This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE ICC 2010 proceedings

Damped Belief Propagation Based Near-Optimal Equalization of Severely Delay-Spread UWB MIMO-ISI Channels

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***Abstract*— In this paper, we present a *belief propagation* (BP) based equalizer for ultrawideband (UWB) multiple-input multiple-output (MIMO) inter-symbol interference (ISI) channels characterized by *severe delay spreads*. We employ a *Markov random field* (MRF) graphical model of the system on which we carry out message passing. The proposed BP equalizer is shown to perform increasingly closer to optimal performance for increasing number of multipath components (MPC) at a much lesser complexity than that of the optimum equalizer. The proposed equalizer performs close to within 0.25 dB of SISO AWGN performance at 10^{-3} bit error rate on a severely delay-spread MIMO-ISI channel with 20 equal-energy MPCs. We point out that, although MIMO/UWB systems are characterized by fully/densely connected graphical models, the following two proposed features are instrumental in achieving near-optimal performance for large number of MPCs at low complexities: i) use of *pairwise compatibility functions* in densely connected MRFs, and ii) use of *damping* of messages.**

Keywords – *Ultrawideband MIMO-ISI channels, severe delay spreads, equalization, near-optimal performance, belief propagation, damping.*

I. INTRODUCTION

Wireless communication systems using ultrawideband (UWB) techniques typically use high transmission bandwidths to accommodate very high data rates [1]. Such UWB channels are highly frequency-selective, and are characterized by severe inter-symbol interference (ISI) due to large delay spreads [2]-[6]. The number of multipath components (MPC) in such channels in indoor/industrial environments has been observed to be of the order of several tens to hundreds; number of MPCs ranging from 12 to 120 are common in UWB channel models [2],[5]. Such large number of MPCs is often viewed as a source of severe ISI that hurts performance. On the other hand, these MPCs, if carefully exploited, can provide the opportunity to achieve increased time-diversity benefits [2]. But the complexity of receivers (e.g., RAKE [7]) that combine signal energies in all the MPCs can get prohibitive in complexity for large number of MPCs. Consequently, achieving near-optimum receiver/equalizer performance for channels with large number of MPCs at low complexities has been a challenging issue. We address this important issue in this paper. Particularly, we propose a novel equalizer algorithm, based on *belief propagation*, that achieves near-optimum performance for large number of MPCs.

Belief propagation (BP) is a technique that solves inference problems using graphical models [8]. BP is a simple, yet highly effective, technique that has been successfully employed in a variety of applications including computational biology, statistical signal/image processing, etc. BP is well suited in several communication problems as well [9]; e.g., decoding of turbo codes and LDPC codes [10],[11], multiuser detection [12],[13], signal detection in ISI channels [14],[15], and MIMO detection [16],[17]. Recently, in [18], Wo and Hoeher has adopted BP for equalization in severely delay-spread MIMO-ISI channels, using message passing on factor graphs. However, in terms of performance, the algorithm exhibited high error floors. In [19], an equalizer for severely delay-spread MIMO-ISI channels, based on a reactive tabu search (RTS) algorithm, which is a local neighborhood search algorithm that minimized the maximum-likelihood (ML) criterion, has been proposed. The RTS algorithm was shown to achieve near-ML performance for large number of MPCs at low complexities.

Our aim in this paper is to adopt BP for achieving near-optimal equalization in severely delay-spread MIMO-ISI channels. To do that, we employ a Markov random field (MRF) graphical model of the MIMO/UWB system on which we perform message passing. Although MIMO/UWB systems are characterized by fully/densely connected MRFs, we adopt two key features which become instrumental in achieving near-optimal performance for large number of MPCs at low complexities: they include 1) use of pairwise compatibility function contributed towards reduced complexity, and 2) damping of messages (where messages are computed as a weighted average of the messages in the previous iteration and the current iteration) contributed to achieving improved performance. Simulation results show that the proposed message damped BP equalizer achieves close to single-input single-output (SISO) AWGN performance within 0.25 dB at 10^{-3} bit error rate (BER) in a MIMO-ISI channel with 20 equal-energy MPCs.

The rest of the paper is organized as follows. In Section II, we present the considered system model. The proposed BP based equalization algorithm is presented in Section III. Simulated BER results and discussions are presented in Section IV. Section V presents the conclusions.

II. SYSTEM MODEL

Consider a frequency-selective MIMO channel with N_t transmit and N_r receive antennas as shown in Fig. 1. Let L denote the number of MPCs. Data is transmitted in frames, where each frame has K data symbols preceded by a cyclic prefix (CP) of length L symbols, $K \geq L$. While CP avoids inter-frame interference, there will be ISI within the frame. Let $\mathbf{x}_q \in \{\pm 1\}^{N_t}$ be the transmitted symbol vector at time q, $0 \leq q \leq K-1$, The received signal vector at time q can be written as

$$\mathbf{y}_q \;=\; \sum_{l=0}^{L-1} \mathbf{H}_l \, \mathbf{x}_{q-l} + \mathbf{w}_q, \qquad q = 0, \cdots, K-1, \quad (1)$$

where $\mathbf{y}_q \in \mathbb{C}^{N_r}$, $\mathbf{H}_l \in \mathbb{C}^{N_r \times N_t}$ is the channel gain matrix for the lth MPC such that $H_{j,i}^{(l)}$ denotes the entry on the

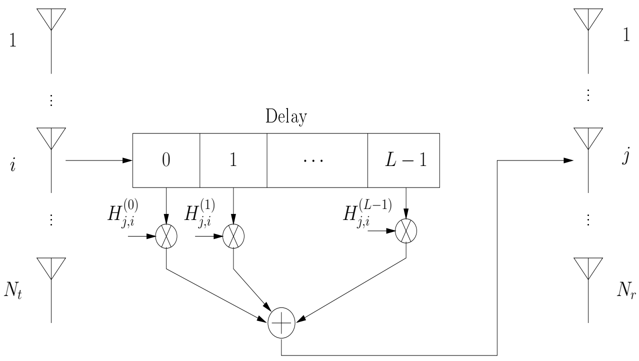

Fig. 1. MIMO-ISI Channel Model.

jth row and ith column of the $\mathbf{H}_l$ matrix, i.e., $H_{j,i}^{(l)}$ is the channel from ith transmit antenna to the jth receive antenna on the lth MPC. The entries of $\mathbf{H}_l$ are assumed to be random with distribution $\mathcal{CN}(0,1)$. It is further assumed that $\mathbf{H}_l$, $l = 0, \cdots, L-1$ do not change for one frame duration. $\mathbf{w}_q \in \mathbb{C}^{N_r}$ is the additive white Gaussian noise vector at time q, whose entries are independent, each with variance σ^2. The CP will render the linearly convolving channel to a circularly convolving one, and so the channel will be multiplicative in frequency domain. Because of the CP, the received signal in frequency domain, for the ith frequency index ($0 \leq i \leq K-1$), can be written as

$$\mathbf{r}_i \;=\; \mathbf{G}_i\, \mathbf{u}_i + \mathbf{v}_i, \tag{2}$$

where $\mathbf{r}_i = \frac{1}{\sqrt{K}} \sum_{q=0}^{K-1} e^{\frac{-2\pi \mathbf{j} q i}{K}} \mathbf{y}_q$, $\mathbf{u}_i = \frac{1}{\sqrt{K}} \sum_{q=0}^{K-1} e^{\frac{-2\pi \mathbf{j} q i}{K}} \mathbf{x}_q$, $\mathbf{v}_i = \frac{1}{\sqrt{K}} \sum_{q=0}^{K-1} e^{\frac{-2\pi \mathbf{j} q i}{K}} \mathbf{w}_q$, $\mathbf{G}_i = \sum_{l=0}^{L-1} e^{\frac{-2\pi \mathbf{j} l i}{K}} \mathbf{H}_l$, and $\mathbf{j} = \sqrt{-1}$. Stacking the K vectors $\mathbf{r}_i$, $i = 0, \cdots, K-1$, we write

$$\mathbf{r} \;=\; \underbrace{\mathbf{GF}}_{\triangleq\, \mathbf{H}_{eff}}\, \mathbf{x}_{eff} + \mathbf{v}_{eff}, \tag{3}$$

where

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_0 \\ \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_{K-1} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{G}_0 & & & \mathbf{0} \\ & \mathbf{G}_1 & & \\ & & \ddots & \\ \mathbf{0} & & & \mathbf{G}_{K-1} \end{bmatrix},$$

$$\mathbf{x}_{eff} = \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{K-1} \end{bmatrix}, \quad \mathbf{v}_{eff} = \begin{bmatrix} \mathbf{v}_0 \\ \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_{K-1} \end{bmatrix},$$

$$\begin{aligned} \mathbf{F} &= \frac{1}{\sqrt{K}} \begin{bmatrix} \rho_{0,0}\mathbf{I}_{N_t} & \rho_{1,0}\mathbf{I}_{N_t} & \cdots & \rho_{K-1,0}\mathbf{I}_{N_t} \\ \rho_{0,1}\mathbf{I}_{N_t} & \rho_{1,2}\mathbf{I}_{N_t} & \cdots & \rho_{K-1,1}\mathbf{I}_{N_t} \\ \vdots & \vdots & \cdots & \vdots \\ \rho_{0,K-1}\mathbf{I}_{N_t} & \rho_{1,K-1}\mathbf{I}_{N_t} & \cdots & \rho_{K-1,K-1}\mathbf{I}_{N_t} \end{bmatrix} \\ &= \frac{1}{\sqrt{K}} \mathbf{D}_K \otimes \mathbf{I}_{N_t}, \end{aligned}$$

where $\rho_{q,i} = e^{\frac{-2\pi \mathbf{j} q i}{K}}$, $\mathbf{D}_K$ is the K-point DFT matrix and $\otimes$ denotes the Kronecker product. Equation (3) can be written in an equivalent linear vector channel model of the form

$$\mathbf{r} \;=\; \mathbf{Hx} + \mathbf{v}, \tag{4}$$

where $\mathbf{H} = \mathbf{H}_{eff}$, $\mathbf{x} = \mathbf{x}_{eff}$, and $\mathbf{v} = \mathbf{v}_{eff}$.

The goal is to obtain an estimate of vector $\mathbf{x}$, given $\mathbf{r}$ and the knowledge of $\mathbf{H}$. The optimal maximum aposteriori probability (MAP) detector enumerates the joint posterior distribution

$$p(\mathbf{x}|\mathbf{r}, \mathbf{H}) \;\propto\; p(\mathbf{r}|\mathbf{x}, \mathbf{H})\, p(\mathbf{x}), \tag{5}$$

and marginalizes out each variable as $p(x_i|\mathbf{r}, \mathbf{H}) = \sum_{x_{-i}} p(\mathbf{x}|\mathbf{r}, \mathbf{H})$, where x_{-i} stands for all entries of $\mathbf{x}$ except x_i. The MAP estimate of the bit x_i, $i = 1, \cdots, KN_t$, is then given by

$$\widehat{x}_i \;=\; \underset{a \in \{\pm 1\}}{\arg\max}\; p\big(x_i = a \,|\, \mathbf{r}, \mathbf{H}\big), \tag{6}$$

whose complexity is exponential in KN_t. In the following section, we present the proposed low-complexity BP equalization algorithm for the linear vector channel model in (4).

III. Proposed Equalization Using BP on MRFs

Among the well known graphical models including Bayesian belief networks, factor graphs and MRFs, we use pairwise MRF representation [8] of the linear vector channel in (4).

Consider a pairwise MRF [8] in which the x_i's denote underlying *hidden* variables on which the observed variables y_i's are dependent. Let the dependence between the hidden variable x_i and the explicit variable y_i be represented by a *joint* compatibility function $\phi_i(x_i, y_i)$ (also called as 'evidence' of x_i). The dependence between x_i and x_j is represented by the function $\psi_{i,j}(x_i, x_j)$, where x_i, x_j are variables connected by an edge in the MRF. The joint distribution of the hidden and explicit variables is

$$\mathrm{p}(\mathbf{x}, \mathbf{y}) \;\propto\; \prod_{i,j} \psi_{i,j}(x_i, x_j) \prod_i \phi_i(x_i, y_i). \tag{7}$$

Since y_i's are fixed, we can drop them in the above equation and write

$$p(\mathbf{x}) \;\propto\; \prod_{i,j} \psi_{i,j}(x_i, x_j) \prod_i \phi_i(x_i). \tag{8}$$

We need expressions for the clique potentials, $\psi_{i,j}(x_i, x_j)$, and the joint compatibility functions, $\phi_i(x_i)$, for the system of interest, namely the linear vector channel in (4). For an MRF with V nodes, E edges, and ± 1 variables with pairwise interactions, the joint distribution of the variables can be written in the form [21]

$$p(\mathbf{x}) \;\propto\; \exp\left\{ \left(\sum_{\{i,j\} \in E} J_{i,j} x_i x_j \;+\; \sum_{i \in V} \theta_i x_i \right) \right\}, \tag{9}$$

for some choice of the parameters $J_{i,j} \in \mathbb{R}$ (referred to as 'couplings') and $\theta_i \in \mathbb{R}$ (referred to as the 'local fields'). The choice of the coupling and local field parameters is domain (i.e., system) specific. Comparing the expressions in (8) and (9), we can write the compatibility functions in terms of the coupling and local field parameters as

$$\psi_{i,j}(x_i, x_j) \;\propto\; \exp\left(J_{i,j} x_i, x_j\right), \tag{10}$$

$$\phi_i(x_i) \;\propto\; \exp\left(\theta_i x_i\right). \tag{11}$$

Consequently, we need to choose expressions for parameters $J_{i,j}$ and θ_i in order to obtain the compatibility functions.

For the linear vector channel in (4) with Gaussian noise $\mathbf{v} \sim \mathbb{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$, given $\mathbf{x}$ and $\mathbf{H}$, $\mathbf{r}$ is a complex Gaussian random vector with mean $\mathbf{Hx}$ and covariance $\sigma^2 \mathbf{I}$. Therefore,

$$\begin{aligned} p(\mathbf{r}|\mathbf{x}, \mathbf{H}) &\propto \exp\left\{\frac{-(\mathbf{r}-\mathbf{Hx})^H(\mathbf{r}-\mathbf{Hx})}{\sigma^2}\right\} \\ &= \exp\left(-\frac{1}{\sigma^2}\mathbf{r}^H\mathbf{r}\right)\exp\left(\frac{2}{\sigma^2}\Re\left\{\mathbf{x}^H\mathbf{H}^H\mathbf{r}\right\}\right) \\ &\quad \exp\left(-\frac{1}{\sigma^2}\mathbf{x}^H\mathbf{H}^H\mathbf{Hx}\right). \end{aligned} \tag{12}$$

Also, assuming the symbols in $\mathbf{x}$ are all independent,

$$p(\mathbf{x}) = \prod_i p(x_i). \tag{13}$$

Substituting (12) and (13) in (5),

$$\begin{aligned} p(\mathbf{x}|\mathbf{r}, \mathbf{H}) &\propto \exp\left(\frac{2}{\sigma^2}\Re\left\{\mathbf{x}^H\mathbf{H}^H\mathbf{r}\right\}\right) \\ &\quad \cdot \exp\left(-\frac{1}{\sigma^2}\mathbf{x}^H\mathbf{H}^H\mathbf{Hx}\right)\prod_i \mathrm{p}(x_i). \end{aligned} \tag{14}$$

Defining

$$\mathbf{R} \triangleq \frac{1}{\sigma^2}\mathbf{H}^H\mathbf{H}, \tag{15}$$

$$\mathbf{z} \triangleq \frac{1}{\sigma^2}\mathbf{H}^H\mathbf{r}, \tag{16}$$

we can write

$$\begin{aligned} p(\mathbf{x}|\mathbf{r}, \mathbf{H}) &\propto \exp\left(2\,\mathbf{x}^H\mathbf{z} - \mathbf{x}^H\mathbf{Rx}\right)\prod_i p(x_i) \\ &= \prod_i \exp\left(2x_i^H z_i + \ln\{p(x_i)\}\right)\prod_j \exp\left(x_i^H R_{i,j} x_j\right), \end{aligned} \tag{17}$$

where z_i and $R_{i,j}$ are the elements of $\mathbf{z}$ and $\mathbf{R}$, respectively. As in [16], comparing (17) and the joint probability distribution in a MRF given by (8), we adopt the definitions of the coupling and local field parameters as $J_{i,j} = -\Re\{R_{i,j}\}$ and $\theta_i = \Re\{z_i\}$, and the compatibility functions as

$$\phi_i(x_i) = \exp\left(x_i^H \Re\{z_i\} + \ln\{\mathrm{p}(x_i)\}\right) \tag{18}$$

$$\psi_{i,j}(x_i, x_j) = \exp\left(-x_i^H \Re\{R_{i,j}\} x_j\right). \tag{19}$$

A. Message Passing

The values of ϕ and ψ given by (18) and (19) define, respectively, the variable and edge potentials of an undirected graphical model to which message passing algorithms (such as BP) can be applied to compute the marginal probabilities of the variables. BP tries to estimate the marginal probabilities of all the variables by way of passing messages between the local nodes.

A *message* from node j to node i is denoted as $m_{j,i}(x_i)$, and belief at node i is denoted as $\mathrm{b}_i(x_i)$, $x_i \in \{\pm 1\}$. The $\mathrm{b}_i(x_i)$ is proportional to how likely x_i was transmitted. On the other hand, $m_{ji}(x_i)$ is proportional to how likely x_j thinks x_i was transmitted. The belief at node i is

$$\mathrm{b}_i(x_i) \propto \phi_i(x_i)\prod_{j\in\mathcal{N}(i)} m_{j,i}(x_i), \tag{20}$$

where $\mathcal{N}(i)$ denotes the neighboring nodes of node i, and the messages are defined as [8]

$$m_{j,i}(x_i) \propto \sum_{x_j}\phi_j(x_j)\psi_{j,i}(x_j, x_i)\prod_{k\in\mathcal{N}(j)\setminus i} m_{k,j}(x_j). \tag{21}$$

Equation (21) actually constitutes an iteration, as the message is defined in terms of the other messages. So, BP essentially involves computing the outgoing messages from a node to each of its neighbors using the local joint compatibility function and the incoming messages and transmitting them.

B. Improvement through Message Damping

In systems characterized by fully/highly connected graphical models, BP based algorithms may fail to converge, and if they do converge, the estimated marginals may be far from exact [21],[22]. It may be expected that BP might perform poorly in MIMO/UWB graphs due to the high density of connections. However, several methods are known in the literature, including *double loop methods* [23],[24] and *damping* [25],[26] which can be applied to improve things if BP does not converge (or converges too slowly).

In [25], Pretti proposed a modified version of BP with over-relaxed BP dynamics. At each step of the algorithm, the evaluation of messages is taken to be a weighted average between the old estimate and the new estimate. The weighted average could either be applied to the messages (resulting in *message damped BP*) or to the estimate of the probability distribution/beliefs of the variables (*probability/belief damped BP*), or to both messages and beliefs (*hybrid damped BP*). It is shown, in [25], that the probability damping BP can be derived as a limit case in which the double-loop algorithm becomes a single-loop one.

Message Damped BP: Denoting $\widetilde{m}_{i,j}^{(t)}(x_j)$ as the updated message in iteration t obtained by message passing, the new message from node i to node j in iteration t, denoted by $m_{i,j}^{(t)}(x_j)$, is computed as a convex combination of the old message and the updated message as

$$\widetilde{m}_{i,j}^{(t)}(x_j) \propto \sum_{x_i}\phi_i(x_i)\psi_{i,j}(x_i, x_j)\prod_{k\in\mathcal{N}(i)\setminus j} m_{k,i}^{(t-1)}(x_i), \tag{22}$$

$$m_{i,j}^{(t)}(x_j) = \alpha_m\, m_{i,j}^{(t-1)}(x_j) + (1-\alpha_m)\,\widetilde{m}_{i,j}^{(t)}(x_j), \tag{23}$$

where $\alpha_m \in [0, 1)$ is referred as the *message damping factor*.

Belief Damping: Instead of damping the messages in each iteration, the beliefs of the variables can be computed in each iteration as a weighted average, as

$$\widetilde{\mathrm{b}}_i^{(t)}(x_i) \propto \phi_i(x_i)\prod_{j\in\mathcal{N}(i)} m_{j,i}^{(t)}(x_i), \tag{24}$$

$$\mathrm{b}_i^{(t)}(x_i) = \alpha_b\,\mathrm{b}_i^{(t-1)}(x_i) + (1-\alpha_b)\,\widetilde{\mathrm{b}}_i^{(t)}(x_i), \tag{25}$$

where $\alpha_b \in [0, 1)$ is referred to as the *belief damping factor*.

Hybrid Damping: As a more general damping strategy, we can update both the messages as well as the beliefs according to (23) and (25), respectively, in each iteration. Different combinations of (α_m, α_b) values specializes to different strategies; for e.g., $(\alpha_m = \alpha_b = 0)$ corresponds to Undamped BP, $(\alpha_m \neq 0, \alpha_b = 0)$ corresponds to Message damped BP, $(\alpha_m = 0, \alpha_b \neq 0)$ corresponds to Belief damped BP, and $(\alpha_m \neq 0, \alpha_b \neq 0)$ corresponds to Hybrid damped BP. As we will see in the next section, *message damped BP* significantly improves the convergence and BER performance in the considered MIMO/UWB system.

C. Complexity

The per-symbol complexity of calculating messages and beliefs in a single BP iteration is $O(K^2N_t^2)$ and $O(KN_t)$, respectively. Likewise, the per-symbol complexity of computing ϕ and ψ is $O(1)$ and $O(KN_t)$, respectively. The computation of $\mathbf{z}$ can be carried out with $O(KN_r)$ per-symbol complexity. The computation of $\mathbf{R}$ involves computation of $\mathbf{H}^H\mathbf{H}$, which involves two operations: 1) calculation of $\mathbf{G}^H\mathbf{G}$, and 2) multiplication of $\mathbf{F}^H$ and $\mathbf{F}$ with $\mathbf{G}^H\mathbf{G}$. The operation 1) involves computation of ${\mathbf{G}_i}^H\mathbf{G}_i$ for $i = 0, \cdots, K-1$. The computation of each ${\mathbf{G}_i}^H\mathbf{G}_i$ has complexity $O(N_t^3)$. Therefore, the complexity of K such computation can be done in $O(KN_t^3)$ complexity. Now, the total number of symbols transmitted is KN_t. So, the per-symbol complexity is $O(N_t^2)$. Likewise, the per-symbol complexity corresponding to operation 2) is $O(K^2N_t^2)$. Since the number of BP iterations is much less than KN_t, the overall per-symbol complexity is of the algorithm is given by $O(K^2N_t^2)$.

IV. Simulation Results and Discussions

We evaluated the BER performance of the proposed BP equalizer in a 4×4 MIMO V-BLAST system with BPSK, as a function of average received SNR per receive antenna, γ, through simulations. We have assumed uniform power delay profile (i.e., all the L paths are assumed to be of equal energy). We evaluated the performance for various number of delay paths, L, and frame sizes, K, keeping L/K constant. It is noted that the system becomes a 'large-dimension system' when L and K are increased, keeping L/K constant. As will be seen in Fig. 4, the proposed BP equalizer performs increasingly closer to the optimum performance for increasing number of dimensions (i.e., increasing L and K). In all all the simulations, we have taken the number of MPCs, L, and the CP length to be the same.

Effect of Damping: In Fig. 2, we explore the effect of message damping and belief damping on the BER performance of the BP equalizer. Figure 2 shows the variation of the achieved BER as a function of the damping factor (α_m in the case of message damping, and α_b in the case of belief damping) varied in the range 0 to 1 for $N_t = N_r = 4$, BPSK, $[L = 10, K = 50]$, at an average received SNR of 6 dB. The number of BP iterations used is 10. From Fig. 2 it is can be seen that, message damping, depending on the choice of the value of α_m, can significantly improve the BER performance of the BP algorithm. On the other hand, in case of belief damping, the BER performance fluctuates with varying α_b and is worse than the optimum BER attained using message damping. This indicates that message damping is more effective than belief damping in the considered MIMO-ISI system. Hence, in all other simulations, we consider only message damping, i.e., $\alpha_m \neq 0$, $\alpha_b = 0$.

For the chosen set of system parameters in Fig. 2, the optimum value of α_m is observed to be about 0.45, which gives about an order of BER improvement. This point of the benefit of message damping in terms of BER performance (and also in terms of convergence) is even more clearly brought out in Fig. 3, where we have compared the BER performance without damping ($\alpha_m = 0$) and with damping ($\alpha_m = 0.45$) for $[L = 20, K = 100]$ at an SNR of 7 dB as a function of the number of BP iterations. It is interesting to see that without damping (i.e., with $\alpha_m = 0$), the algorithm indeed shows 'divergence' behavior, i.e., BER increases as number of iterations increases beyond 4. Such divergence behavior is effectively removed by message damping, as can be seen from the BER performance achieved with $\alpha_m = 0.45$. Indeed the algorithm with damping ($\alpha_m = 0.45$) is seen to be converge smoothly. It is also interesting to note that the algorithm converges to a BER which is quite close to the SISO AWGN BER (BER on SISO AWGN at 7 dB SNR is about 7.8×10^{-2} and the converged BER using damped BP is about 1×10^{-3}). This shows the potential of message damping in improving BER performance and convergence of the algorithm when employed for equalization in the considered MIMO system on highly frequency selective channels. It is also noted that damping (as per (23)) does not increase the order of complexity of the algorithm without damping; the order of complexity without and with damping remains the same.

Large-Dimension Behavior of the Algorithm: Another interesting aspect of the proposed BP algorithm is its 'large-dimension behavior', which is illustrated in Fig. 4, where we have compared the BER performance of the algorithm as a function of SNR for three different combination values of L and K, keeping L/K constant. We have shown the BER plots for the following combination values of L and K: $[L = 5, K = 25]$, $[L = 10, K = 50]$, and $[L = 20, K = 100]$. The number of BP iterations is 10 and the value of α_m used is 0.45. We have also plotted the SISO AWGN performance, which can be viewed as a lower bound on the optimum equalizer performance. It can be seen that the performance of the proposed BP equalizer improves as L and K are increased keeping L/K same; a behavior we refer to as large-dimension behavior. The performance gets increasingly closer to SISO AWGN performance for increasing values of L and K; for e.g., the gap between the BP equalizer performance and the SISO AWGN performance is only about 0.25 dB for 20 delay paths at BER of 10^{-3}. This shows the ability of the proposed algorithm to achieve near-optimal performance for severely delay spread MIMO-ISI channels (i.e., large L).

V. Conclusions

Our new contribution in this paper is the successful adoption of BP for equalization on MRF graphical models of MIMO-UWB systems characterized by severe delay spreads. Two key proposed features contributed to achieving low complexity as well as near-optimal performance for large delay spreads: they include i) use of pairwise compatibility function contributed towards reduced complexity, and ii) message damping contributed to achieving improved performance. Simulation results showed that the proposed BP equalizer achieved very close to (within 0.25 dB at 10^{-3} BER) SISO AWGN performance for large number of multipath components.

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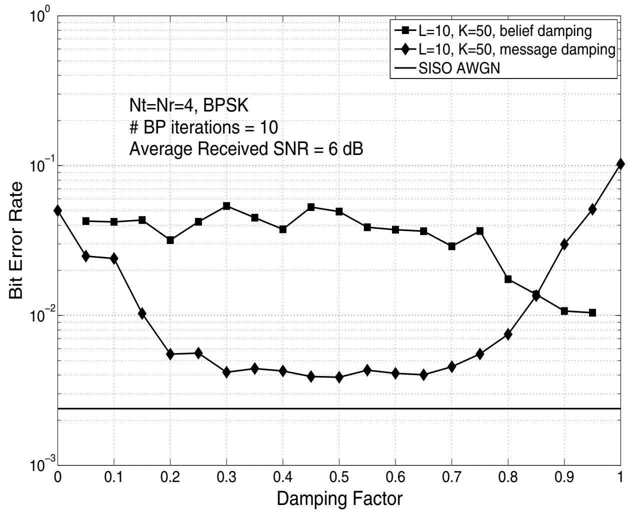

Fig. 2. Effect of message damping and belief damping on the BER performance of the BP equalizer in a MIMO V-BLAST system. $N_t = N_r = 4$, BPSK, $[L = 10, K = 50]$, average rx SNR $\gamma = 6$ dB, # BP iterations = 10.

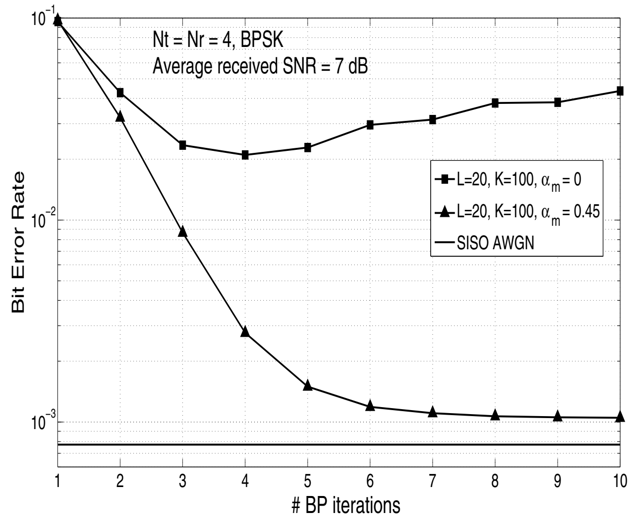

Fig. 3. Comparison of the BER performance of message damped and undamped BP equalizer as a function of number of BP iterations in a MIMO V-BLAST system. $N_t = N_r = 4$, $[L = 20, K = 100]$, BPSK, average received SNR $\gamma = 7$ dB, message damping factor, α_m, for damped BP equalizer = 0.45.

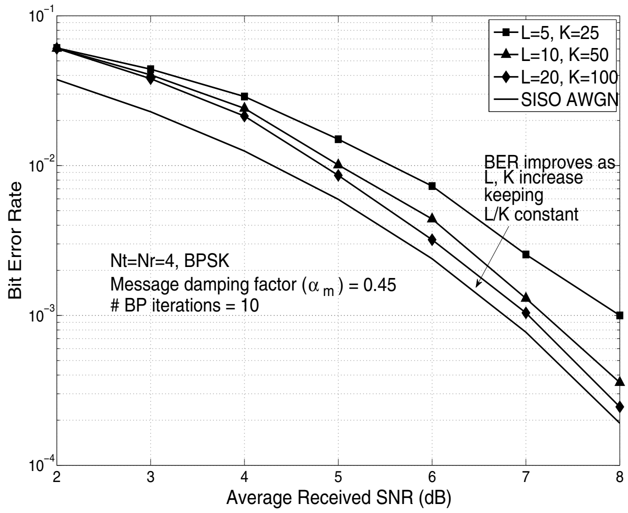

Fig. 4. BER performance of message damped BP equalizer as a function of average SNR in MIMO V-BLAST with $N_t = N_r = 4$ for different values of L and K keeping L/K constant: $[L = 5, K = 25]$, $[L = 10, K = 50]$, and $[L = 20, K = 100]$. BPSK, # BP iterations = 10, message damping factor $\alpha_m = 0.45$.

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