q = 1, \cdots, K]\\ &= \underset{\mathbf{x}_q \in \mathbb{T}_{n_t,M} : q=0,\cdots,K-1}{\arg\max} P(\mathbf{y}|\mathbf{H}, \mathbf{x}_q : q = 0, \cdots, K-1)\\ &= \underset{\mathbf{x}_q \in \Gamma_{n_t,M} : q=0,\cdots,K-1}{\arg\min} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2. \qquad (8)\end{aligned}$$

B. Diversity Analysis

A popular approach to determine the diversity order in a communication system is to find the minimum value of the exponent of SNR in the denominator of the pairwise error probability expression. This approach comes from the consideration of union bound on the error performance, when an exact expression for the error rate of the system becomes difficult to obtain. Establishing the diversity order of CPSC signaling in single-input single-output (SISO) ISI channels has been the topic of investigation in [25]-[28], where it has been shown that, though in the asymptotic regime where SNR $\to \infty$, CPSC does not extract diversity, in the moderate SNRs regime (corresponding to typical BERs used in practice), CPSC extracts some diversity [28]. Here, we carry out a diversity analysis for CPSC on MIMO-ISI channels and obtain the diversity orders for SSK and SM under ML detection.

The pairwise error probability (PEP) of detecting vector $\mathbf{x}$ as $\mathbf{x}'$, given the channel $\mathbf{H}$, is given by

$$P(\mathbf{x} \to \mathbf{x}'|\mathbf{H}) = Q(\sqrt{\mu \|\mathbf{H}(\mathbf{x} - \mathbf{x}')\|^2}), \qquad (9)$$

where μ is a scalar multiple of the average SNR. The block circulant channel matrix $\mathbf{H}$ in (4) can be written in the form

$$\mathbf{H} = \mathbf{D}_K \otimes \mathbf{I}_{n_r} \mathbf{G} \mathbf{D}_K \otimes \mathbf{I}_{n_t}, \qquad (10)$$

where $\mathbf{D}_K$ is the K-point DFT matrix, given by

$$\mathbf{D}_K = \frac{1}{\sqrt{K}} \begin{bmatrix} \rho_{0,0}, & \rho_{0,1} & \cdots & \rho_{0,K-1} \\ \rho_{1,0}, & \rho_{1,1} & \cdots & \rho_{1,K-1} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{K-1,0}, & \rho_{K-1,1} & \cdots & \rho_{K-1,K-1} \end{bmatrix}, \qquad (11)$$

where $\rho_{u,v} = \exp(-j\frac{2\pi uv}{K})$. $\mathbf{G}$ is a block diagonal matrix, given by

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_0 & & & \\ & \mathbf{G}_1 & & \mathbf{0} \\ & & \ddots & \\ & \mathbf{0} & & \mathbf{G}_{K-1} \end{bmatrix}, \qquad (12)$$

where $\mathbf{G}_q$ is given by

$$\begin{aligned}\mathbf{G}_q &= \sum_{l=0}^{L-1} \rho_{q,l} \mathbf{H}_l \\ &= \begin{bmatrix} \rho_{q,0}\mathbf{I}_{n_r} & \rho_{q,1}\mathbf{I}_{n_r} & \cdots & \rho_{q,L-1}\mathbf{I}_{n_r} \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{L-1} \end{bmatrix}}_{\triangleq \overline{\mathbf{H}}} \\ &= \sqrt{K}\mathbf{D}_K^{q,L} \otimes \mathbf{I}_{n_r} \overline{\mathbf{H}}, \end{aligned} \qquad (13)$$

where $\mathbf{D}_K^{q,L}$ denotes the first L elements in the qth row of $\mathbf{D}_K$. From (10), we can write

$$\begin{aligned}\mathbf{H}(\mathbf{x} - \mathbf{x}') &= \mathbf{D}_K \otimes \mathbf{I}_{n_r} \mathbf{G} \mathbf{D}_K \otimes \mathbf{I}_{n_t} (\mathbf{x} - \mathbf{x}') \\ &= \mathbf{D}_K \otimes \mathbf{I}_{n_r} \underbrace{\begin{bmatrix} \mathbf{G}_0 \mathbf{w}_0 \\ \vdots \\ \mathbf{G}_{K-1} \mathbf{w}_{K-1} \end{bmatrix}}_{\triangleq \mathbf{G}\mathbf{w}}, \end{aligned} \qquad (14)$$

where $\mathbf{w} = [\mathbf{w}_0^T \ \mathbf{w}_1^T \ \cdots \ \mathbf{w}_{K-1}^T]^T$ such that $\mathbf{w}_q = \mathbf{D}_K^q \otimes \mathbf{I}_{n_t}(\mathbf{x} - \mathbf{x}')$ and $\mathbf{D}_K^q$ denotes the qth row of $\mathbf{D}_K$. Now, $\mathbf{G}_q\mathbf{w}_q$ can be written as

$$\begin{aligned}\mathbf{G}_q \mathbf{w}_q &= vec(\mathbf{G}_q \mathbf{w}_q) = vec\left(\sqrt{K}(\mathbf{D}_K^{q,L} \otimes \mathbf{I}_{n_r})\overline{\mathbf{H}}\mathbf{w}_q\right) \\ &= \sqrt{K}\left(\mathbf{w}_q^T \otimes (\mathbf{D}_K^{q,L} \otimes \mathbf{I}_{n_r})\right) \underbrace{vec(\overline{\mathbf{H}})}_{\triangleq \mathbf{h}}, \end{aligned} \qquad (15)$$

where $\mathbf{h}$ is the vector of all channel gains obtained by vectorization of $\overline{\mathbf{H}}$. From (15), we can write $\mathbf{G}\mathbf{w}$ as

$$\mathbf{G}\mathbf{w} = \mathbf{V}\mathbf{h}, \qquad (16)$$

where

$$\mathbf{V} = \sqrt{K}\begin{bmatrix} \left(\mathbf{w}_0^T \otimes (\mathbf{D}_K^{0,L} \otimes \mathbf{I}_{n_r})\right) \\ \vdots \\ \left(\mathbf{w}_{K-1}^T \otimes (\mathbf{D}_K^{K-1,L} \otimes \mathbf{I}_{n_r})\right) \end{bmatrix}. \quad (17)$$

From (14) to (17), we have

$$\begin{aligned}\|\mathbf{H}(\mathbf{x}-\mathbf{x}')\|^2 &= \|(\mathbf{D}_K \otimes \mathbf{I}_{n_r})\mathbf{V}\mathbf{h}\|^2 \\ &= \mathbf{h}^H \underbrace{\mathbf{V}^H(\mathbf{D}_K \otimes \mathbf{I}_{n_r})^H(\mathbf{D}_K \otimes \mathbf{I}_{n_r})\mathbf{V}}_{\triangleq \mathbf{B}}\ \mathbf{h}. \quad (18)\end{aligned}$$

$\mathbf{B}$ in (18) is a $Ln_rn_t \times Ln_rn_t$ Hermitian matrix, whose eigen decomposition is

$$\mathbf{B} = \mathbf{U}^H\mathbf{\Lambda}\mathbf{U}, \quad (19)$$

where $\mathbf{\Lambda} = diag\{\lambda_1, \cdots, \lambda_{Ln_rn_t}\}$ and $\mathbf{U}$ is a unitary matrix. Combining (18) and (19), we write

$$\|\mathbf{H}(\mathbf{x}-\mathbf{x}')\|^2 = \tilde{\mathbf{h}}^H\mathbf{\Lambda}\tilde{\mathbf{h}} = \sum_{i=1}^{Ln_rn_t} \lambda_i\|\tilde{\mathbf{h}}_i\|^2, \quad (20)$$

where $\tilde{\mathbf{h}} = \mathbf{U}\mathbf{h}$ is obtained from $\mathbf{h}$, the vector of all channel gains, through unitary transformation, and $\tilde{\mathbf{h}}_i$ is the ith element of $\tilde{\mathbf{h}}$. Therefore, $\tilde{\mathbf{h}}$ is identically distributed as $\mathbf{h}$. Entries of $\mathbf{h}$ are zero-mean independent complex normal random variables. The corresponding covariance matrix is therefore a diagonal matrix. From (9) and (20), the expression for conditional PEP can be written as

$$P(\mathbf{x} \to \mathbf{x}'|\mathbf{H}) = Q\left(\sqrt{\mu \sum_{i=1}^{Ln_rn_t} \lambda_i\|\tilde{\mathbf{h}}_i\|^2}\right), \quad (21)$$

Applying Chernoff bound,

$$P(\mathbf{x} \to \mathbf{x}'|\mathbf{H}) \le \frac{1}{2}\exp\left(-\frac{1}{2}\mu \sum_{i=1}^{Ln_rn_t} \lambda_i\|\tilde{\mathbf{h}}_i\|^2\right), \quad (22)$$

The unconditional PEP $P(\mathbf{x} \to \mathbf{x}')$ can be obtained by taking expectation of this upper bound with respect to $\tilde{\mathbf{h}}$. The number of non-zero λ_i's in (22) will depend on the $\{\mathbf{x}, \mathbf{x}'\}$ pair under consideration since $\mathbf{B}$ is a function of $\{\mathbf{x}, \mathbf{x}'\}$. Let ρ be the number of these non-zero eigenvalues, and, without loss of generality, let λ_i, $i = 1, \cdots, \rho$ be these non-zero eigenvalues. We also observe that each $\|\tilde{\mathbf{h}}_i\|^2$ is a chi-square random variable with 2 degrees of freedom. Let σ_i^2 be the variance of $\tilde{\mathbf{h}}_i$. Then we can write the expectation as [1]

$$\mathbb{E}_{\tilde{\mathbf{h}}}\left[\exp(-\frac{1}{2}\mu\sum_{i=1}^{\rho}\lambda_i\|\tilde{\mathbf{h}}_i\|^2)\right] = \prod_{i=1}^{\rho}\frac{1}{1+\frac{\sigma_i^2\mu\lambda_i}{2}}. \quad (23)$$

Therefore, at very high SNRs, we can write the upper bound of unconditional PEP as

$$P(\mathbf{x} \to \mathbf{x}') \le \frac{2^{\rho-1}}{\mu^\rho}\prod_{i=1}^{\rho}\frac{1}{\sigma_i^2\lambda_i}. \quad (24)$$

Considering the union bound of the probability of error, at very high SNRs, the PEP term corresponding to minimum ρ will dominate other constituent terms of the upper bound expression [1]. Hence, the minimum value of ρ obtained among all possible choice of $\{\mathbf{x}, \mathbf{x}'\}$ pairs gives the diversity of the system. We computed this minimum ρ, and, hence, the diversity order for SSK and SM in MIMO-CPSC systems under ML detection for different n_t, n_r, L, M. The results are presented in Tables I and II for SSK and SM, respectively. The tables show one of the possible combinations of $\{\mathbf{x}, \mathbf{x}'\}$ for which the minimum number of non-zero eigenvalues of $\mathbf{B}$ is obtained, and the corresponding set of eigenvalues of $\mathbf{B}$. From the results in Tables I and II, we conclude that the diversity order achieved in a (n_t, n_r, L) MIMO-CPSC system under SSK and SM signaling is n_r. In the next section, we present BER simulation results which validate this predicted diversity order.

IV. Simulation Results and Discussions

In this section, we present the BER performance of SM and SSK signaling schemes as a function of the average received SNR obtained through simulation. Figures 1 and 2 depict the diversity order achieved by MIMO-CPSC under ML detection for SSK and SM, respectively. Figures 3 and 4 show the comparative performance in MIMO-OFDM versus MIMO-CPSC for SSK and SM, respectively. In all simulations, we have assumed $\beta = 3$ dB, $L = 3$, $n_t = 2$. We consider perfect CSIR, i.e. channel state information is available at the receiver. We also assume quasi-static channel, i.e., the channel remains constant during the transmission of one data block.

Diversity order: Figure 1 shows the uncoded BER performance of SSK for $n_t = 2$, $K = 3$, and $n_r = 1, 2$. We have also plotted the c_1SNR^{-1} (diversity 1) and c_2SNR^{-2} (diversity 2) lines. We have run the BER simulations up to sufficiently lower BERs to observe error rate curves to run parallel to some diversity line. It is seen that for $n_r = 1$ and $n_r = 2$, the simulated BER curves run parallel to c_1SNR^{-1} line and c_2SNR^{-2} line, respectively, at high SNRs. This forms a simulation validation of the prediction in the previous section (Table-I) that the achievable diversity order is n_r. Figure 2 shows a similar set of plots for SM with BPSK modulation. Here again, we see that, as predicted in the previous section (Table-II), the diversity order observed through simulation is also n_r.

Comparison with MIMO-OFDM: In Figs. 3 and 4, we present a ML performance comparison between MIMO-OFDM and MIMO-CPSC with $n_t = 2$, $K = 6$, and $n_r = 2$. Figure 3 is for SSK. Figure 4 is for SM with BPSK modulation. Performance of linear receivers (zero forcing and minimum mean square error receivers) are also shown for comparison. It is seen that MIMO-CPSC scheme achieves significantly better BER performance compared to MIMO-OFDM, particularly in the low-to-moderate SNR regime. For example, at 10^{-3} BER, MIMO-CPSC performs better by about 4 to 5 dB. This performance advantage, coupled with the 'no PAPR problem' advantage, makes MIMO-CPSC to be a credible alternative to

(n_t, n_r)	Space Shift Keying					
	$L = 3, K = 3$			$L = 2, K = 3$		
	Vector pair	Eigenvalues	Diversity	Vector pair	Eigenvalues	Diversity
(2,1)	$\mathbf{x}$: [(1,0),(1,0),(1,0)] $\mathbf{x}'$: [(0,1),(0,1),(0,1)]	6, 5 zeros	1	$\mathbf{x}$: [(1,0),(1,0),(1,0)] $\mathbf{x}'$: [(0,1),(0,1),(0,1)]	4, 3 zeros	1
(2,2)	$\mathbf{x}$: [(1,0),(1,0),(1,0)] $\mathbf{x}'$: [(0,1),(0,1),(0,1)]	6,6, 10 zeros	2	$\mathbf{x}$: [(1,0),(1,0),(1,0)] $\mathbf{x}'$: [(0,1),(0,1),(0,1)]	4,4, 6 zeros	2
(4,3)	$\mathbf{x}$: [(1,0,0,0),(1,0,0,0),(1,0,0,0)] $\mathbf{x}'$: [(0,1,0,0),(0,1,0,0),(0,1,0,0)]	6,6,6, 33 zeros	3	$\mathbf{x}$: [(1,0,0,0),(1,0,0,0),(1,0,0,0)] $\mathbf{x}'$: [(0,1,0,0),(0,1,0,0),(0,1,0,0)]	4,4,4, 21 zeros	3
(4,4)	$\mathbf{x}$: [(1,0,0,0),(1,0,0,0),(1,0,0,0)] $\mathbf{x}'$: [(0,1,0,0),(0,1,0,0),(0,1,0,0)]	6,6,6,6, 44 zeros	4	$\mathbf{x}$: [(1,0,0,0),(1,0,0,0),(1,0,0,0)] $\mathbf{x}'$: [(0,1,0,0),(0,1,0,0),(0,1,0,0)]	4,4,4,4, 28 zeros	4

TABLE I

DIVERSITY ORDER AND CORRESPONDING $(\mathbf{x}, \mathbf{x}')$ PAIR ALONG WITH CORRESPONDING EIGEN VALUES OF $\mathbf{B}$ FOR DIFFERENT CONFIGURATION OF (n_t, n_r, L) FOR SPACE SHIFT KEYING IN MIMO-CPSC SYSTEMS.

(n_t, n_r, Θ_M)	Spatial Modulation					
	$L = 3, K = 3$			$L = 2, K = 3$		
	Vector pair	Eigenvalues	Diversity	Vector pair	Eigenvalues	Diversity
(2,1,BPSK)	$\mathbf{x}$: [(1,0),(1,0),(1,0)], $\mathbf{x}'$: [(-1,0),(-1,0),(-1,0)]	12, 5 zeros	1	$\mathbf{x}$: [(1,0),(1,0),(1,0)] $\mathbf{x}'$: [(-1,0),(-1,0),(-1,0)]	8, 3 zeros	1
(2,2,BPSK)	$\mathbf{x}$: [(1,0),(1,0),(1,0)], $\mathbf{x}'$: [(-1,0),(-1,0),(-1,0)]	12,12, 10 zeros	2	$\mathbf{x}$: [(1,0),(1,0),(1,0)] $\mathbf{x}'$: [(-1,0),(-1,0),(-1,0)]	8,8, 6 zeros	2
(2,3,4-QAM)	$\mathbf{x}$: [(1+j,0),(1+j,0),(1+j,0)] $\mathbf{x}'$: [(1-j,0),(1-j,0),(1-j,0)]	12,12,12, 15 zeros	3	$\mathbf{x}$: [(1+j,0),(1+j,0),(1+j,0)] $\mathbf{x}'$: [(1-j,0),(1-j,0),(1-j,0)]	8,8,8, 9 zeros	3
(4,4,4-QAM)	$\mathbf{x}$: [(1+j,0,0,0),(1+j,0,0,0), (1+j,0,0,0)] $\mathbf{x}'$: [(1-j,0,0,0),(1-j,0,0,0), (1-j,0,0,0)]	12,12,12,12, 44 zeros	4	$\mathbf{x}'$: [(1+j,0,0,0),(1+j,0,0,0), (1+j,0,0,0)], $\mathbf{x}'$: [(1-j,0,0,0),(1-j,0,0,0), (1-j,0,0,0)]	8,8,8,8, 28 zeros	4

TABLE II

DIVERSITY ORDER AND CORRESPONDING $(\mathbf{x}, \mathbf{x}')$ PAIR ALONG WITH CORRESPONDING EIGEN VALUES OF $\mathbf{B}$ FOR DIFFERENT CONFIGURATION OF (n_t, n_r, L, Θ_M) FOR SPATIAL MODULATION IN MIMO-CPSC SYSTEMS.

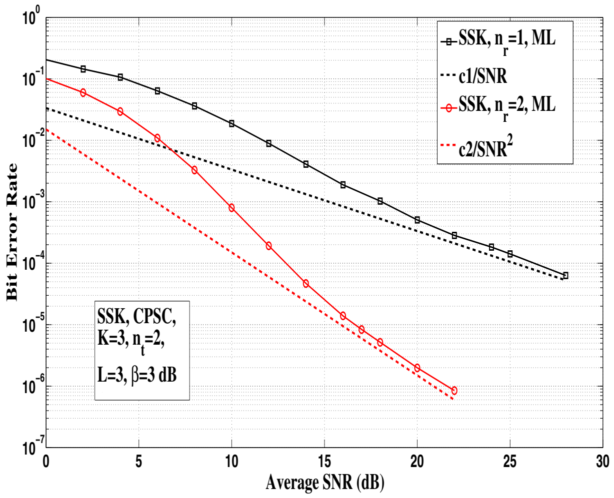

Fig. 1. Diversity order of MIMO CPSC system with SSK. $K = 3$, $n_t = 2$, $L = 3$, $\beta = 3$ dB, $n_r = 1, 2$.

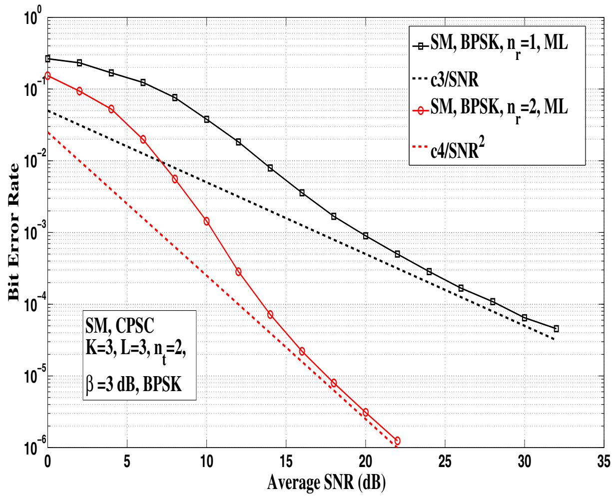

Fig. 2. Diversity order of MIMO CPSC system with SM. $K = 3$, $n_t = 2$, $L = 3$, $\beta = 3$ dB, BPSK, $n_r = 1, 2$.

MIMO-OFDM in MIMO-ISI channels. We note that the cardinality of the search space in the ML detection is of the order of $M^K {n_t}^K$ in both SSK and SM, resulting in exponential computation complexity in K. So for the MIMO-CPSC approach to be feasible in SSK and SM for large K (which will be needed in MIMO-ISI channels with large delay spreads), there is a need to devise low-complexity near-optimal algorithms that scale well for large K, which is being pursued as future extension to this work.

V. CONCLUSIONS

We studied spatial modulation and space shift keying in the context of cyclic-prefixed single carrier systems on MIMO-ISI channels. Our diversity analysis revealed that the diversity order achieved by SM and SSK in MIMO-CPSC systems

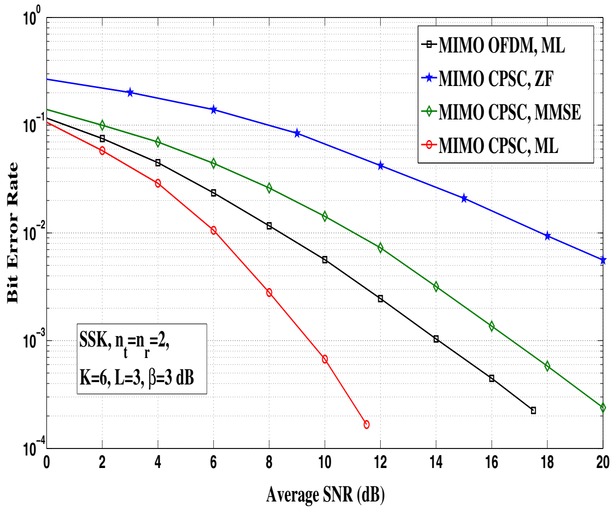

Fig. 3. Comparison of BER performance of ML detection in MIMO-OFDM and MIMO CPSC systems with SSK. $K = 6, n_t = n_r = 2, L = 3, \beta = 3$ dB.

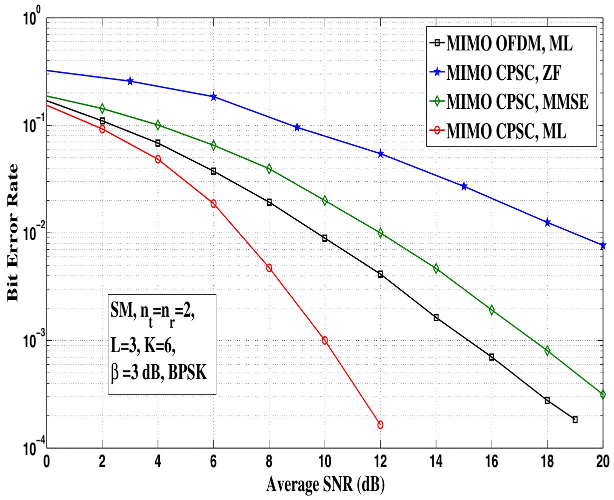

Fig. 4. Comparison of BER performance of ML detection in MIMO-OFDM and MIMO CPSC systems with SM for $K = 6$, $n_t = n_r = 2$, $L = 3$, $\beta = 3$ dB.

is n_r. Simulation results validated this prediction of diversity order. Simulation results also revealed that MIMO-CPSC with SM and SSK performed significantly better than MIMO-OFDM with SM and SSK, making CPSC signaling to be a credible alternative to multicarrier signaling like OFDM in MIMO-ISI channels.

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