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Downlink Power Allocation with Pricing in CDMA

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Abstract— **In this paper, we present an approach for power allocation to maximize the effective system throughput with *pricing* on the downlink in a CDMA system. We present a pricing policy and obtain the optimum powers for the users to maximize the effective system throughput incorporting this pricing policy. We also study the asymptotic behavior of the system (i.e., when the number of users and the available bandwidth are large). We show that in such a system, all users obtain equal SIR at the optimum point irrespective of their locations and processing gains. We also derive an expression for the maximum asymptotic spectral efficiency.**

Keywords – Cellular CDMA, downlink, power allocation, utility, pricing.

I. Introduction

The developments in micro-economics [1], have motivated utility function based resource allocation in cellular CDMA systems [2]-[7]. In [2], Famolari *et al* considered a utility function based approach to allocate powers to the users in a single-cell CDMA system, where they considered a utility function which is a function of the transmit power and the signal-to-interference ratio (SIR). In [4], Song and Mandayam presented a throughput maximization approach to satisfy SIR constraints in a time slotted system. An optimal utility function was derived to maximize the throughput. In [3], Liu *et al* considered utility functions based on residual service times and obtained scheduling policies and rate allocation to users. Resource allocation incorporating *pricing* has been studied in [5]-[7]. In [5], Saraydar *et al* extended the formulation in [2] with supermodular game theoretic models and incorporating pricing. They obtained more socially optimal solutions compared to [2]. In [6], Mau *et al* studied pricing as a trade off mechanism between user centric and network centric requirements. In [7], Matbach and Berry discussed differential pricing for users. It is of interest to study a pricing policy which results in a simplified solution for the optimum powers.

In this paper, we study power allocation on the downlink with pricing in a CDMA system. Each user receives data from the base station at a fixed rate, and powers are allocated to users to maximize the effective throughput of the system. We propose a pricing policy to price users and obtain the optimum powers for the users to maximize the effective system throughput incorporating this pricing policy. We then study the asymptotic behavior of the system with pricing, i.e., the behavior the system with large number of users and large amount of resources. We show that in such a system all users obtain equal SIR at the optimum point irrespective of their locations and processing gains.

The rest of the paper is organized as follows. In Section II, we present the optimum power allocation to the users without and with pricing. Section III presents the numerical results. Section IV provides the conclusion.

II. Power Allocation

In this section, we study optimal power allocation to users on the downlink. The i^{th} user in the cell receives data from the base station at rate r_i, which is assigned apriori by the base station. However, the data can be received in error due to interference from the other users in the cell. Therefore, the effective data rate of user i (i.e., the rate of "error-free" data transmission) falls below r_i. This decrease in the effective data rate of user i is a function of the bit error rate (BER) of user i, which, in turn, depends on the SIR seen by user i. The SIR of the i^{th} user is a function of the power, P_i, at which the base station transmits to user i. The objective is to allocate powers to users to maximize the "effective throughput" of the system, i.e., the sum of the effective data rates of all users in the system.

Consider a cell with M users, where the channel gain from the base station to user i is h_i, and $\mathcal{H} = [h_i]_{1 \leq i \leq M}$ is the channel gain vector. The SIR seen by the i^{th} user, Γ_i, is given by

$$\Gamma_i = \frac{P_i h_i G_i}{N_0 W + \sum_{l \neq i} P_l (1-\nu) h_i}, \quad (1)$$

where N_0 is the power spectral density of additive white Gaussian noise, W is the system bandwidth, $G_i = W/r_i$ is the processing gain of user i, and $\nu \in [0,1]$ is the orthogonality factor. When $\nu = 1$, the users are said to be perfectly orthogonal to each other, i.e., users do not interfere with each other. When $\nu = 0$, the users cause maximum interference to each other.

A. Power Allocation without Pricing

In this subsection, we present the power allocation to users without pricing. Let U_i denote the utility of user i. We define U_i as the effective throughput of user i, i.e.,

$$U_i = r_i f(\Gamma_i), \quad (2)$$

where $f(\Gamma_i) \in [0,1]$ denotes the factor by which the effective rate of user i is below the actual rate r_i due to interference from the other users. The function $f(\Gamma_i)$, for example, can be the mutual information transfer rate or the probability of error-free reception. To perform the power allocation, we consider the function $f(\Gamma_i)$ having the following properties:

- A user i whose SIR, Γ_i, is zero, has a BER of 0.5, and hence an effective throughput of zero. Similarly, if $\Gamma_i = \infty$, then the BER of the i^{th} user is zero, and hence, $U_i = r_i$. Therefore, $f(\Gamma_i)$ satisfies $f(0) = 0$ and $\lim_{\Gamma_i \to \infty} f(\Gamma_i) = 1$.
- $f(\Gamma_i)$ is an increasing function of Γ_i, i.e., $f'(\Gamma_i) > 0$. This property implies that a user derives more utility when its SIR increases.
- To satisfy the law of diminishing marginal utility [1], $\lim_{\Gamma_i \to \infty} f'(\Gamma_i) = 0$ and $f''(\Gamma_i) < 0$, i.e., $f(\Gamma_i)$ is a concave function of Γ_i.

The power allocation problem can be formulated as follows:

$$\text{Maximize} \sum_i U_i \quad (3)$$

subject to

$$0 \leq P_i \leq P_{max} \quad \forall i, \quad (4)$$

and

$$\sum_i P_i \leq P_{tot}. \quad (5)$$

In the absence of constraints (4) and (5), the maximum value of $\sum_i U_i$ is $\sum_i r_i$, which occurs at $\Gamma_i = \infty \ \forall\ i$. From the SIR vector, $\underline{\Gamma} = [\Gamma_i]_{1 \leq i \leq M}$, and defining $S(\underline{\Gamma})$ as

$$S(\underline{\Gamma}) \triangleq \sum_{k=1}^{M} \frac{(1-\nu)\Gamma_k}{(1-\nu)\Gamma_k + G_k}, \quad (6)$$

P_i can be obtained by algebric simplification as

$$P_i = N_0 W \left[\frac{\Gamma_i}{\Gamma_i(1-\nu) + G_i}\right] \left[\frac{1}{h_i} + \frac{T(\underline{\Gamma}, \mathcal{H})}{[1 - S(\underline{\Gamma})]}\right], \quad (7)$$

where

$$T(\underline{\Gamma}, \mathcal{H}) = \sum_{l=1}^{M} \frac{(1-\nu)\Gamma_l}{h_l\left[(1-\nu)\Gamma_l + G_l\right]}. \quad (8)$$

Therefore, if $\Gamma_i = \infty \ \forall\ i$, it leads to $P_i = \infty \ \forall\ i$. When constraints (4) and (5) are incorporated, the power allocation problem is solved as follows. Let i_1, i_2, $\cdots$, i_M be the users arranged in the non-increasing order of rates, i. e., $r_{i_1} \geq r_{i_2} \geq \cdots \geq r_{i_M}$. Let the powers corresponding to these users be P_{i_1}, P_{i_2}, $\cdots$, P_{i_M}, respectively. Then,

1) $P_{i_1} = \min(P_{max}, P_{tot})$.
2) $P_{i_j} = \min(P_{max}, P_{tot} - \sum_{l=1}^{j-1} P_{i_l})$, $2 \leq j \leq M$.

Consider a system in which $r_i = r \ \forall\ i$. If $P_{tot} \geq MP_{max}$, then at the optimum point, $P_i = P_{max} \ \forall\ i$, which leads to unequal SIRs for the users. This, in turn, results in largest utility for the nearest user (i.e., the user with largest value of h_i), and smallest utility for the farthest user (i.e., the user with the least value of h_i). In other words, this power allocation without pricing results in near-far unfairness. This near-far unfairness can be combated by incorporating pricing, which is explained in the following subsection.

B. Power Allocation incorporating Pricing

In this subsection, we present a framework for allocating powers to users by incorporating pricing. We present a pricing function where the price paid by user i, C_i, is defined as follows.

$$C_i = \lambda \frac{P_i h_i}{P_i h_i + I_i}, \quad (9)$$

where λ is the pricing parameter and $I_i = N_0 W + \sum_{l \neq i} P_l (1-\nu) h_i$ is the interference seen by user i. The price, C_i, in (9) represents the ratio of the signal power of user i to the total power received by user i (i.e., the sum of the signal power and interference). This pricing policy results in users with higher SIR paying more than users with lower SIR. Hence, in a system in which all users have same r_i, the pricing policy results in a higher price for users who receive larger SIR, and hence they obtain larger utility (i.e., larger effective data rate). The net utility derived by each user, U_i^{net}, is then defined as

$$U_i^{net} \triangleq U_i - C_i. \quad (10)$$

The power allocation problem including the pricing can then be formulated as an optimization problem to maximize $\sum_i U_i^{net}$, subject to the constraints (4) and (5). We first maximize $\sum_i U_i^{net}$ without the constraints (4) and (5), which, in effect, is to solve for P_i's to maximize U_i^{net} $\forall$ i. The optimum power, $\tilde{P}_i$, to maximize U_i^{net} is obtained by solving

$$\frac{\partial U_i^{net}}{\partial P_i} = 0 \quad \forall i. \tag{11}$$

From (1) and (7), it is observed that there is a one-to-one mapping between the optimum power vector, $\tilde{\mathbf{P}}$, and the corresponding SIR vector, $\underline{\Gamma}$. Hence, we formulate the power allocation problem as an SIR allocation problem, and obtain the optimal values of Γ_i for user i which maximize U_i^{net}. From $\underline{\Gamma}$, the optimum power vector, $\mathbf{P}$, can then be obtained from (7). Therefore, solving (11) is equivalent to solving for $\underline{\Gamma}$ to satisfy

$$g(\Gamma_i) = \frac{\lambda}{W} \quad \forall i, \tag{12}$$

where

$$g(\Gamma_i) \triangleq f'(\Gamma_i)\left(1 + \frac{\Gamma_i}{G_i}\right)^2. \tag{13}$$

Let Γ_i^* be the value of Γ_i that satisfies (12). Let $\underline{\Gamma}^* = [\Gamma_i^*]_{1\leq i\leq M}$. The following theorem presents a necessary and sufficient condition for the feasibility of an SIR vector, $\underline{\Gamma}$[1].

Theorem 2.1: The necessary and sufficient condition for the SIR vector, $\underline{\Gamma} = [\Gamma_i]_{1\leq i\leq M}$, to be feasible is $S(\underline{\Gamma}) < 1$.

From (13), it is observed that

$$g'(\Gamma_i) = \left(\frac{1}{G_i}\right)\left(1+\frac{\Gamma_i}{G_i}\right)\left[2f'(\Gamma_i) + (\Gamma_i + G_i)f''(\Gamma_i)\right]. \tag{14}$$

We choose $f(\Gamma_i)$ such that, in addition to satisfying the conditions mentioned in Section II-A,

$$2f'(\Gamma_i) + (\Gamma_i + G_i)f''(\Gamma_i) < 0 \quad \forall\Gamma_i \tag{15}$$

so that $g(\Gamma_i)$ is a decreasing function of Γ_i[2]. Therefore, the maximum value of $g(\Gamma_i)$ occurs when $\Gamma_i = 0$. From (13), it is observed that $g(0) = f'(0)$. Hence, if $\lambda > Wf'(0)$, then (12) cannot be solved for $\Gamma_i \geq 0$. Therefore, when $f(\Gamma_i)$ is chosen to satisfy (15), it fixes an upper limit on the pricing parameter, λ (given by $Wf'(0)$), to obtain a feasible solution to the power allocation problem incorporating pricing. It can be shown [8] that if $f(\Gamma_i)$ is chosen to be the mutual information in a binary symmetric channel (BSC) with binary phase shift keying (BPSK), then $f(\Gamma_i)$ satisfies the properties mentioned in Section II-A and (15). When $g(\Gamma_i)$ is a decreasing function of Γ_i, it is also possible to find a lower limit on the pricing parameter λ to solve the power allocation problem with pricing. This is explained in the following theorem.

Theorem 2.2: There exists a λ^* such that, for any $\lambda \in (\lambda^*, Wf'(0))$, it is possible to find a feasible SIR vector, $\underline{\Gamma}^*$, to solve the power control problem incorporating pricing.

Let $\tilde{\mathbf{P}} = [\ \tilde{P}_1 \ \ \tilde{P}_2 \ \ \tilde{P}_3 \ \ \cdots \ \ \tilde{P}_M\ ]$ be the power vector which forms the solution of the power allocation problem incorporating pricing without constraints (4) and (5). The solution to the power allocation problem incorporating pricing including constraints (4) and (5), $\mathbf{P}^* = [\ P_1^* \ \ P_2^* \ \ P_3^* \ \ \cdots \ \ P_M^*\ ]$, is then obtained as follows.

1) Let $r_{i_1} \geq r_{i_2} \geq \cdots \geq r_{i_M}$.
2) $P_{i_1}^* = \min(\tilde{P}_{i_1}, P_{max}, P_{tot})$.
3) $P_{i_j}^* = \min(\tilde{P}_{i_j}, P_{max}, P_{tot} - \sum_{l=1}^{j-1} P_{i_l}^*)$, $2 \leq j \leq M$.

We present the numerical results (i.e., the powers allocated to different users and the effective data rates of different users) for the system without and with pricing in Section III.

C. Asymptotic Behavior

From the formulation in the previous subsection, it is observed that the optimum SIR allocation leads to solving (12) M times for M users. In this subsection, we present the SIR allocation with pricing in a system with large number of users and large amount of resources, (i.e., large values of W and P_{tot}).

Theorem 2.3: Let $\lambda \in (\lambda^*, Wf'(0))$, where λ^* is obtained from Theorem 2.2. If $r_{min} \leq r_i \leq r_{max}$ $\forall$ i, and $M/W = \rho$, then, for large values of M and W, all the users obtain equal SIR at the optimum point of the power allocation problem with pricing. The SIR, β, is given by

$$\beta = \frac{[1 - v(\lambda)]\,W}{(1-\nu)\sum_i r_i}, \tag{16}$$

where $v(\lambda)$ is an increasing function of λ such that $v(\lambda^*) = 0$ and $v(Wf'(0)) = 1$.

From Theorem 2.3, it is oberved that in a system with large number of users and large bandwidth, all users

[1] We omit the proofs of theorems due to lack of space. The detailed proofs are available in [8].

[2] Although it appears that (15) is restrictive, it is usually satisifi ed if $f(\Gamma)$ is chosen to be the probability of correct reception in differential phase shift keying (DPSK) over additive whit Gaussian noise (AWGN) channels or mutual information in a binary symmetric channel (BSC) with binary phase shift keying (BPSK).

obtain equal SIR, β, at the optimum point, where β is given by (16).

The spectral efficiency, η, is defined as the effective throughput per unit bandwidth, i.e., $\eta \triangleq \frac{1}{W}\sum_{i=1}^{M} r_i f(\Gamma_i)$. The asymptotic spectral efficiency, $\hat{\eta}$ is defined as $\hat{\eta} \triangleq \lim_{\substack{M\to\infty \\ W\to\infty}} \eta$. From the value of β in (16), the asymptotic spectral efficiency is obtained from the following theorem.

Theorem 2.4: If $\hat{\beta} \triangleq \sup_{\lambda} \beta$, and

$$\hat{\eta} \triangleq \sup_{\hat{\beta}} \frac{f(\hat{\beta})}{(1-\nu)\hat{\beta}}, \quad (17)$$

then the maximum asymptotic spectral efficiency, η^*, is given by

$$\eta^* = \min(\hat{\eta}, \rho r_{max}). \quad (18)$$

III. Results and Discussion

In this section, we present the numerical results for the power allocation on the downlink without and with pricing. The following values are used in the numerical computations. The cell has 10 users, i.e., $M = 10$. The system bandwidth, $W = 5$ MHz, $P_{max} = 2$ Watts, $P_{tot} = 25$ Watts, and $N_0 = 10^{-8}$ W/Hz. We consider channel gain vector, $\mathcal{H} = [\ 0.9\ \ 0.85\ \ 0.8\ \ 0.75\ \ 0.6\ \ 0.5\ \ 0.4\ \ 0.3\ \ 0.2\ \ 0.1\ ]$, which represents a scenario with both near users and far users to the base station, and $r_i = 20$ Kbps $\forall\ i$, We take $f(\Gamma_i)$ to be the mutual information in a binary symmetric channel (BSC) with binary phase shift keying (BPSK). It can be shown that $f(\Gamma)$ satisfies the properties mentioned in Section II-A and (15). We consider $\lambda = Wf'(0)/10$.

Table I presents the power allocation to users to maximize $\sum_i U_i$ without pricing. It is observed that $P_i = P_{max} = 2$ Watts $\forall\ i$, and the SIR, Γ_i, is larger for the nearest user (i.e., the user with the largest h_i) and least for the farthest user (i.e., the user with least h_i). Therefore, the nearest user obtains a larger utility (i.e., larger effective data rate) than the farthest user. Let $\tau_i \triangleq U_i/r_i$ be the fraction of data successfully received by user i. It is observed that $\tau_i = 1$ for the nearest user (due to larger value of SIR), and $\tau_i = 0.71$ for the farthest user (due to lower value of SIR).

The power allocation to users incorporating the pricing function given by (9), is presented in Table II. It is observed that the SIRs of all the users are equal at the optimum point. This is because, at the optimum point, Eqn. (12) is satisfied for all the users. From (12) and (13), it is observed that if $G_i = G_l$, then $\Gamma_i = \Gamma_l$ at the optimum point. Therefore, all users obtain the same SIR, and hence the same utility. Therefore, τ_i is also same ($\tau_i = 0.94$) for all the users.

We also study the power allocation in a system with different processing gain for each user. Tables III and IV present the power allocation to users in this system without and with pricing, respectively. The values of the rate, r_i, and the processing gain, G_i, are also provided in Table IV. From Table IV, it is observed that the SIRs of users are unequal, and that the user with higher rate obtains larger SIR. This is because, from (12) and (13), it is observed that if $G_i > G_l$, then at the optimum point, $\Gamma_i < \Gamma_l$[3]. However, the variation in the SIR is small, i.e., the SIR for different users varies between 2.61 and 2.51. According to Theorem 2.3 in Section II-C, all users obtain equal SIR asymptotically. However,from Table IV, it is observed that for 10 users, the variation in the allocated SIR for users is very small (between 2.61 and 2.51) when the proposed pricing policy is incorporated. This also results in about the same value of τ_i for all the users. From Table IV it is also observed that the nearest user (i.e., the users with largest value of h_i) may require more power than the farther users (i.e., users with lower values of h_i). This is because, the SIR and rate is larger for the nearest user and hence, from (7), the power may be larger. However, from (7), it is also observed that the powers need not be monotonic, i.e., $G_i > G_l$ need not result in $P_i < P_l$. The spectral efficiency for the system with 10 users with unequal processing gains discussed above, is observed to be 0.099. From Theorem 2.4, the asymptotic spectral efficiency is found to be 0.1. However, in this case, $\hat{\eta} > \rho r_{max}$.

We also studied the power allocation in a system with $M = 32$ users and $W = 1.25$ MHz [8], in which $\hat{\eta} < \rho r_{max}$. In such a system, the spectral efficiency was found to be 1.20 and the asymptotic spectral efficiency by applying Theorem 2.4 was found to be 1.25, i.e., it was observed that the system with 32 users with 1.25 MHz bandwidth showed values of spectral efficiency, η, close to the asymptotic spectral efficiency, $\hat{\eta}$ (e.g., $\eta = 1.20$ and $\hat{\eta} = 1.25$). Results similar to those obtained using the $f(\Gamma_i)$ for BSC with BPSK have been observed when $f(\Gamma_i)$ is chosen to be the probability of error-free reception in a system with differential phase shift keying (DPSK).

IV. Conclusion

We studied power allocation on the downlink in CDMA systems, without and with pricing. We allocated powers

[3]This also uses the property of $f(\Gamma_i)$ as given by (15).

to users to maximize the effective system throughput. We presented a pricing function to price users, and studied power allocation incorporating this pricing. We showed that when the processing gains of users are equal, the power allocation with pricing resulted in equal utility for all users at the optimum point. We also studied the asymptotic behavior of the system and showed that when both the available bandwidth as well as the number of users become large, all users obtain equal SIR at the optimum point. We obtained an expression for the maximum asymptotic spectral efficiency of the system. A similar approach for power allocation in a multi-cell environment is also of significant interest, and is a topic for further investigation.

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i	h_i	r_i (kbps)	G_i	Γ_i	P_i (W)	U_i (kbps)	τ_i
1	0.9	20	250	6.80	2	20.0	1.00
2	0.85	20	250	6.51	2	20.0	1.00
3	0.8	20	250	6.21	2	20.0	1.00
4	0.75	20	250	5.91	2	20.0	1.00
5	0.6	20	250	4.93	2	19.9	0.99
6	0.5	20	250	4.23	2	19.7	0.99
7	0.4	20	250	3.50	2	19.5	0.98
8	0.3	20	250	2.71	2	18.8	0.94
9	0.2	20	250	1.87	2	17.5	0.88
10	0.1	20	250	0.97	2	14.1	0.71

TABLE I

OPTIMAL POWERS TO MAXIMIZE $\sum_i U_i$ WITHOUT PRICING. $\tau_i = U_i/r_i$, AND $\eta \triangleq (1/W)\sum_i r_i f(\Gamma_i) \approx 0.038$.

i	h_i	r_i (kbps)	G_i	Γ_i	P_i (mW)	U_i (kbps)	τ_i
1	0.9	20	250	2.52	57.5	18.7	0.94
2	0.85	20	250	2.52	60.8	18.7	0.94
3	0.8	20	250	2.52	64.5	18.7	0.94
4	0.75	20	250	2.52	68.7	18.7	0.94
5	0.6	20	250	2.52	85.5	18.7	0.94
6	0.5	20	250	2.52	102.3	18.7	0.94
7	0.4	20	250	2.52	127.5	18.7	0.94
8	0.3	20	250	2.52	169.5	18.7	0.94
9	0.2	20	250	2.52	253.5	18.7	0.94
10	0.1	20	250	2.52	505.5	18.7	0.94

TABLE II

OPTIMAL POWERS TO MAXIMIZE $\sum_i U_i^{net}$ INCORPORATING PRICING. $\eta \triangleq (1/W)\sum_i r_i f(\Gamma_i) \approx 0.037$.

i	h_i	r_i (kbps)	G_i	Γ_i	P_i (W)	U_i (kbps)	τ_i
1	0.90	100	50.0	1.36	2	80.3	0.80
2	0.85	90	55.5	1.45	2	73.6	0.82
3	0.80	80	62.5	1.55	2	66.8	0.84
4	0.75	70	71.4	1.69	2	59.8	0.85
5	0.60	60	83.3	1.64	2	50.9	0.85
6	0.50	50	100.0	1.69	2	42.8	0.86
7	0.40	40	125.0	1.74	2	34.5	0.86
8	0.30	30	166.7	1.81	2	26.1	0.87
9	0.20	20	250.0	1.87	2	17.5	0.88
10	0.10	10	500.0	1.93	2	8.8	0.88

TABLE III

OPTIMAL POWERS TO MAXIMIZE $\sum_i U_i$ WITHOUT PRICING. $\tau_i = U_i/r_i$, AND $\eta \triangleq (1/W)\sum_i r_i f(\Gamma_i) \approx 0.092$.

i	h_i	r_i (kbps)	G_i	Γ_i	P_i (mW)	U_i (kbps)	τ_i
1	0.90	100	50.0	2.61	302.4	93.9	0.94
2	0.85	90	55.5	2.59	286.3	84.4	0.94
3	0.80	80	62.5	2.58	268.7	75.0	0.94
4	0.75	70	71.4	2.57	249.2	65.6	0.94
5	0.60	60	83.3	2.56	264.0	56.2	0.94
6	0.50	50	100.0	2.55	261.6	46.8	0.94
7	0.40	40	125.0	2.54	259.3	37.4	0.94
8	0.30	30	166.7	2.53	257.0	28.0	0.94
9	0.20	20	250.0	2.52	254.7	18.7	0.94
10	0.10	10	500.0	2.51	252.4	9.3	0.93

TABLE IV

OPTIMAL POWERS TO MAXIMIZE $\sum_i U_i^{net}$ INCORPORATING PRICING. $\eta \triangleq (1/W)\sum_i r_i f(\Gamma_i) \approx 0.099$.