Performance Analysis of a Parallel Interference Canceller on Rayleigh Fading Channels

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***Abstract*— An exact bit error rate (BER) expression, in closed-form, for parallel interference cancellation (PIC) with a decorrelator based initial data estimates has been derived earlier by Verdu [1], for non-fading channels. In this paper, we extend this work to Rayleigh fading channels. Specifically, we derive an exact BER expression for a PIC with decorrelator based initial data estimates on flat Rayleigh fading channels. The BER expression is obtained in terms of the elliptic integral of the third kind. It is noted that this BER expression is one of the rare closed-form expressions one can obtain for the BER of a nonlinear multiuser detector.**

Keywords — Multiuser detection, parallel interference cancellation, decorrelator, fading channels.

I. Introduction

Parallel interference cancellation (PIC) is a multistage, non-linear multiuser detection scheme that can be used to separate and demodulate signals in a multiuser system [1],[2]. The multistage approach can be used with tentative decisions provided by either the conventional single-user matched filters (MF) or other multiuser detectors (for example, a decorrelating detector can serve as the first stage of the PIC receiver). Several papers have reported the bit error rate (BER) performance of PIC receivers, based on mainly simulations. Because of the complexity involved in the derivation, exact analytical BER expressions obtained for PIC are rather limited. Even the available exact analytical BER expressions are for simplified system models (e.g., two-user systems, non-fading channels, BER expressions only for the first few stages, and so on). In [2],[3], exact analytical BER expressions for the first and second stage of a PIC, which obtains initial data estimates from conventional MFs, on non-fading channels are presented. It has been pointed out that generalizing the analysis for any arbitrary stage in the PIC is extremely complex, which would involve volume integrals over a K-dimensional jointly Gaussian density function, where K is the number of users in the system. In [1] (Ch. 7.3.2), Verdu derived an exact BER expression for a PIC, which uses initial data estimates from a decorrelating detector, on non-fading channels in a two-user system.

In this paper, we present the BER performance analysis of a PIC *on Rayleigh fading channels*. Specifically, we derive an exact BER expression for a PIC, which uses decorrelator based initial data estimates, on flat Rayleigh fading in a two-user system. The BER expression is obtained in terms of the elliptic integral of the third kind.

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The rest of the paper is organized as follows. In Section II, we present the system model. In Section III, we present exact BER analysis of a PIC with decorrelator as the first stage on Rayleigh fading channels. Section IV provides the numerical results and Section V gives the conclusions.

II. System Model

Consider a synchronous multiuser system with K users. Let $\mathbf{y} = (y_1, y_2, \cdots, y_K)$ denote the received signal vector at the output of the MFs at the receiver. The k^{th} user's MF output is given by

$$y_k = A_k h_k b_k + \sum_{j=1 j\neq k}^{K} \rho_{jk} A_j h_j b_j + z_j, \qquad (1)$$

where A_k and $b_k \in \{+1, -1\}$ denote the transmit amplitude and data bit, respectively, of the k^{th} user, h_k denotes the complex channel fade coefficient corresponding to the k^{th} user, ρ_{jk} is the correlation coefficient between the signature waveforms of the j^{th} and the k^{th} users, and z_i denotes the complex Gaussian noise for the k^{th} user with zero mean and variance $2\sigma^2$. The fade coefficients h_k's are assumed to be i.i.d complex Gaussian r.v's (i.e., fade amplitudes are Rayleigh distributed) with zero mean and $E[h_{kI}^2] = E[h_{kQ}^2] = 1$, where h_{kI} and h_{kQ} are the real and imaginary parts of h_k. For a two-user system, the MF outputs for users 1 and 2 are given by

$$y_1 = A_1 h_1 b_1 + \rho A_2 h_2 b_2 + z_1 \qquad (2)$$
$$y_2 = A_2 h_2 b_2 + \rho A_1 h_1 b_1 + z_2, \qquad (3)$$

where $\rho = \rho_{12} = \rho_{21}$.

We consider a PIC with decorrelator as the first stage. The initial bit estimates from this decorrelator is used by the second stage of the PIC for interference cancellation. We are interested in obtaining an expression for the BER at the output of the second stage of the PIC.

Let $\hat{b}_1^{(i)}$ and $\hat{b}_2^{(i)}$ denote the bit decisions made by the i^{th} stage for users 1 and 2, respectively. Assuming perfect estimates of the channel coefficients at the receiver, the bit decisions made by the first stage (i.e., the decorrelator stage), $\hat{b}_1^{(1)}$ and $\hat{b}_2^{(1)}$, are given by [1]

$$\hat{b}_1^{(1)} = sgn\Big(Re(h_2^* y_2 - \rho h_2^* y_1)\Big) \quad (4)$$

$$\hat{b}_2^{(1)} = sgn\Big(Re(h_1^* y_1 - \rho h_1^* y_2)\Big), \quad (5)$$

where $(\cdot)^*$ denotes the complex conjugate operation. The probability of bit error for user k at the output of the decorrelator stage, $P_k^{(1)}$, $k = 1, 2$, can be obtained as [1]

$$P_k^{(1)} = \frac{1}{2}\left(1 - \frac{A_k\sqrt{1-\rho^2}}{\sqrt{\sigma^2 + A_k^2(1-\rho^2)}}\right), \qquad k = 1, 2. \quad (6)$$

The bit estimates $\hat{b}_1^{(1)}$ and $\hat{b}_2^{(1)}$ are given as inputs to the second stage of the PIC. After reconstruction and cancellation of the interference at the second stage, the bit decisions at the output of the second stage for the users 1 and 2, $\hat{b}_1^{(2)}$ and $\hat{b}_2^{(2)}$, are given by

$$\hat{b}_1^{(2)} = sgn\Big(Re\left(h_1^* y_1 - h_1^* h_2 A_2\, \rho\, \hat{b}_2^{(1)}\right)\Big) \quad (7)$$

$$\hat{b}_2^{(2)} = sgn\Big(Re\left(h_2^* y_2 - h_2^* h_1 A_1\, \rho\, \hat{b}_1^{(1)}\right)\Big). \quad (8)$$

III. BER Analysis

We derive the probability of bit error for the k^{th} user at the output of the second stage, $P_k^{(2)}$, as follows. Let user 1 be the desired user. The probability of bit error for the desired user, $P_1^{(2)}$, can be written as

$$P_1^{(2)} = \Pr\left(\hat{b}_1^{(2)} \neq b_1 \middle| \hat{b}_2^{(1)} = b_2\right)\Pr\left(\hat{b}_2^{(1)} = b_2\right) + \Pr\left(\hat{b}_1^{(2)} \neq b_1 \middle| \hat{b}_2^{(1)} \neq b_2\right)\Pr\left(\hat{b}_2^{(1)} \neq b_2\right). (9)$$

The first term in the above Eqn. (9) is easy to obtain, whereas the derivation of the second term is rather involved. The first term in (9) can be written as

$$\Pr\left(\hat{b}_1^{(2)} \neq b_1 \middle| \hat{b}_2^{(1)} = b_2\right)\Pr\left(\hat{b}_2^{(1)} = b_2\right) = \left(1 - E\left[Q\left(\frac{A_2|h_2|^2(1-\rho^2)}{\sigma\sqrt{|h_2|^2(1-\rho^2)}}\right)\right]\right)\cdot E\left[Q\left(\frac{A_1|h_1|^2}{\sigma\sqrt{|h_1|^2}}\right)\right], (10)$$

which results because of the fact that the correct bit decision in the previous stage (i.e., $\hat{b}_2^{(1)} = b_2$) makes the current stage decision $\hat{b}_1^{(2)}$ independent of bit b_2 (i.e., independent of interference). The expectations of the Q-functions in (10) are w.r.t the channel fades, which can be obtained as [1]

$$E\left[Q\left(\frac{A_2|h_2|^2(1-\rho^2)}{\sigma\sqrt{|h_2|^2(1-\rho^2)}}\right)\right] = \frac{1}{2}\left(1 - \frac{A_2\sqrt{1-\rho^2}}{\sqrt{\sigma^2 + A_2^2(1-\rho^2)}}\right), (11)$$

and

$$E\left[Q\left(\frac{A_1|h_1|^2}{\sigma\sqrt{|h_1|^2}}\right)\right] = \frac{1}{2}\left(1 - \frac{A_1}{\sqrt{\sigma^2 + A_1^2}}\right). \quad (12)$$

Note that the expression in (11) corresponds to the BER expression for the simple decorrelating detector on flat Rayleigh fading, and the expression in (12) corresponds to the BER expression for a single-user scheme on flat Rayleigh fading.

As mentioned earlier, the derivation of the second term in (9) is rather involved, since an error in the bit estimation in the previous stage (i.e., $\hat{b}_2^{(1)} \neq b_2$) makes the current decision of the first user's bit $\hat{b}_1^{(2)}$ depend on the second user's bit estimate $\hat{b}_2^{(1)}$. Consequently, the evaluation of the second term in (9) involves evaluating the expectation of a product of two Q-functions, as

$$\Pr\left(\hat{b}_1^{(2)} \neq b_1 \middle| \hat{b}_2^{(1)} \neq b_2\right)\Pr\left(\hat{b}_2^{(1)} \neq b_2\right) = E\left[Q\left(\frac{A_2|h_2|^2(1-\rho^2)}{\sigma\sqrt{|h_2|^2(1-\rho^2)}}\right)Q\left(\frac{A_1|h_1|^2 + 2A_2Re(h_1^* h_2)\rho}{\sigma\sqrt{|h_1|^2}}\right)\right] (13)$$

where the expectation is over h_1 and h_2. Averaging over h_1, (13) can be simplified to

$$\Pr\left(\hat{b}_1^{(2)} \neq b_1 \middle| \hat{b}_2^{(1)} \neq b_2\right)\Pr\left(\hat{b}_2^{(1)} \neq b_2\right) = E\left[Q\left(\frac{A_2|h_2|^2\sqrt{1-\rho^2}}{\sigma\sqrt{|h_2|^2}}\right)Q\left(\frac{2h_{2I}\rho A_2}{\sigma}\right)\right] - \frac{A_1}{\sqrt{A_1^2+\sigma^2}} \cdot E\left[Q\left(\frac{A_2|h_2|^2\sqrt{1-\rho^2}}{\sigma\sqrt{|h_2|^2}}\right)Q\left(\frac{2h_{2I}\rho A_2 A_1}{\sigma\sqrt{A_1^2+\sigma^2}}\right)e^{\frac{-4h_{2I}^2\rho^2A_2^2}{2(A_1^2+\sigma^2)}}\right], (14)$$

where h_{2I} denotes the real part of h_2. To average over h_2, we convert h_2 into polar form, i.e., $h_2 = re^{j\theta}$, where r is Rayleigh distributed and θ is uniformly distributed in $[0, 2\pi]$. The first expectation on the RHS of (14) can be obtained as

$$E\left[Q\left(\frac{A_2|h_2|^2\sqrt{1-\rho^2}}{\sigma\sqrt{|h_2|^2}}\right)Q\left(\frac{2h_{2I}\rho A_2}{\sigma}\right)\right] = \frac{1}{4}\left(1 - A_2\frac{\sqrt{1-\rho^2}}{\sqrt{\sigma^2 + A_2^2(1-\rho^2)}}\right). \quad (15)$$

The second expectation on the RHS of (14) is obtained as follows. Substituting $h_{2I} = r\cos\theta$, we can write

$$E\left[Q\left(\frac{A_2|h_2|^2\sqrt{1-\rho^2}}{\sigma\sqrt{|h_2|^2}}\right)Q\left(\frac{2A_2A_1h_{2I}\rho}{\sigma\sqrt{A_1^2+\sigma^2}}\right)e^{\frac{-4h_{2I}^2\rho^2A_2^2}{2(A_1^2+\sigma^2)}}\right] = \frac{1}{2\pi}\int_0^\infty e^{-r^2/2}rQ\left(\frac{A_2r\sqrt{1-\rho^2}}{\sigma}\right)dr \cdot\int_0^{2\pi}Q\left(\frac{2A_2A_1r\cos\theta\rho}{\sigma\sqrt{A_1^2+\sigma^2}}\right)e^{\frac{-4r^2\cos^2\theta\rho^2A_2^2}{2(A_1^2+\sigma^2)}}d\theta. \quad (16)$$

Substituting $\cos^2\theta = p/2$, we can write

$$\int_0^{2\pi}Q\left(\frac{2A_2A_1r\cos\theta\rho}{\sigma\sqrt{A_1^2+\sigma^2}}\right)e^{\frac{-4r^2A_2^2\cos^2\theta\rho^2}{2(A_1^2+\sigma^2)}}d\theta = \int_0^2\frac{e^{-r^2A_2^2\rho^2p/(1+\sigma^2)}}{\sqrt{p(2-p)}}dp, \quad (17)$$

From [4], (Eqn. GW(312)(7a), pp. 341), we get

$$\int_0^2\frac{e^{-r^2A_2^2\rho^2p/(1+\sigma^2)}}{\sqrt{p(2-p)}}dp = \pi e^{-\frac{A_2^2r^2\rho^2}{1+\sigma^2}}I_o\left(\frac{A_2^2r^2\rho^2}{1+\sigma^2}\right), \quad (18)$$

where $I_o(x) = \frac{1}{\pi}\int_0^\pi e^{x\cos\theta}d\theta$. Using (18) in (16), we can write

$$\begin{aligned} E&\left[Q\left(\frac{A_2|h_2|^2\sqrt{1-\rho^2}}{\sigma\sqrt{|h_2|^2}}\right)Q\left(\frac{2A_2A_1h_{2I}\rho}{\sigma\sqrt{A_1^2+\sigma^2}}\right)e^{\frac{-4h_{2I}^2\rho^2A_2^2}{2(A_1^2+\sigma^2)}}\right] \\ &= \frac{1}{2\pi}\int_0^\infty\int_0^\pi re^{-\frac{r^2}{2}\left[\frac{2A_2^2\rho^2}{A_1^2+\sigma^2}+1-\frac{2A_2^2r^2\rho^2\cos\theta}{A_1^2+\sigma^2}\right]} \cdot Q\left(\frac{A_2r\sqrt{1-\rho^2}}{\sigma}\right)drd\theta \\ &= \frac{1}{4\pi}\int_0^\pi\frac{1}{a(\theta)}\left(1-\sqrt{\frac{a(\theta)\sigma^2}{A_2^2(1-\rho^2)}+1}\right)d\theta, \end{aligned} \quad (19)$$

where the second step in (19) is due to [1] (Eqn. 3.61), and

$$a(\theta) = \frac{2A_2^2\rho^2(1-\cos\theta)}{A_1^2+\sigma^2}+1. \quad (20)$$

Also,

$$\int_0^\pi\frac{1}{a(\theta)}d\theta = \frac{\pi}{\sqrt{1+\frac{4A_2^2\rho^2}{A_1^2+\sigma^2}}}. \quad (21)$$

Defining $b = \frac{2A_2^2\rho^2}{A_1^2+\sigma^2}$ and $c = \frac{A_2^2(1-\rho^2)}{\sigma^2}$, and by substituting $\cos\theta = t$, we get

$$\int_0^\pi\frac{d\theta}{a(\theta)\sqrt{\frac{a(\theta)\sigma^2}{A_2^2(1-\rho^2)}+1}} = \frac{\sqrt{c}}{b\sqrt{b}}\int_{-1}^1\frac{dt}{(1-t+\frac{1}{b})\sqrt{(1-t^2)(1-t+\frac{1+c}{b})}}. \quad (22)$$

Using ([4], Eqn. BY(233.02), pp. 258), we can write

$$\int_{-1}^1\frac{dt}{(1-t+\frac{1}{b})\sqrt{(1-t^2)(1-t+\frac{1+c}{b})}} = \frac{2}{(2+\frac{1}{b})\sqrt{2+\frac{1+c}{b}}}\Pi\left(\frac{\pi}{2},\frac{2}{2+\frac{1}{b}},\sqrt{\frac{2}{2+\frac{1+c}{b}}}\right), \quad (23)$$

where $\Pi(\theta, u, v)$ is the elliptic integral of the third kind given by

$$\Pi(\theta, u, v) = \int_0^\theta\frac{d\alpha}{(1-u\sin^2\alpha)\sqrt{1-v^2\sin^2\alpha}}, \quad (24)$$

which can be directly computed using the 'EllipticPi' function in Mathematica.

Using (23), (22), (21), (19), (15), (14), (12), (11), (10), and (9), the expression for the BER at the second stage output, $P_1^{(2)}$, can be obtained as

$$\begin{aligned} P_1^{(2)} = &\frac{1}{2}\left(1-\frac{A_1}{\sqrt{\sigma^2+A_1^2}}\right) \\ &-\frac{1}{4}\left(1-\frac{A_1}{\sqrt{\sigma^2+A_1^2}}\right)\left(1-\frac{A_2\sqrt{1-\rho^2}}{\sqrt{\sigma^2+A_2^2(1-\rho^2)}}\right) \\ &+\frac{1}{4}\left(1-\frac{A_2\sqrt{1-\rho^2}}{\sqrt{\sigma^2+A_2^2(1-\rho^2)}}\right) \\ &-\frac{A_1}{4\pi\sqrt{\sigma^2+A_1^2}}\frac{\pi}{\sqrt{1+\frac{4\rho^2A_2^2}{A_1^2+\sigma^2}}} \\ &+\frac{A_1}{4\pi\sqrt{\sigma^2+A_1^2}}\frac{2\sqrt{c}}{b\sqrt{b}(2+\frac{1}{b})\sqrt{2+\frac{1+c}{b}}} \\ &\cdot\Pi\left(\frac{\pi}{2},\frac{2}{2+\frac{1}{b}},\sqrt{\frac{2}{2+\frac{1+c}{b}}}\right), \end{aligned} \quad (25)$$

which is one of the rare closed-form expressions one can get for the BER of a nonlinear multiuser detector. It is noted that, because of the symmetry in the PIC receiver structure, the BER for user 2 is the same as (25) with A_1 and A_2 interchanged.

IV. Numerical Results

Fig. 1 shows the BER performance of the PIC with decorrelator as the first stage for user 1 as a function of the average

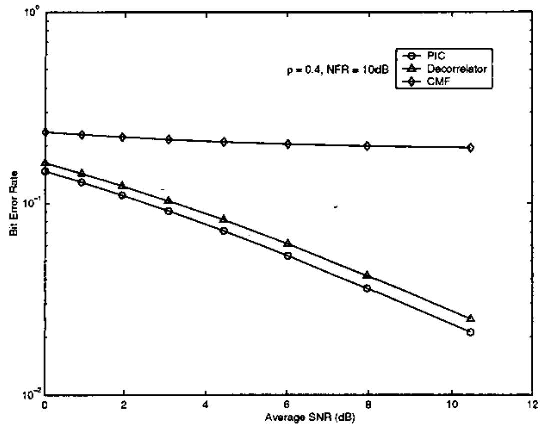

Fig. 1. BER performance of PIC with decorrelator as the first stage on Rayleigh fading channels. $\rho = 0.4$, NFR=10 dB. Performance of decorrelating detector as well as conventional MF detector are also shown.

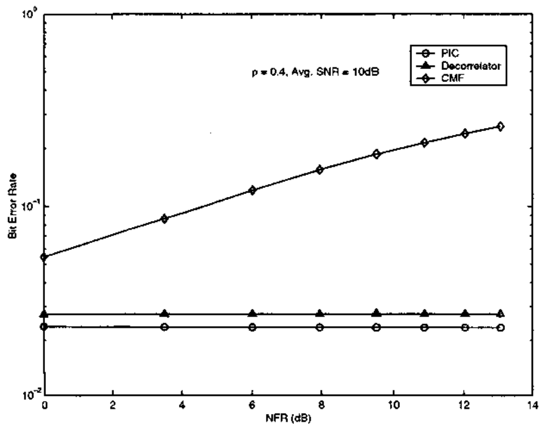

Fig. 2. BER performance of PIC with decorrelator as the first stage as a function of NFR on Rayleigh fading channels. $\rho = 0.4$, Avg. SNR = 10 dB. Performance of decorrelating detector as well as conventional MF detector are also shown.

SNR, $\overline{\gamma} = A_1^2 E[|h_1|^2]/2\sigma^2 = A_1^2/\sigma^2$, evaluated from (25), for $\rho = 0.4$ and near-far ratio, NFR = 10 dB (we define the near-far ratio as NFR = $20\log(A_2/A_1)$). The performance of the conventional MF (CMF) detector as well as the decorrelating detector are also shown for comparison. The BER plots for the decorrelating and MF detectors are from the corresponding BER expressions in [1]. As expected, the PIC performs better than the linear decorrelating detector and the MF detector. Fig. 2 shows the user 1 BER performance as a function of NFR, for $\rho = 0.4$ and average SNR, $\overline{\gamma} = 10$ dB. We observe that both the PIC (with decorrelator as the first stage) as well as the decorrelating detector are near-far resistant, with the PIC performing better than the decorrelating detector as expected. The conventional MF detector however is not near-far resistant (i.e., performance degrades as the NFR is increased).

V. Conclusion

We derived an exact BER expression, in closed-form, for parallel interference cancellation (PIC) with decorrelator based initial data estimates on Rayleigh fading channels. The BER expression was obtained in terms of the elliptic integral of the third kind. It is noted that this BER expression is one of the rare closed-form expressions one can obtain for the BER of a nonlinear multiuser detector.

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