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BER Analysis of Uplink OFDMA in the Presence of Carrier Frequency and Timing Offsets on Rician Fading Channels

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***Abstract*— In uplink orthogonal frequency division multiple access (OFDMA), carrier frequency offsets (CFO) and/or timing offsets (TO) of other users with respect to a desired user can cause significant multiuser interference (MUI). In this paper, we derive an *analytical bit error rate (BER) expression* that quantify the degradation in BER due to the *combined effect of both CFOs and TOs* in uplink OFDMA *on Rician fading channels*. Such an analytical BER derivation for uplink OFDMA with CFOs and TOs on Rician fading channels has not been reported so far. For the case of non-zero CFOs/TOs, we obtain an approximate BER expression involving a single integral. Analytical and simulation BER results are shown to match very well.**

Keywords – *Uplink OFDMA, carrier frequency offset, timing offset, multiuser interference, Rician fading, BER analysis.*

I. Introduction

Orthogonal frequency division multiple access (OFDMA) is attractive in wireless communications due its high spectral efficiency, robustness to multipath and less complexity [1]. The performance of OFDMA on the uplink depends to a large extent on how well the orthogonality among different subcarriers from different users is maintained at the receiver. In uplink OFDMA, factors including $i)$ timing offsets (TO) of different users caused due to path delay differences between different users and imperfect timing synchronization, and $ii)$ carrier frequency offsets (CFO) of different users induced by Doppler effects and/or poor oscillator alignments, can destroy the orthogonality among subcarriers at the receiver and cause multiuser interference (MUI). Several techniques have been proposed to alleviate the loss in performance due to CFOs and TOs; they include $i)$ tight closed-loop frequency/timing correction between mobile transmitters and the base station receiver [2], $ii)$ providing adequate guard interval and use of GPS timing, and $iii)$ interference cancellation techniques at the receiver [3]-[6].

Analytical characterization of the bit/symbol error performance of uplink OFDMA in the presence of large CFOs and TOs has not been addressed adequately in the literature. Most bit error rate (BER) evaluations in uplink OFDMA are based on simulations, e.g., [3]-[6]. In terms of analytical evaluation, an approximate analysis of the SNR degradation and BER of 'single user OFDM' with CFO on AWGN channels was introduced in [7]. Later, in [8], Santhanam and Tellambura presented an exact BER analysis of single user OFDM with CFO on AWGN channels. Further, making a Gaussian approximation of the inter-carrier interference (ICI), Rugini and Banelli extended the BER analysis of OFDM to frequency-selective Rayleigh and Rician fading with CFO in [9]. However, the analyses in [7]-[9] do not consider TOs. In [10], an approximate average signal-to-interference (SIR) analysis for OFDM with TO alone (assuming zero CFO) was presented. In [11], an approximate symbol error rate (SER) analysis of OFDM with both CFO and TO is presented. However, papers [7]-[11] do not consider uplink OFDMA (i.e., 'multiuser OFDM' on the uplink).

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In terms of performance analysis of uplink OFDMA, [12] and [6] derived analytical expressions for average SIR at the receiver; [12] derived SIR expressions considering only TO (assuming zero CFO), whereas [6] derived SIR expressions considering both CFOs as well as TOs. However, to our knowledge, analytical derivation of BER expressions for uplink OFDMA in the presence of both CFO as well as TO on Rician fading has not been reported. Our contribution in this paper aims to fill this gap. In particular, we derive an *analytical BER expression* that quantify the degradation in BER due to the *combined effect of both CFOs and TOs* in uplink OFDMA *on Rician fading channels*, which has not been reported so far. For non-zero CFOs/TOs, we obtain an approximate BER expression involving a single integral. Analytical and simulation BER results are shown to match very well.

II. Uplink OFDMA System Model

Consider an uplink OFDMA system with K users, where each user communicates with a base station (BS) through an independent multipath *Rician* fading channel. We assume that there are N subcarriers in each OFDM symbol and one subcarrier can be allocated to only one user. The information symbol for the uth user on the kth subcarrier is denoted by X_k^u, $k \in S_u$, where S_u is the set of subcarriers assigned to the uth user and $\mathbb{E}\Big[|X_k^u|^2\Big] = 1$, where $\mathbb{E}[.]$ denote the expectation operator. Then, $\bigcup_{u=1}^{K} S_u = \{0, 1, \ldots, N-1\}$ and $S_u \bigcap S_v = \phi$ for $u \neq v$. The length of the cyclic prefix (CP) added is N_g sampling periods, and is assumed to be equal to the maximum channel delay spread, $L-1$, normalized by the sampling period (i.e., $N_g \geq L-1$). After IDFT processing and CP insertion at the transmitter, the time-domain sequence of the uth user, x_n^u, is given by

$$x_n^u = \frac{1}{N} \sum_{k \in S_u} X_k^u e^{\frac{j2\pi nk}{N}}, \quad -N_g \leq n \leq N-1. \qquad (1)$$

The uth user's signal at the receiver input, after passing through the channel, in the case of perfect synchronization, is given by

$$s_n^u = x_n^u \star h_n^u, \qquad (2)$$

where $\star$ denotes linear convolution and h_n^u is the uth user's channel impulse response. It is assumed that h_n^u is non-zero only for $n = 0, \ldots, L-1$, and that all users' channels are statistically independent. We assume that h_n^u's are i.i.d. complex Gaussian with $h_{n,I}^u$ and $h_{n,Q}^u$ as their real and imaginary parts. The first path's channel coefficient, which contains the line-of-sight component, is of non-zero mean with $\mathbb{E}\Big[h^u_{0,I}\Big] = m_I$ and $\mathbb{E}\Big[h^u_{0,Q}\Big] = m_Q$, so that the average power of the specular component, denoted by $|s|^2$, is $m_I^2 + m_Q^2$, and

$$\text{var}(h^u_{n,I}) \;=\; \text{var}(h^u_{n,I}) \;=\; \sigma^2 \;=\; \frac{1-s^2}{2L}, \;\; \forall n. \qquad (3)$$

The uth user's channel coefficient on kth subcarrier in frequency domain, H^u_k, is given by

$$H^u_k \;=\; \sum_{n=0}^{L-1} h^u_n e^{\frac{-j2\pi nk}{N}}, \qquad (4)$$

with $\mathbb{E}\Big[H^u_k\Big] = m_I + \mathbf{j}m_Q$ where $\mathbf{j} = \sqrt{-1}$, and $\text{var}(H^u_k) = 1 - s^2$, such that $\mathbb{E}\Big[\,|H^u_k|^2\,\Big] = 1$ and the Rician factor, $K_r = \frac{s^2}{1-s^2}$. Note that the average powers of the specular and the random components are assumed to be the same for all the subcarriers. The DFT output at the receiver on kth carrier is

$$Y^u_k \;=\; H^u_k X^u_k \,+\, Z^u_k, \qquad (5)$$

where Z^u_k is the output noise of variance σ_n^2.

III. BER analysis with CFOs and TOs

Let ϵ_u, $u = 1, 2, \cdots, K$ denote uth user's residual CFO normalized by the subcarrier spacing, $|\epsilon_u| \leq 0.5, \forall\, u$, and let μ_u, $u = 1, 2, \cdots, K$ denote uth user's residual TO in number of sampling periods at the receiver. The DFT output on the kth carrier of the uth user at the receiver in the presence of CFOs and TOs can be written in the form

$$Y^u_k \;=\; \underbrace{H^u_{k,k}X^u_k + \sum_{\substack{q\in\mathcal{S}_u\\ q\neq k}} H^u_{k,q}X^u_q + \sum^{u}_{q\in\mathcal{S}_u} H^{u,I}_{k,q}X^{u,I}_q}_{\text{self interference (SI)}}$$

$$+ \underbrace{\sum_{\substack{v=1,\\ v\neq u}}^{K} \sum_{q\in\mathcal{S}_v} H^v_{k,q}X^v_q + H^{v,I}_{k,q}X^{v,I}_q}_{\text{MUI}} + Z^u_k, \qquad (6)$$

where X^u_q and $X^{u,I}_q$ are the symbols from the current and interfering frames, respectively, of uth user, and Z^u_k is the output noise of variance σ_n^2. If the TO is negative, the interfering frame will be the the previous frame; if the TO is positive, interfering frame will be the next frame. The coefficients $H_{k,q}$'s depend on the CFO and TO values. To write the expressions for these coefficients for $N_g = L - 1$, we need to consider four different cases of TOs, referred to as *Cases* $a)$ to $d)$ of TOs [6], where $0 < -\mu_u \leq N_g$ for *Case a)*, $-\mu_u > N_g$ for *Case b)*, $0 < \mu_u < L$ for *Case c)*, and $\mu_u \geq L$ for *Case d)*. Using l to denote the path index, and defining

$$\Gamma^{u,l}_{qk}(n_1, n_2) \;\;\triangleq\;\; \frac{1}{N}\sum_{n=n_1}^{n_2} e^{\frac{j2\pi n(q-k+\epsilon_u)}{N}}, \qquad (7)$$

the expressions for $H^u_{k,q}$ for different cases of TOs can be written as [6]

$$H^u_{k,q} \;=\; e^{\frac{i2\pi\mu_u q}{N}} \sum_{l=0}^{L-1} h^u_l e^{\frac{-i2\pi lq}{N}} \Gamma^{u,l}_{qk}(n_{\alpha_1}, n_{\alpha_2}), \qquad (8)$$

where $(n_{\alpha_1}, n_{\alpha_2})$ corresponding to different cases $a)$ to $d)$ are given by

$$(n_{a_1}, n_{a_2}) \;=\; \begin{cases} (0, N-1), & \text{for } l \leq N_g + \mu_u, \\ (l - \mu_u - N_g, N-1), & \text{for } l > N_g + \mu_u, \end{cases} \qquad (9)$$

$$(n_{b_1}, n_{b_2}) \;=\; (l - \mu_u - N_g, N-1), \;\; \forall l, \qquad (10)$$

$$(n_{c_1}, n_{c_2}) \;=\; \begin{cases} (0, N-1-\mu_u+l), & \text{for } 0 \leq l \leq \mu_u - 1, \\ (0, N-1), & \text{for } l \geq \mu. \end{cases}, \qquad (11)$$

and

$$(n_{d_1}, n_{d_2}) \;=\; (0, N-1+l-\mu_u), \;\; \forall l. \qquad (12)$$

It is noted that in *Cases a)* and *b)*, interference is only due to previous frame, and in *Cases c)* and *d)*, interference is only due to next frame. Based on this observation, the expressions for $H^{u,I}_{k,q}$'s for *Cases a)* and *b)* can be written as

$$H^{u,I}_{k,q} \;=\; e^{\frac{i2\pi(\mu_u+N_g)q}{N}} \sum_{l=N_g+\mu_u+1}^{L-1} h^u_l e^{\frac{-i2\pi lq}{N}} \Gamma^{u,l}_{qk}(0, n_{\alpha_1}-1), \qquad (13)$$

where n_{α_1} in (13) is n_{a_1} for *Case a)* and n_{b_1} for *Case b)*. Likewise, the expressions for $H^{u,I}_{k,q}$'s for *Cases c)* and *d)* can be written as

$$H^{u,I}_{k,q} \;=\; e^{\frac{-i2\pi(N_g-\mu_u)q}{N}} \sum_{l=0}^{\mu_u-1} h^u_l e^{\frac{-i2\pi lq}{N}} \Gamma^{u,l}_{qk}(n_{\alpha_2}+1, N-1), \qquad (14)$$

where n_{α_2} in (14) is n_{c_2} for *Case c)* and n_{d_2} for *Case d)*. We note that, due to the combined effect of CFOs and TOs,

1) the means of different coefficients are given by

$$s^u_{kq} = \mathbb{E}\Big[H^u_{k,q}\Big] \;=\; s\,\Gamma^{u,0}_{kq}(n_{\alpha_1}, n_{\alpha_2}),$$

$$s^{u,I}_{kq} = \mathbb{E}\Big[H^{u,I}_{k,q}\Big] \;=\; \begin{cases} \Gamma^{u,0}_{kq}(n_{\alpha_1}, N-1), & \text{for } \mu_u > 0 \\ \Gamma^{u,0}_{kq}(0, n_{\alpha_2}-1), & \text{for } \mu_u < 0, \end{cases} \qquad (15)$$

2) the coefficients of any given user u, (i.e., $H^u_{k,q}$'s) are correlated, whereas the coefficients of any two different users (i.e., $H^u_{k,q}$'s and $H^v_{k,q}$'s, $u \neq v$) are uncorrelated.

So, computation of the exact BER would involve M-fold integral in the case of the system with only CFOs and $2M$-fold integral for the system with both CFOs and TOs (where M is the number of subcarriers allotted to each user). To reduce this computational complexity, we proceed to obtain an analytical expression for the BER using the following 3 steps:

- $i)$ since $H^u_{k,q}$'s are correlated, we obtain an estimate of each $H^u_{k,q}$ and $H^{u,I}_{k,q}$, in terms of $H^u_{k,k}$,
- $ii)$ obtain expressions for the variances of SI/MUI and the SINR, conditioned on $|H^u_{k,k}|$,
- $iii)$ obtain expression for the BER conditioned on $|H^u_{k,k}|$ by assuming the estimation errors in $H^u_{k,q}$'s and $H^{u,I}_{k,q}$'s to be Gaussian, and uncondition it to obtain the unconditional BER.

Step i): To obtain an estimate of $H^u_{k,q}$ in terms of $H^u_{k,k}$, we use the fact that, if two non-zero mean complex Gaussian random variables X and Y having the means m_x and m_y, respectively, are correlated, an estimate of one variable (say, Y) can be obtained, in terms of the other variable, as

$$\widehat{Y} = \frac{C_{X,Y}}{\sigma_X^2}X + \left(m_y - \frac{C_{X,Y}}{\sigma_X^2}m_x\right), \quad (16)$$

with an estimation error, $\mathcal{E}_Y = Y - \widehat{Y}$, of variance $\sigma_Y^2 - \frac{C_{X,Y}^2}{\sigma_X^2}$, where $C_{X,Y}$ is the covariance of X and Y, and σ_X^2 and σ_Y^2 are the variances of X and Y, respectively. Using this, we can write all $H_{k,q}$'s in (6) in terms of $H_{k,k}$, to get

$$\begin{aligned} Y_k^u &= H_{k,k}^u X_k^u + \sum_{\substack{q\in\mathcal{S}_u\\ q\neq k}} \left(\frac{C_{k,q}^u}{(\sigma_k^u)^2}H_{k,k}^u + (s_{kq}^u - \frac{C_{k,q}^u}{(\sigma_k^u)^2}s_{kk}^u) + \mathcal{E}_q^u\right)X_q^u \\ &+ \sum_{q\in\mathcal{S}_u}\left(\frac{C_{k,q}^{u,I}}{(\sigma_k^u)^2}H_{k,k}^u + (s_{kq}^{u,I} - \frac{C_{k,q}^{u,I}}{(\sigma_k^u)^2}s_{kk}^u) + \mathcal{E}_q^{u,I}\right)X_q^{u,I} \\ &+ \sum_{\substack{v=1,\\ v\neq u}}^{K}\sum_{q\in\mathcal{S}_v} H_{k,q}^v X_q^v + H_{k,q}^{v,I}X_q^{v,I} + Z_k^u, \quad (17)\end{aligned}$$

where

$$\begin{aligned} C_{k,q}^u &= \mathbb{E}[(H_{k,k}^u - s_{kk}^u)(H_{k,q}^u - s_{kq}^u)^*] \\ C_{k,q}^{u,I} &= \mathbb{E}[(H_{k,k}^u - s_{kk}^u)(H_{k,q}^{u,I} - s_{kq}^{u,I})^*] \\ (\sigma_k^u)^2 &= \mathbb{E}[(H_{k,k}^u - s_{kk}^u)(H_{k,k}^u - s_{kk}^u)^*], \quad (18)\end{aligned}$$

where $(.)^*$ denotes the conjugate operation.

Step ii): Now, in (17), the total variance of all the terms which are interference to the uth user's symbol on kth subcarrier, conditioned on $H_{k,k}^u$, is obtained as

$$\begin{aligned} \sigma_{I|H_{k,k}}^2 &= \underbrace{|H_{k,k}^u|^2\left(\sum_{\substack{q\in\mathcal{S}_u\\ q\neq k}}\frac{|C_{k,q}^u|^2}{(\sigma_k^u)^4} + \sum_{q\in\mathcal{S}_u}\frac{|C_{k,q}^{u,I}|^2}{(\sigma_k^u)^4}\right)}_{\triangleq\,\mathcal{A}} \\ &+ \underbrace{\left(\sum_{\substack{q\in\mathcal{S}_u\\ q\neq k}}(\sigma_q^u)^2 - \frac{|C_{k,q}^u|^2}{(\sigma_k^u)^2} + \sum_{q\in\mathcal{S}_u}(\sigma_q^{u,I})^2 - \frac{|C_{k,q}^{u,I}|^2}{(\sigma_k^u)^2}\right)}_{\triangleq\,\mathcal{B}_1} \\ &+ \underbrace{\left(\sum_{\substack{q\in\mathcal{S}_u\\ q\neq k}}\left|s_{kq}^u - \frac{C_{k,q}^u}{(\sigma_k^u)^2}s_{kk}^u\right|^2 + \sum_{q\in\mathcal{S}_u}\left|s_{kq}^{u,I} - \frac{C_{k,q}^{u,I}}{(\sigma_k^u)^2}s_{kk}^u\right|^2\right)}_{\triangleq\,\mathcal{B}_2} \\ &+ \underbrace{\sum_{\substack{v=1,\\ v\neq u}}^{K}\sum_{q\in\mathcal{S}_v}(\sigma_q^v)^2 + (\sigma_{q,I}^v)^2}_{\triangleq\,\mathcal{B}_3} + \underbrace{\sum_{\substack{v=1,\\ v\neq u}}^{K}\sum_{q\in\mathcal{S}_v}|s_{kq}^v|^2 + |s_{kq}^{v,I}|^2}_{\triangleq\,\mathcal{B}_4}, \quad (19)\end{aligned}$$

where

$$\begin{aligned} (\sigma_q^u)^2 &= \mathbb{E}[(H_{k,q}^u - s_{kq}^u)(H_{k,q}^u - s_{kq}^u)^*] \\ (\sigma_{\mathcal{E}_q^u})^2 &= (\sigma_q^u)^2 - \frac{|C_{k,q}^u|^2}{(\sigma_k^u)^2}, \\ (\sigma_{q,I}^u)^2 &= \mathbb{E}[(H_{k,q}^{u,I} - s_{kq}^{u,I})(H_{k,q}^u - s_{kq}^{u,I})^*]. \quad (20)\end{aligned}$$

Assuming that among K users in the system, K_λ users belong to *Case* λ), $\lambda \in \{a,b,c,d\}$, the expressions for terms $\mathcal{A}$ and $\mathcal{B}_1$ in (19) for different TO cases are given in Table-I, where

$$(\sigma_k^u)^2 = \frac{1-s^2}{L}\sum_{l=0}^{L}|\Gamma_{kk}^{u,l}(n_{\lambda_1}, n_{\lambda_2})|^2, \quad (21)$$

$$(\sigma_q^u)^2 = \frac{1-s^2}{L}\sum_{l=0}^{L}|\Gamma_{qk}^{u,l}(n_{\lambda_1}, n_{\lambda_2})|^2, \quad (22)$$

$$(\sigma_{q,I}^u)^2 = \begin{cases} \frac{1-s^2}{L}\sum_{l=0}^{L}|\Gamma_{qk}^{u,l}(0, n_{\lambda_1}-1)|^2, & \text{for } \lambda = a,b \\ \frac{1-s^2}{L}\sum_{l=0}^{L}|\Gamma_{qk}^{u,l}(n_{\lambda_2}+1, N-1)|^2, & \text{for } \lambda = c,d. \end{cases} \quad (23)$$

Similarly, the expressions for the term $\mathcal{B}_2$ in (19) for the different TO cases are given in Table-II.

Now, defining $\mathcal{B} = \mathcal{B}_1 + \mathcal{B}_2 + \mathcal{B}_3 + \mathcal{B}_4 + \sigma_n^2$, the SINR at the kth subcarrier of the uth user, conditioned on $H_{k,k}^u$, denoted by $\gamma_{H_{k,k}^u}$, is given by

$$\gamma_{H_{k,k}^u} = \frac{|H_{k,k}^u|^2}{\mathcal{A}|H_{k,k}^u|^2 + \mathcal{B}}. \quad (24)$$

Here, $\mathbb{E}[H_{k,k}^u] = (m_I + \mathbf{j}m_Q)\Gamma_{qk}^{u,0}(n_{\lambda_1}, n_{\lambda_2})$ and the variance of real and imaginary parts of $H_{k,k}^u$, denoted by σ_e^2, is $\frac{1}{2}(\sigma_k^u)^2$. The effective Rician factor, $K_r^{'}$, is $\frac{s^2|\Gamma_{qk}^{u,0}(n_{\lambda_1},n_{\lambda_2})|^2}{\sigma_e^2}$.

Step iii): Now, assuming $\mathcal{E}_q^u$ and $\mathcal{E}_q^{u,I}$ to be Gaussian, the conditional BER, denoted by $P_e(\gamma_{H_{k,k}^u})$, can be written as

$$P_e(\gamma_{H_{k,k}^u}) = \frac{1}{\sqrt{2\pi}}\int_{\sqrt{p\gamma_{H_{k,k}^u}}}^{\infty} e^{\frac{-y^2}{2}}dy, \quad (25)$$

where $p = 1$ for QPSK and $p = 2$ for BPSK. Unconditioning over the Rician pdf of $R = |H_{k,k}^u|$, we get the unconditional BER expression as

$$P_e = \frac{1}{\sqrt{2\pi}}\int_0^\infty\int_{\sqrt{p\gamma_{H_{k,k}^u}}}^{\infty} e^{\frac{-y^2}{2}}\,dy\, f_R(r)\,dr, \quad (26)$$

where

$$f_R(r) = e^{-K_r}\frac{r}{\sigma^2}e^{\frac{-r^2}{2\sigma^2}}I_0\left(\frac{rs}{\sigma^2}\right), \quad r \geq 0, \quad (27)$$

and

$$I_0\left(\frac{rs}{\sigma^2}\right) = \sum_{c=0}^{\infty}\frac{r^{2c}s^{2c}}{(2\sigma^2)^{2c}(c!)^2}. \quad (28)$$

Equation (26) can be further simplified as [14]

$$P_e = 1 - e^{-K_r^{'}}\sum_{c=0}^{\infty}\frac{(s_{kk}^u)^{2c}}{(2\sigma_e^2)^c c!}\sum_{l=0}^{c}\frac{1}{l!}\left(\frac{\mathcal{B}}{2\sigma_e^2}\right)^l I_l, \quad (29)$$

where

$$I_l = \frac{1}{\sqrt{2\pi}}\int_0^\infty\frac{y^{2l}}{(p-\mathcal{A}y^2)^l}e^{-y^2\left(\frac{1}{2}+\frac{\mathcal{B}}{2\sigma_e^2(p-\mathcal{A}y^2)}\right)}dy. \quad (30)$$

where $p = 1$ for QPSK and $p = 2$ for BPSK. The integral in (30) can be evaluated using Simpson's rule. We see that the integral in (29) has infinite discontinuity at $y = \sqrt{\frac{p}{\mathcal{A}}}$, so it is enough to evaluate this integral from zero to $\lfloor\sqrt{\frac{p}{\mathcal{A}}}\rfloor$.

We can consider the following two special cases.

1) Zero CFO and TO for All Users: For the special case of perfect synchronization (i.e., zero CFO and TO for all users), the BER expression can be obtained as

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Case λ	$\mathcal{A}$
$\lambda=a,b$	$\frac{(1-s^2)^2}{L^2(\sigma_k^u)^4}\Big[\sum_{q\in S_u, q\neq k}\big\lvert\sum_{l=0}^{L-1}e^{\frac{i2\pi(k-q)(\mu_u-l)}{N}}\Gamma_{kk}^{u,l}(n_{\lambda_1},n_{\lambda_2})\Gamma_{qk}^{u,l^*}(n_{\lambda_1},n_{\lambda_2})\big\rvert^2$ $+\big\lvert e^{\frac{-i2\pi N_g q}{N}}\sum_{q\in S_u}\sum_{l=0}^{L-1}e^{\frac{i2\pi(k-q)(\mu_u-l)}{N}}\Gamma_{kk}^{u,l}(n_{\lambda_1},n_{\lambda_2})\Gamma_{qk}^{u,l^*}(0,n_{\lambda_1}-1)\big\rvert^2\Big]$
$\lambda=c,d$	$\frac{(1-s^2)^2}{L^2(\sigma_k^u)^4}\Big[\sum_{q\in S_u, q\neq k}\big\lvert\sum_{l=0}^{L-1}e^{\frac{i2\pi(k-q)(\mu_u-l)}{N}}\Gamma_{kk}^{u,l}(n_{\lambda_1},n_{\lambda_2})\Gamma_{qk}^{u,l^*}(n_{\lambda_1},n_{\lambda_2})\big\rvert^2$ $+\big\lvert e^{\frac{i2\pi N_g q}{N}}\sum_{q\in S_u}\sum_{l=0}^{L-1}e^{\frac{i2\pi(k-q)(\mu_u-l)}{N}}\Gamma_{kk}^{u,l}(n_{\lambda_1},n_{\lambda_2})\Gamma_{qk}^{u,l^*}(n_{\lambda_2}+1,N-1)\big\rvert^2\Big]$
Case λ	$\mathcal{B}_1$
$\lambda=a,b$	$\sum_{q\in S_u, q\neq k}(\sigma_q^u)^2-\frac{1-s^2}{(\sigma_k^u)^2}\big\lvert\sum_{l=0}^{L-1}e^{\frac{i2\pi(k-q)(\mu_u-l)}{N}}\Gamma_{kk}^{u,l}(n_{\lambda_1},n_{\lambda_2})\Gamma_{qk}^{u,l^*}(n_{\lambda_1},n_{\lambda_2})\big\rvert^2$ $+\sum_{q\in S_u}(\sigma_{q,I}^u)^2-\frac{1-s^2}{(\sigma_k^u)^2}\big\lvert e^{\frac{-i2\pi N_g q}{N}}\sum_{q\in S_u}\sum_{l=0}^{L-1}e^{\frac{i2\pi(k-q)(\mu_u-l)}{N}}\Gamma_{kk}^{u,l}(n_{\lambda_1},n_{\lambda_2})\Gamma_{qk}^{u,l^*}(0,n_{\lambda_1}-1)\big\rvert^2$
$\lambda=c,d$	$\sum_{q\in S_u, q\neq k}(\sigma_q^u)^2-\frac{1-s^2}{(\sigma_k^u)^2}\big\lvert\sum_{l=0}^{L-1}e^{\frac{i2\pi(k-q)(\mu_u-l)}{N}}\Gamma_{kk}^{u,l}(n_{\lambda_1},n_{\lambda_2})\Gamma_{qk}^{u,l^*}(n_{\lambda_1},n_{\lambda_2})\big\rvert^2$ $+\sum_{q\in S_u}(\sigma_{q,I}^u)^2-\frac{1-s^2}{(\sigma_k^u)^2}\big\lvert e^{\frac{i2\pi N_g q}{N}}\sum_{q\in S_u}\sum_{l=0}^{L-1}e^{\frac{i2\pi(k-q)(\mu_u-l)}{N}}\Gamma_{kk}^{u,l}(n_{\lambda_1},n_{\lambda_2})\Gamma_{qk}^{u,l^*}(n_{\lambda_2}+1,N-1)\big\rvert^2$

TABLE I

EXPRESSIONS FOR $\mathcal{A}$ AND $\mathcal{B}_1$ FOR DIFFERENT TIME OFFSET CASES *a*) TO *d*).

Case λ	$\mathcal{B}_2$
$\lambda=a$	$\frac{1-s^2}{L}\sum_{v=1, v\neq u}^{K_a}\Big[\sum_{q\in S_v, v\neq u}\sum_{l=0}^{L-1}\big\lvert\Gamma_{qk}^{v,l}(n_{a_1},n_{a_2})\big\rvert^2$ $+\sum_{q\in S_v, v\neq u}\sum_{l=N_g+\mu_v+1}^{L-1}\big\lvert\Gamma_{qk}^{v,l}(0,n_{a_1}-1)\big\rvert^2\Big]$
$\lambda=b$	$\frac{1-s^2}{L}\sum_{v=1, v\neq u}^{K_b}\Big[\sum_{q\in S_v, v\neq u}\sum_{l=0}^{L-1}\big\lvert\Gamma_{qk}^{v,l}(n_{b_1},n_{b_2})\big\rvert^2$ $+\sum_{q\in S_v, v\neq u}\sum_{l=0}^{L-1}\big\lvert\Gamma_{qk}^{v,l}(0,n_{b_1}-1)\big\rvert^2\Big]$
$\lambda=c$	$\frac{1-s^2}{L}\sum_{v=1, v\neq u}^{K_c}\Big[\sum_{q\in S_v, v\neq u}\sum_{l=0}^{L-1}\big\lvert\Gamma_{qk}^{v,l}(n_{c_1},n_{c_2})\big\rvert^2$ $+\sum_{q\in S_v, v\neq u}\sum_{l=0}^{\mu_v-1}\big\lvert\Gamma_{qk}^{v,l}(n_{c_2}+1,N-1)\big\rvert^2\Big]$
$\lambda=d$	$\frac{1-s^2}{L}\sum_{v=1, v\neq u}^{K_d}\Big[\sum_{q\in S_v, v\neq u}\sum_{l=0}^{L-1}\big\lvert\Gamma_{qk}^{v,l}(n_{d_1},n_{d_2})\big\rvert^2$ $+\sum_{q\in S_v, v\neq u}\sum_{l=0}^{L-1}\big\lvert\Gamma_{qk}^{v,l}(n_{d_2}+1,N-1)\big\rvert^2\Big]$

TABLE II

EXPRESSIONS FOR $\mathcal{B}_2$ FOR DIFFERENT TIME OFFSET CASES *a*) TO *d*).

$$P_e = \frac{1}{2}\left[1-e^{-K_r}\sum_{c=0}^{\infty}\frac{s^{2c}}{(2\sigma^2)^c c!}\sqrt{\frac{2p\sigma^2}{\sigma_n^2+p\sigma^2}}\left(\sum_{l=0}^{c}\frac{(2l)!}{(l!)^2}\left(\frac{\sigma_n^2}{\sigma_n^2+p\sigma^2}\right)^l\left(\frac{1}{2}\right)^{(2l+0.5)}\right)\right]. \quad (31)$$

Though (31) contains an infinite sum, only first few terms are significant, as $c!$ increases rapidly with increase in c. Also, when s is very small and tends to zero, K_r also tends to zero, and σ^2 tends to $\frac{1}{2}$ making

$$P_e = \frac{1}{2}\left[(1-\sqrt{\frac{p}{\sigma_n^2+\frac{p}{2}}}\left(\frac{1}{2}\right)^{0.5}\right], \quad (32)$$

which on simplification, and defining SNR as $\frac{1}{\sigma_n^2}$, gives $P_e = \frac{1}{2}\left[1-\sqrt{\frac{\frac{p}{2}SNR}{1+\frac{p}{2}SNR}}\right]$, which is the well known BER expression in Rayleigh fading for BPSK ($p=2$) and QPSK ($p=1$).

2) Zero CFO and TO for Desired User Alone: If the desired user CFO and TO are zero, and the other users' CFOs and TOs are non-zero, then there will not be any SI (so, no Gaussian approximation is needed) and only MUI occurs. For this case, $\mathcal{A}=0$, $\mathcal{B}=\mathcal{B}_3+\mathcal{B}_4+\sigma_n^2$, and $(\sigma_k^u)^2=2\sigma_e^2=1$ and $s_{kk}^u=s$, leading to the simplification of (30) as $I_l = \frac{1}{\sqrt{2\pi}}\int_0^{\infty}\frac{y^{2l}}{p^l}e^{-y^2(\frac{1}{2}+\frac{\mathcal{B}}{2\sigma_e^2 p})}dy$. The BER in this case is given by (31), with σ_n^2 replaced by $\mathcal{B}_3+\mathcal{B}_4+\sigma_n^2$, which is an exact closed-form BER expression.

IV. NUMERICAL RESULTS

In this section, we present the analytical and simulation results on the BER performance of uplink OFDMA without and with CFOs and TOs. We consider a system with $N=64$, $K=4$, number of subcarriers allocated to each user, $M=16$, $L=2$, $N_g=1$, and interleaved allocation of subcarriers. BPSK and QPSK modulations are considered. We take the first user as the desired user and plot its BER performance.

Figure 1 shows the BER performance of uplink OFDMA, obtained using both analysis as well as simulation, as a function of SNR for BPSK modulation with no CFOs and TOs. For this parameter setting, where $\epsilon_u=\mu_u=0$ for all the users, the system is not affected by interference, and the analysis becomes exact with (31) giving the exact BER. This can be verified by the very close match between the analysis and simulation plots of the BER in Fig. 1 for various values of the Rician factor, K_r. In this figure, we can also see that, as the Rician factor is increased from zero towards infinity, the BER of the system improves from Rayleigh fading BER towards AWGN BER.

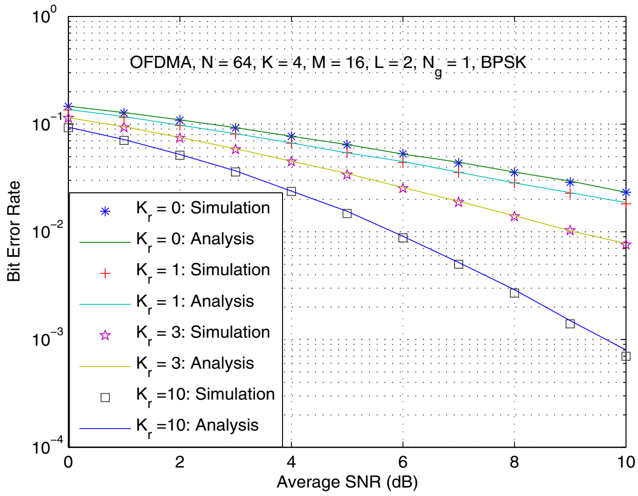

Fig. 1. BER performance of uplink OFDMA with no CFOs and TOs for different values of Rice factors, K_r. $N = 64$, $K = 4$, $M = 16$, $L = 2$, $N_g = 1$, BPSK.

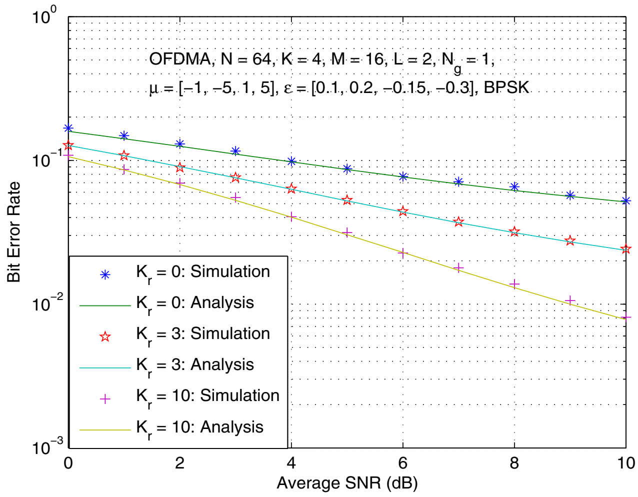

Fig. 2. BER performance of uplink OFDMA in the presence of CFOs and TOs for different values of Rice factors, K_r. $N = 64$, $K = 4$, $L = 2$, $N_g = 1$, BPSK. 1st user is desired user; all users' CFOs and TOs are non-zero: $\epsilon = [0.1, 0.2, -0.15, -0.3]$, $\mu = [-1, -5, 1, 5]$.

Figure 2 shows the analytical and simulated BER performance of uplink OFDMA with CFOs and TOs for BPSK modulation, where the CFO and TO values are taken to be $\mu = [-1, -5, 1, 5]$ and $\epsilon = [0.1, 0.2, -0.15, -0.3]$. Comparing the BER performance in Figs. 1 and 2, we see that the BER degrades due to CFOs and TOs, which is expected. Also, for a given set of values of CFOs and TOs, BER improves with increasing value of K_r, which is also expected. Also, it can be observed that, even with the approximation made to handle the correlation in the channel coefficients of subcarriers of the same user, there is an almost exact match between the analytical and simulated BER. A similar set of observations can be made for QPSK with CFOs and TOs in Fig. 3.

V. Conclusions

We presented a BER analysis of uplink OFDMA in the presence of both CFOs as well as TOs on Rician fading channels, which has not been reported before. For the cases when i) all the users are perfectly synchronized (i.e., no CFO and TO), and ii) only the desired user is perfectly aligned in frequency/time (i.e., zero CFO/TO for desired user) while the other users have non-zero CFOs and TOs, we obtained an exact closed-form expressions for the BER. For the case when all the users (including the desired user) have non-zero CFOs and TOs, we obtained an approximate expression for the BER, which involved the computation of a single integral. Analytical and simulated BER results matched very well.

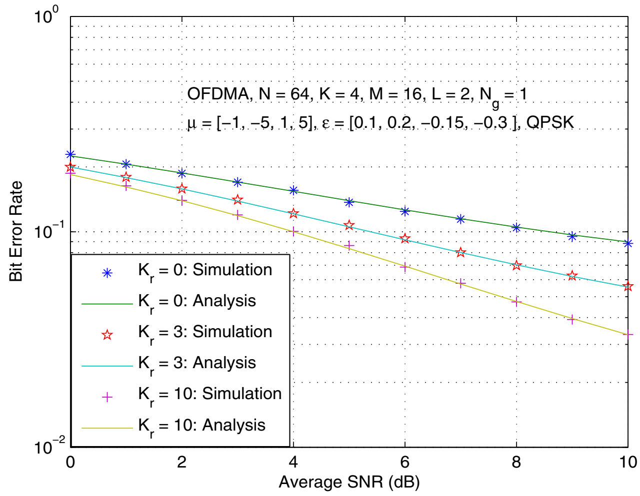

Fig. 3. BER performance of uplink OFDMA in the presence of CFOs and TOs for different values of Rice factors, K_r. $N = 64$, $K = 4$, $L = 2$, $N_g = 1$, 1st user is desired user; all users' CFOs and TOs are non-zero: $\epsilon = [0.1, 0.2, -0.15, -0.3]$, $\mu = [-1, -5, 1, 5]$, QPSK.

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