Robust Transceiver Design for Multiuser MIMO Downlink

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***Abstract*—In this paper, we consider robust joint linear precoder/receive filter design for multiuser multi-input multi-output (MIMO) downlink that minimizes the sum mean square error (SMSE) in the presence of imperfect channel state information (CSI). The base station is equipped with multiple transmit antennas, and each user terminal is equipped with multiple receive antennas. The CSI is assumed to be perturbed by estimation error. The proposed transceiver design is based on jointly minimizing a modified function of the MSE, taking into account the statistics of the estimation error under a total transmit power constraint. An alternating optimization algorithm, wherein the optimization is performed with respect to the transmit precoder and the receive filter in an alternating fashion, is proposed. The robustness of the proposed algorithm to imperfections in CSI is illustrated through simulations.**

Keywords – *multiuser MIMO downlink, multiuser interference, imperfect CSI, alternating optimization.*

I. Introduction

There has been considerable interest in multiuser multiple-input multiple-output (MIMO) wireless communication systems in view of their potential to offer the benefits of transmit diversity and increased channel capacity [1], [2]. In multiuser MIMO systems, multiuser interference at the receiver is a crucial issue. As a means to mitigate multiuser interference, transmit-side processing in the form of precoding has been studied widely [2]. Because of the difficulty in providing user terminals with several antennas due to space constraints, multiuser multiple-input single-output (MISO) wireless communication on the downlink, where the base station is equipped with multiple transmit antennas and each user terminal is equipped with a single receive antenna, is of interest. Several studies on precoding for such multiuser MISO systems have been reported [3]–[5]. There has been an increased interest to provide user terminals with two receive antennas, i.e., multiple-input double-output (MIDO) downlink. This necessitates the development and analysis of precoder designs for multiuser MIDO downlink, and more generally for multiuser MIMO downlink.

An important criterion that has been frequently used in precoder designs for multiuser MIMO downlink is sum mean square error (SMSE) [6]–[9]. Iterative algorithms that minimize SMSE with a constraint on total transmit power, where the minimization is done alternately between the transmit precoder and receive filter, are reported in [6], [7]. These algorithms are not guaranteed to converge to the global minimum. Minimum SMSE precoder and receiver designs based on uplink-downlink duality have been proposed in [8], [9]. These algorithms are guaranteed to converge to the global minimum.

However, the studies in [6]–[9] assume availability of perfect CSI at the transmitter. But, in practice, CSI at the transmitter suffers from inaccuracies caused by errors in channel estimation and/or limited, delayed or erroneous feedback. The performance of precoding schemes is sensitive to such inaccuracies [10]. Hence, it is of interest to develop *transceiver designs that are robust to errors in CSI*. Two approaches to robust designs are generally adopted. One approach is based on minimax or worst case performance [11], [12]. The other approach is based on a stochastic measure of the performance. In the former case, the design is conservative but it ensures a minimum performance for all values of the uncertain parameter belonging to a predefined uncertainty set. This approach is applicable when the parameter uncertainties belong to a predefined uncertainty set. In the latter case, robustness is achieved by optimizing an average or some other appropriate stochastic measure of the performance metric. This approach is possible if the distribution of the parameter uncertainty is available. A few studies on robust precoding for multiuser MISO downlink with imperfect CSI have been reported in the literature [13]–[15]. The studies on robust precoder design in [13]–[15], however, are only for user terminals with single receive antenna.

We, in this paper, consider downlink users having more than one receive antenna, i.e., we consider MIMO downlink instead of MISO downlink. Specifically, we propose a robust joint design of the precoder and receive filter for multiuser MIMO downlink with imperfect CSI. The proposed transceiver design is based on minimizing a modified function of MSE under a total transmit power constraint. We propose an alternating optimization algorithm to solve this constrained minimization problem. In this approach, the joint optimization with respect to the precoder matrix and receive filter is replaced by optimization over precoder and receiver in an alternating fashion. We note that the proposed robust design encompasses transceiver designs proposed in [6] and [7] as special cases when the CSI at transmitter is perfect. The robustness of the proposed algorithm to imperfections in CSI is illustrated through simulations.

The rest of the paper is organized as follows. In Section II, we present the multiuser MIMO system model and the CSI error model. The proposed robust transceiver design is presented in Section III. Simulation results and comparisons are presented in Section IV. Conclusions are presented in Section V.

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II. SYSTEM MODEL

We consider a multiuser MIMO downlink, where a base station (BS) communicates with M users on the downlink. The BS employs N_t transmit antennas and the kth user is equipped with N_{r_k} receive antennas, $1 \leq k \leq M$. Let $\mathbf{u}_k$ denote[1] the $L_k \times 1$ data symbol vector for the kth user, where L_k, $k = 1, 2, \cdots, M$, is the number of data streams for the kth user. Stacking the data vectors for all the users, we get the global data vector $\mathbf{u} = [\mathbf{u}_1^T, \cdots, \mathbf{u}_M^T]^T$. Let $\mathbf{B}_k \in \mathcal{C}^{N_t \times L_k}$ represent the precoding matrix for the kth user. The global precoding matrix $\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_M]$. The transmit vector is given by

$$\mathbf{x} \;=\; \mathbf{B}\mathbf{u}. \tag{1}$$

The kth component of the transmit vector $\mathbf{x}$ is transmitted from the kth transmit antenna. The overall channel matrix is

$$\mathbf{H} \;=\; [\mathbf{H}_1^T \quad \mathbf{H}_2^T \cdots \mathbf{H}_M^T]^T, \tag{2}$$

where $\mathbf{H}_k$ is the $N_{r_k} \times N_t$ channel matrix of the kth user. The entries of the channel matrices are assumed to be zero-mean, unit-variance complex Gaussian random variables. The received signal vectors are given by

$$\mathbf{y}_k \;=\; \mathbf{H}_k\mathbf{B}\mathbf{u} + \mathbf{n}_k, \quad 1 \leq k \leq M. \tag{3}$$

The users estimate the data vector meant for them as

$$\begin{aligned} \widehat{\mathbf{u}}_k &= \mathbf{C}_k\mathbf{y}_k \;=\; \mathbf{C}_k\mathbf{H}_k\mathbf{B}\mathbf{u} + \mathbf{C}_k\mathbf{n}_k \\ &= \mathbf{C}_k\mathbf{H}_k\Big(\sum_{j=1}^{M}\mathbf{B}_j\mathbf{u}_j\Big) + \mathbf{C}_k\mathbf{n}_k, \;\; 1 \leq k \leq M, \end{aligned} \tag{4}$$

where $\mathbf{n}_k$ is the noise vector at the kth user, $\mathbf{C}_k$ is the $L_k \times N_{r_k}$ dimensional receive filter of the kth user. Stacking the estimated vectors of all users, the global estimate can be written as

$$\widehat{\mathbf{u}} \;=\; \mathbf{C}\mathbf{H}\mathbf{B}\mathbf{u} + \mathbf{C}\mathbf{n}, \tag{5}$$

where $\mathbf{C}$ is a block diagonal matrix with $\mathbf{C}_k, 1 \leq k \leq M$ on the diagonal, and $\mathbf{n} = [\mathbf{n}_1^T, \cdots, \mathbf{n}_M^T]^T$. The global receive matrix $\mathbf{C}$ has block diagonal structure as the receivers are non-cooperative. The MSE between the symbol vector $\mathbf{u}_k$ and the estimate $\widehat{\mathbf{u}}_k$ at the kth user is given by

$$\epsilon_k \;=\; \mathbb{E}\{\|\widehat{\mathbf{u}}_k - \mathbf{u}_k\|^2\}, \;\; 1 \leq k \leq M, \tag{6}$$

and the sum-MSE (SMSE) is given by

$$\texttt{smse} \;=\; \mathbb{E}\{\|\widehat{\mathbf{u}} - \mathbf{u}\|^2\} \;=\; \sum_{k=1}^{M}\epsilon_k, \tag{7}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

[1] Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$, $[.]^H$, and $[.]^\dagger$, denote transpose, Hermitian, and pseudo-inverse operations, respectively. $[\mathbf{A}]_{ij}$ denotes the element on the ith row and jth column of the matrix $\mathbf{A}$. vec(.) operator stacks the columns of the input matrix into one column-vector.

A. CSI Error Model

The transceiver design in this paper is based on a statistical model for the error in CSI at the transmitter. In this model, the true channel matrix of the kth user $\mathbf{H}_k$ is represented as

$$\mathbf{H}_k \;=\; \widehat{\mathbf{H}}_k + \mathbf{E}_k, \quad 1 \leq k \leq M, \tag{8}$$

where $\widehat{\mathbf{H}}_k$ is the estimated channel matrix of the kth user and $\mathbf{E}_k$ is the estimation error matrix. The error matrix $\mathbf{E}_k$ is assumed to be Gaussian distributed with zero mean and $\mathbb{E}\{\mathbf{E}_k\mathbf{E}_k^H\} = \sigma_{\mathbf{E}}^2\mathbf{I}_{N_{r_k}N_{r_k}}$. The overall channel matrix can be written as

$$\mathbf{H} \;=\; \widehat{\mathbf{H}} + \mathbf{E}, \tag{9}$$

where $\widehat{\mathbf{H}} = [\widehat{\mathbf{H}}_1^T \quad \widehat{\mathbf{H}}_2^T \cdots \widehat{\mathbf{H}}_M^T]^T$, and $\mathbf{E} = [\mathbf{E}_1^T \quad \mathbf{E}_2^T \cdots \mathbf{E}_M^T]^T$. This statistical model is suitable for systems with uplink-downlink reciprocity.

III. PROPOSED ROBUST TRANSCEIVER DESIGN

When the transmitter has perfect knowledge of CSI, the problem of designing the transmit precoder $\mathbf{B}$ and receive filter $\mathbf{C}$ which minimizes the SMSE under a transmit power constraint can be written as

$$\begin{aligned} &\min_{\mathbf{B},\mathbf{C}} && \texttt{smse} \\ &\text{subject to} && \text{Tr}(\mathbf{B}\mathbf{B}^H) \leq P_T, \end{aligned} \tag{10}$$

where P_T is the maximum allowed transmit power, and Tr$(\cdot)$ is the trace operator. Using (4), the MSE of the kth user given in (6) can be written as

$$\begin{aligned} \epsilon_k &= \mathbb{E}\{\|\widehat{\mathbf{u}}_k - \mathbf{u}_k\|^2\} \\ &= \text{Tr}\Big(\mathbb{E}\{(\widehat{\mathbf{u}}_k - \mathbf{u}_k)(\widehat{\mathbf{u}}_k - \mathbf{u}_k)^H\}\Big) \\ &= \text{Tr}\Big(\mathbf{C}_k\mathbf{H}_k\Big(\sum_{j=1}^{M}\mathbf{B}_j\mathbf{B}_j^H\Big)\mathbf{H}_k\mathbf{C}_k^H\Big) \\ &\quad -\text{Tr}\Big(2\Re\left(\mathbf{C}_k\mathbf{H}_k\mathbf{B}_k\right) + \mathbf{I} + \mathbf{C}_k\mathbf{R}_{\mathbf{n}_k}\mathbf{C}_k^H\Big), \end{aligned} \tag{11}$$

where $\mathbf{R}_{\mathbf{n}_k} = \mathbb{E}\{\mathbf{n}_k\mathbf{n}_k^H\}$. Different algorithms for solving the problem in (10) with perfect CSI have been reported in the literature. Optimization based on alternating design of precoding and receive filter matrices is reported in [7]. This algorithm is not guaranteed to converge the global optimum. Algorithms based on uplink-downlink duality reported in [9] and [8] converge to the global optimum. But when the CSI at the transmitter is imperfect, the use of precoders and receive filters designed based on these algorithms that assume perfect CSI results in performance degradation.

A. Robust Design with imperfect CSI

In order to incorporate the CSI imperfections in the transceiver design to make it robust, we consider an appropriately modified objective function for minimization. If the error in CSI is bounded, then the worst-case SMSE can be taken as the new objective function. But when the error in CSI is modeled as stochastic, as in II-A, it is appropriate to consider the SMSE averaged over the CSI error as the objective function. Following this approach, the robust transceiver design problem can be written as

$$\begin{aligned} \min_{\mathbf{B},\mathbf{C}} \quad & \mathbb{E}_{\mathbf{E}}\left\{\texttt{smse}(\mathbf{B},\mathbf{C},\mathbf{E})\right\} \\ \text{subject to} \quad & \operatorname{Tr}\left(\mathbf{B}\mathbf{B}^H\right) \le P_T. \end{aligned} \tag{12}$$

Substituting $\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}$ in (5), the SMSE can be written as

$$\begin{aligned} \texttt{smse}(\mathbf{B},\mathbf{C},\mathbf{E}) &= \mathbb{E}\{\|\hat{\mathbf{u}} - \mathbf{u}\|^2\} \\ &= \operatorname{Tr}\left((\mathbf{CHB} - \mathbf{I})(\mathbf{CHB} - \mathbf{I})^H + \operatorname{Tr}\left(\mathbf{C}\mathbf{R}_{\mathbf{n}}\mathbf{C}^H\right)\right) \\ &= \operatorname{Tr}\left(\left(\mathbf{C}\left(\widehat{\mathbf{H}} + \mathbf{E}\right)\mathbf{B} - \mathbf{I}\right)\left(\mathbf{C}\left(\widehat{\mathbf{H}} + \mathbf{E}\right)\mathbf{B} - \mathbf{I}\right)^H\right) \\ &\quad + \operatorname{Tr}\left(\mathbf{C}\mathbf{R}_{\mathbf{n}}\mathbf{C}^H\right), \end{aligned} \tag{13}$$

where $\mathbf{R}_{\mathbf{n}} = \mathbb{E}\{\mathbf{n}\mathbf{n}^H\}$.

Let $\widehat{\mathbf{D}} = \mathbf{I} \otimes (\mathbf{C}\widehat{\mathbf{H}})$, $\widetilde{\mathbf{D}} = \mathbf{I} \otimes (\mathbf{CE})$, $\mathbf{b} = \text{vec}(\mathbf{B})$, and $\mathbf{f} = \text{vec}(\mathbf{I})$. The SMSE can be written in terms of $\mathbf{b}$, the vectorized form of $\mathbf{B}$, as

$$\begin{aligned} \texttt{smse}(\mathbf{b},\mathbf{C},\sigma_{\mathbf{E}}^2) &= \left((\widehat{\mathbf{D}} + \widetilde{\mathbf{D}})\mathbf{b} - \mathbf{f}\right)^H \left((\widehat{\mathbf{D}} + \widetilde{\mathbf{D}})\mathbf{b} - \mathbf{f}\right) \\ &\quad + \operatorname{Tr}\left(\mathbf{C}\mathbf{R}_{\mathbf{n}}\mathbf{C}^H\right). \end{aligned} \tag{14}$$

Defining $\mu(\mathbf{B},\mathbf{C}) \triangleq \mathbb{E}_{\mathbf{E}}\left\{\texttt{smse}(\mathbf{B},\mathbf{C},\mathbf{E})\right\}$ as the new objective function, we have

$$\begin{aligned} \mu(\mathbf{b},\mathbf{C}) &= \mathbb{E}_{\mathbf{E}}\left\{\texttt{smse}(\mathbf{B},\mathbf{C},\mathbf{E})\right\} \\ &= \mathbb{E}_{\mathbf{E}}\left\{\left(\left(\widehat{\mathbf{D}} + \widetilde{\mathbf{D}}\right)\mathbf{b} - \mathbf{f}\right)^H \left(\left(\widehat{\mathbf{D}} + \widetilde{\mathbf{D}}\right)\mathbf{b} - \mathbf{f}\right)\right\} \\ &\quad + \operatorname{Tr}(\mathbf{C}\mathbf{R}_{\mathbf{n}}\mathbf{C}^H) \\ &= \left\|\mathbf{D}\mathbf{b} - \mathbf{f}\right\|^2 + \sigma_{\mathbf{E}}^2 \operatorname{Tr}(\mathbf{C}\mathbf{C}^H)\left\|\mathbf{b}\right\|^2 \\ &\quad + \operatorname{Tr}(\mathbf{C}\mathbf{R}_{\mathbf{n}}\mathbf{C}^H). \end{aligned} \tag{15}$$

The robust transceiver design problem can now be written as

$$\begin{aligned} \min_{\mathbf{b},\mathbf{C}} \quad & \mu(\mathbf{b},\mathbf{C}) \\ \text{subject to} \quad & \|\mathbf{b}\|^2 \le P_T. \end{aligned} \tag{16}$$

We solve this problem by minimizing the objective function over $\mathbf{b}$ and $\mathbf{C}$ in an alternating manner. When perfect CSI is available, the precoder and receive filter computed by our proposed design and those presented in [6], [7] will be identical. The following subsections describe our proposed design of robust precoder $\mathbf{b}$, receive filter $\mathbf{C}$, and the alternating optimization algorithm.

1) Robust Receiver Design: In this subsection, we consider the design of a robust receive filter that minimizes μ for a given precoder $\mathbf{B}$. Towards this end, we represent μ as the sum of MSEs of individual receivers averaged over the CSI error. Substituting the CSI error, (11) can be written as

$$\begin{aligned} \epsilon_k &= \operatorname{Tr}\left(\mathbf{C}_k\left(\widehat{\mathbf{H}}_k + \mathbf{E}_k\right)\mathbf{B}\mathbf{B}^H\left(\widehat{\mathbf{H}}_k + \mathbf{E}_k\right)^H \mathbf{C}_k^H\right) \\ &\quad + \operatorname{Tr}\left(-2\Re(\mathbf{C}_k\mathbf{H}_k\mathbf{B}_k) + \mathbf{I} + \mathbf{C}_k\mathbf{R}_{\mathbf{n}k}\mathbf{C}_k^H\right), 1 \le k \le M. \end{aligned} \tag{17}$$

Averaging ϵ_k over the CSI error, we have

$$\begin{aligned} \widetilde{\epsilon}_k &\triangleq \mathbb{E}_{\mathbf{E}_k}\{\epsilon_k\} \\ &= \operatorname{Tr}\left(\mathbf{C}_k\widehat{\mathbf{H}}_k\mathbf{B}\mathbf{B}^H\widehat{\mathbf{H}}_k^H\mathbf{C}_k^H - 2\Re(\mathbf{C}_k\widehat{\mathbf{H}}_k\mathbf{B}_k) + \mathbf{I}\right) \\ &\quad + \sigma_{\mathbf{E}}^2 \operatorname{Tr}\left(\mathbf{B}\mathbf{B}^H \operatorname{Tr}\left(\mathbf{C}_k\mathbf{C}_k^H\right) + \mathbf{C}_k\mathbf{R}_{\mathbf{n}k}\mathbf{C}_k^H\right). \end{aligned} \tag{18}$$

From (15) and (18)

$$\mu(\mathbf{B},\mathbf{C}) = \sum_{k=1}^{M} \widetilde{\epsilon}_k. \tag{19}$$

The objective function $\mu(\mathbf{B},\mathbf{C})$ is convex in $\mathbf{B}$ for a fixed value of $\mathbf{C}$ and vice versa, but is not jointly convex in $\mathbf{B}$ and $\mathbf{C}$. For a given $\mathbf{B}$, we can find the optimal receiver $\mathbf{C}_k^{\mathbf{B}}$ for the kth user by finding the stationary point of $\mu(\mathbf{B},\mathbf{C})$ with respect to $\mathbf{C}_k$. Differentiating $\mu(\mathbf{B},\mathbf{C})$ with respect to $\mathbf{C}_k$ and equating the result to zero, we have

$$\begin{aligned} \mathbf{B}_k^H\mathbf{H}_k^H &= \mathbf{C}_k^{\mathbf{B}}\left(\mathbf{H}_k\mathbf{B}\mathbf{B}^H\mathbf{H}_k^H + \mathbf{R}_{\mathbf{n}k} + \sigma_{\mathbf{E}}^2\operatorname{Tr}\left(\mathbf{B}\mathbf{B}^H\right)\right), \\ &\quad 1 \le k \le M. \end{aligned} \tag{20}$$

From the above equation, we get

$$\begin{aligned} \mathbf{C}_k^{\mathbf{B}} &= \mathbf{B}_k^H\mathbf{H}_k^H\left(\mathbf{H}_k\mathbf{B}\mathbf{B}_k^H + \mathbf{R}_{\mathbf{n}k} + \sigma_{\mathbf{E}}^2\operatorname{Tr}\left(\mathbf{B}\mathbf{B}^H\right)\right)^{-1}, \\ &\quad 1 \le k \le M. \end{aligned} \tag{21}$$

For a given precoder matrix $\mathbf{B}$, the global receive filter matrix $\mathbf{C}^{\mathbf{B}}$ is obtained as a block-diagonal matrix with $\mathbf{C}_k^{\mathbf{B}},\ 1 \le k \le M$ on the diagonal.

2) Robust Transmit Precoder Design: For a given receive filter $\mathbf{C}$, the problem of designing a robust precoder $\mathbf{B}^{\mathbf{C}}$ can be written as

$$\begin{aligned} \min_{\mathbf{B}} \quad & \mu \\ \text{subject to} \quad & \|\mathbf{b}\|^2 \le P_T. \end{aligned} \tag{22}$$

As the last term in (15) does not depend on $\mathbf{B}$, we can drop that term form the objective function while computing $\mathbf{B}$. We can formulate this robust design problem as

$$\min_{\mathbf{B}} \quad \|\mathbf{D}\mathbf{b} - \mathbf{f}\|^2 + \sigma_{\mathbf{E}}^2\operatorname{Tr}(\mathbf{C}\mathbf{C}^H)\|\mathbf{b}\|^2 \tag{23}$$

$$\text{subject to} \quad \|\mathbf{b}\|^2 \le P_T. \tag{24}$$

Introducing dummy variables t_1 and t_2, (23) can be written as the following rotated second order cone program [16]

$$\min_{\mathbf{b}} \quad t_1 + \sigma_{\mathbf{E}}^2\operatorname{Tr}(\mathbf{C}\mathbf{C}^H)t_2 \tag{25}$$

$$\text{subject to} \quad \|\mathbf{D}\mathbf{b} - \mathbf{f}\|^2 \le t_1, \tag{26}$$

$$\|\mathbf{b}\|^2 \le t_2, \tag{27}$$

$$t_2 \le P_T. \tag{28}$$

This convex optimization problem can be efficiently solved using interior point methods.

3) Alternating Optimization: In this subsection, we consider the alternating optimization (AO) approach to the minimization of the SMSE averaged over the CSI error. As the objective function $\mu(\mathbf{B}, \mathbf{C})$ is not jointly convex in both $\mathbf{B}$ and $\mathbf{C}$, this alternating optimization algorithm is not guaranteed to converge to the global minimum.

In the AO method, the entire set of optimization parameters is partitioned into non-overlapping subsets, and an iterative sequence of optimizations on these subsets is carried out, which is often simpler compared to simultaneous optimization over all parameters. In the present problem, we partition the optimization set $\{\mathbf{B},\mathbf{C}\}$ into the non-overlapping subsets $\{\mathbf{B}\}$ and $\{\mathbf{C}\}$ and perform the optimization with respect to these subsets in an alternating fashion.

TABLE I

N_{max}: Maximum number of iterations
T_{th}: Convergence threshold

1) Initialize $\mathbf{B}$ and $\mathbf{C}$
2) $n = 0$
3) **while** $n \leq N_{max}$
4) Compute $\mathbf{B}^{n+1}$ using (25) and $\mathbf{C}^{n}$
5) Compute $\mathbf{C}^{n+1}$ using (21) and $\mathbf{B}^{n+1}$
6) **if** $\left\| \mu\left(\mathbf{B}^{n+1}, \mathbf{C}^{n+1}\right) - \mu\left(\mathbf{B}^{n}, \mathbf{C}^{n}\right) \right\| \leq T_{th}$ **then**
7) **break**
8) **endif**
9) $n \leftarrow n+1$
10) **endwhile**

The algorithmic form of the alternating optimization for the computation of the matrices $\mathbf{B}$ (or equivalently $\mathbf{b}$) and $\mathbf{G}$ is shown in Table-I. At the $(n+1)$th iteration, the value of $\mathbf{B}$ is the solution to the following problem:

$$\mathbf{B}^{n+1} = \underset{\mathbf{B}:\mathrm{Tr}(\mathbf{B}\mathbf{B}^H) \leq P_T}{\operatorname{argmin}} \quad \mu(\mathbf{B}, \mathbf{C}^n), \tag{29}$$

which, as formulated in (25), can be solved efficiently. Having computed $\mathbf{B}^{n+1}$, $\mathbf{C}^{n+1}$ is the solution to the following problem:

$$\mathbf{C}^{n+1} = \underset{\mathbf{C}}{\operatorname{argmin}}\, \mu(\mathbf{B}^{n+1}, \mathbf{C}), \tag{30}$$

and its solution is given in (21). This alternating optimization over $\{\mathbf{B}\}$ and $\{\mathbf{C}\}$ can be repeated till convergence of the optimization variables. As the objective in (15) is monotonically decreasing after each iteration and is lower bounded, convergence is guaranteed. The iteration is terminated when the norm of the difference in the results of consecutive iterations is below a threshold or when the maximum number of iterations is reached.

B. Transceiver Design With Per-Antenna Power Constraint

As each antenna at the base station usually has its own amplifier, it is important to consider transceiver design with constraints on power transmitted from each antenna. A precoder design for multiuser MISO downlink with per-antenna power constraint with perfect CSI at the transmitter was considered in [17].

Here, we incorporate per-antenna power constraint in the proposed robust transceiver design. For this, only the precoder matrix design (23) has to be modified by including the constraints on power transmitted from each antenna as given below:

$$\begin{aligned} \min_{\mathbf{B}} \quad & \|\mathbf{D}\mathbf{b} - \mathbf{f}\|^2 + \sigma_{\mathbf{E}}^2 \mathrm{Tr}(\mathbf{C}\mathbf{C}^H)\|\mathbf{b}\|^2 \\ \text{subject to} \quad & \|\underline{\mathbf{b}}_k\|^2 \leq P_k, \quad 1 \leq k \leq M, \end{aligned} \tag{31}$$

where $\underline{\mathbf{b}}_k$ is the kth row of $\mathbf{B}$. The receive filter can be computed using (21).

IV. Simulation Results

In this section, we illustrate the performance of the proposed robust transceiver algorithm, evaluated through simulations. We compare the performance of the proposed design with the transceiver designs reported in the literature. The minimum SMSE designs proposed in [8] and [9] converge to the global minimum, whereas the designs reported in [6] and [7] do not converge to the global minimum. We compare the performance of the proposed robust transceiver design with that of the non-robust design in [9], which has better performance compared to the other non-robust designs in [6], [7]. The comparison is based on the symbol error rate (SER) averaged over all users versus the SNR defined as P_{Tr}/σ^2, where $P_{Tr} = \mathrm{Tr}\left(\mathbf{B}\mathbf{B}^H\right)$ is the total transmit power.

The channel fading is modeled as Rayleigh, with the channel matrices $\mathbf{H}_k$, $1 \leq k \leq M$, comprising of independent and identically distributed (i.i.d) samples of a complex Gaussian process with zero mean and unit variance. The noise at each antenna of each user terminal is assumed to be zero-mean unit-variance complex Gaussian random variable. The elements of the error matrices $\mathbf{E}_k$, $1 \leq k \leq M$, are zero-mean complex Gaussian random variables with variance $\sigma_{\mathbf{E}}^2$. QPSK modulation is employed on each data stream.

In the first experiment, we consider a system with the base station equipped with $N_t = 2$ transmit antennas, transmitting $L = 1$ one data stream to each user. There are $M = 2$ users, each equipped with $N_r = 2$ receive antennas. The simulation results are shown in Fig. 1. SER performances of the proposed robust design and the non-robust design proposed in [9] for $\sigma_{\mathbf{E}}^2 = 0.15$, 0.3 are compared. The proposed robust design is seen to outperform the non-robust design in [9]. It is found that the difference between the performance of these algorithms increase as the SNR increases. This is observable in (15), where the second term shows the effect of the CSI error variance amplified by the transmit power.

In the second experiment, we consider a system with $N_t = 4$ transmit antennas and $M = 2$ users, each equipped with $N_r = 2$ receive antennas. Here, we study the robustness of the proposed algorithm by transmitting $L = 2$ data streams to one user and one data stream to the other user. The simulation results for $\sigma_{\mathbf{E}}^2 = 0.15$, 0.3 are shown in Fig. 2. Here again, we

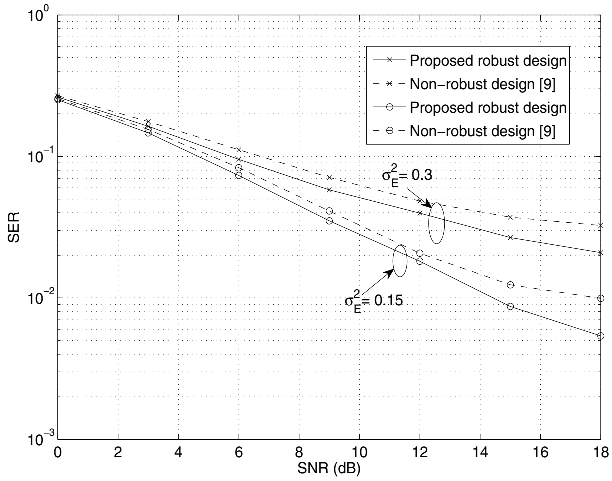

Fig. 1. Symbol error rate performance of the proposed robust transceiver design for $N_t = 2$, $M = 2$, $N_r = 2$, $L = 1$, $\sigma^2_{\mathbf{E}} = 0.15$, 0.3, QPSK.

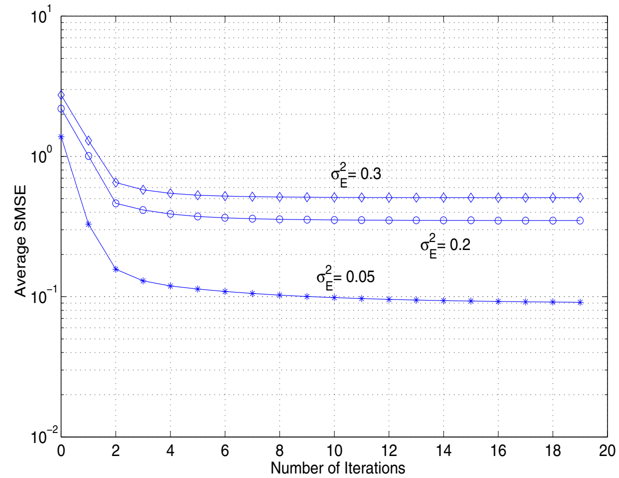

Fig. 3. Convergence behavior of the proposed robust transceiver design for different CSI error variances, $\sigma^2_{\mathbf{E}} = 0.05, 0.2, 0.3$.

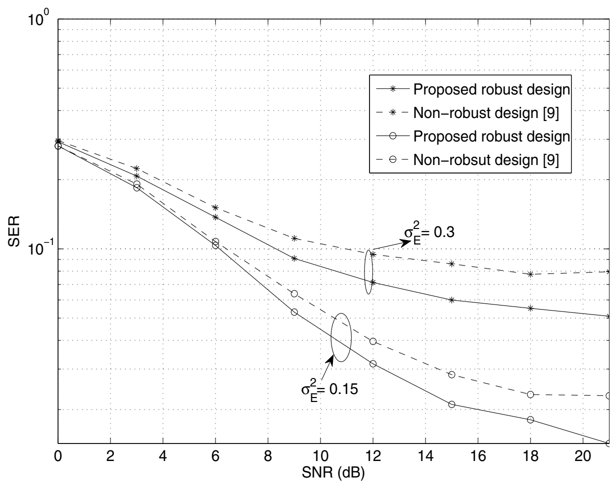

Fig. 2. Symbol error rate performance of the proposed robust transceiver design for $N_t = 4$, $M = 2$, $N_r = 2$, $L_1 = 2, L_2 = 1$, $\sigma^2_{\mathbf{E}} = 0.15, 0.3$, QPSK.

observe improved performance of the proposed robust design compared to that of the non-robust design in [9].

In the third experiment, we study the convergence behavior of the proposed robust design. Figure 3 shows the convergence behavior of the proposed algorithm. Here, we consider a system with $N_t = 6$, and $M = 4$. Each user is equipped with $N_r = 2$ antennas and receives $L = 1$ data stream. Simulation results for different values of $\sigma^2_{\mathbf{E}}$ are shown in Fig. 3. It can be seen that the proposed algorithm exhibits fast convergence.

V. Conclusions

We presented a robust joint design of linear precoder and receive filter for multiuser MIMO downlink with imperfect CSI. We proposed an alternating optimization algorithm, wherein the optimization is performed with respect to the transmit precoder and the receive filter in an alternating fashion. The proposed robust design is based on minimization of a modified function of MSE, taking into account the statistics of the CSI error under a constraint on total transmit power. We illustrated the robustness of the proposed design to the errors in CSI through simulations.

References

[1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2006.

[2] H. Bolcskei, D. Gesbert, C. B. Papadias, and A.-J. van der Veen, *Space-time Wireless Systems: From Array Processing to MIMO Communications*. Cambridge University Press, 2006.

[3] K. Kusume, M. Joham, W. Utschick, and G. Bauch, "Efficient tomlinson-harashima precoding for spatial multiplexing on flat MIMO channel," in *Proc. IEEE ICC'2005*, May 2005, pp. 2021–2025.

[4] A. Wiesel, Y. C. Eldar, and Shamai, "Linear precoder via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 52, pp. 161–176, Jan. 2006.

[5] R. Fischer, C. Windpassinger, A. Lampe, and J. Huber, "MIMO precoding for decentralized receivers," in *Proc. IEEE ISIT'2002*, 2002, p. 496.

[6] B. Bandemer, M. Haardt, and S. Visuri, "Liner MMSE multi-user MIMO downlink precoding for users with multple antennas," in *Proc. PIMRC'06*, Sep. 2006, pp. 1–5.

[7] J. Zhang, Y. Wu, S. Zhou, and J. Wang, "Joint linear transmitter and receiver design for the downlink of multiuser MIMO systems," *IEEE Commun. Lett.*, vol. 9, pp. 991–993, Nov. 2005.

[8] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE transceiver optimization for multiuser MIMO systems: Duality and sum-mse minimization," *IEEE Trans. Signal Process.*, vol. 55, pp. 5436–5446, Nov. 2007.

[9] A. Mezghani, M. Joham, R. Hunger, and W. Utschick, "Transceiver design for multi-user MIMO systems," in *Proc. WSA 2006*, Mar. 2006.

[10] N. Jindal, "MIMO broadcast channels with finite rate feed-back," in *Proc. IEEE GLOBECOM'2005*, Nov. 2005.

[11] S. A. Kassam and H. V. Poor, "Robust techniques for signal processing: A survey," *Proceeding of The IEEE*, vol. 3, pp. 433–481, Mar. 1985.

[12] A. Ben-Tal and A. Nemirovsky, "Robust convex optimization," *Mathematics of Operations Research*, vol. 23, no. 4, pp. 769–805, Nov. 1998.

[13] M. B. Shenouda and T. N. Davidson, "Robust linear precoding for uncertain MISO broadcast channels," in *Proc. IEEE ICASSP'2006*, vol. 4, 2006, pp. 37–40.

[14] R. Hunger, F. Dietrich, M. Joham, and W. Utschick, "Robust transmit zero-forcing filters," in *Proc. ITG Workshop on Smart Antennas*, Munich, Mar. 2004, pp. 130–137.

[15] M. Biguesh, S. Shahbazpanahi, and A. B. Gershman, "Robust downlink power control in wireless cellular systems," *EURASIP Jl. Wireless Commun. Networking*, vol. 2, pp. 261–272, 2004.

[16] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[17] W. Yu and T. Lan, "Transmitter optimization for multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, pp. 2646–2660, Jun. 2007.